

Mortgage Refinancing for Distracted Consumers

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ABSTRACT. We solve the optimal refinancing problem of a distracted consumer, who only sporadically has the time to attend to his financial business. The optimal rule engenders both *late* and *early* mortgage refinancing. Distraction produces late refinancing, since interest rates can move a long way before the distracted consumer has an opportunity to refinance. Distraction produces early refinancing, since distracted consumers optimally downweight option value considerations that would lead normal consumers to wait to refinance. Our model predicts that late refinancing should have dominated in the 1980's and that early refinancing should have become common in the 1990's. Using a unique panel data set from a large financial institution we show that early refinancing accounted for around 1/3 of all refinancing in the 1990's. The model also explains why so many financial advisors recommend following an NPV rule for refinancing decisions. Such a rule is disastrous for a household who continuously monitors interest rates. But the NPV rule is approximately optimal for a distracted consumer who only reconsiders her refinancing decision from time to time.

JEL classification: G11, G21 .

Keywords: Mortgage, refinance, option value, infrequent adjustment.

1. INTRODUCTION

Households are faced daily with many complicated, time-consuming choices. Given a binding time-budget constraint, distracted consumers may only be able to make certain decisions at stochastic intervals; i.e., put less formally, when they have a spare moment. The idea that people may only make decisions infrequently has long been used by economists to explain apparent deviations from optimal behavior. Calvo (1983) modeled monopolistically-competitive firms as setting prices at some constant hazard rate, generating price stickiness; a version of this model is now the basis for the commonly used New Keynesian aggregate supply curve. Gabaix and Laibson (2001) show that assuming agents can only adjust infrequently helps explain the equity premium puzzle. Mankiw and Reis (2002) alter that model by assuming that price-setters can change prices continuously, but are only able to gather information at random intervals, generating persistence in inflation; Ball, Mankiw and Reis (2003) use this framework to study monetary policy.

In this paper, we introduce infrequent adjustment into a model of another important financial choice: the decision to refinance one's mortgage. Households in the US hold over \$10 trillion in real estate assets.¹ For two-thirds of households, real estate accounts for the majority of their assets. Almost all home buyers obtain mortgages and the total value of outstanding home mortgages approaches \$5 trillion, near the value of US government debt. Hence decisions about mortgage refinancing both have large macroeconomic effects and are among the largest households make.

Borrowers refinance their mortgages for several reasons. First, some households refinance to either pay down their mortgage or to increase their mortgage. The latter group are typically motivated by a desire to increase current consumption. Second, households refinance to take advantage of lower interest rates and thereby save on

¹Flow of Funds Accounts of the United States, Board of Governors of the Federal Reserve System, September 15, 2000.

future interest payments.

In modeling refinancing for the second reason, it has long been known that the borrower's refinancing decision reduces to the choice of an optimal interest rate differential. We first replicate this finding of Dunn and McConnell (1981a, 1981b), Dunn and Spatt (1986) and Chen and Ling(1989) in a simple, analytically tractable model. At this optimal differential, the value of the interest saved equals the sum of refinancing costs and the difference between the old 'in the money' refinancing option that is implicitly given up at the refinancing and the new 'out of the money' refinancing option that is implicitly acquired. Solving for these option values yields an equation for the optimal interest differential. The solution depends on the discount factor, refinancing costs, mortgage size, the standard deviation of the innovation in the mortgage interest rate, and an exogenous probability of moving. For conservative choices of these parameter values, the optimal refinancing differentials range between 100 and 200 basis points.

It has also long been recognized that the actual behavior of mortgagees differs in significant ways from the predictions of this model. In the 1980s, when mortgage rates dropped by large amounts, academic researchers and issuers of mortgage-backed securities noted that many individuals failed to refinance despite holding significantly 'in the money' options (1988 Journal of Real Estate Literature Special Issue, Giliberto and Thibodeau, 1989). Stanton (1995), Deng and Quigley (2000) and Deng, Quigley and Van Order (2000) try to explain the behavior of these 'woodheads' through transactions costs.

Using a unique panel data set from a large financial institution, we find that this late refinancing behavior persists today. But we also document a new empirical puzzle: a substantial number of people (on the order of one-third of refinancers) refinance *too early*, when the option is not yet in the money. Many do so at differentials smaller than 25 basis points, despite paying substantial refinancing costs. The

relative frequency of this early refinancing rose over the 1990s.

Our paper builds on the option pricing literature by showing that adding a single mechanism — distraction — simultaneously predicts the following set of phenomena.

1. **Late Refinancing:** Given a hazard rate of finding the time to refinance, some refinancers will not do so even if rates have fallen substantially.
2. **Early Refinancing:** Distracted consumers should refinance when an “opportunity” presents itself, even if it’s a bit “too early.” Waiting for the next (distant) opportunity is too costly.
3. **Change in Frequency of Late and Early Refinancing:** Distracted consumers will turn out to be late refinancers in an environment in which interest rates rapidly fall (1980s), but early refinancers in an environment in which interest rates fluctuate in a narrow band (1990s).
4. **Net Present Value Rule Advice:** We document that almost all members of a random sample of financial advisers advocate refinancing at the point where the present value of the interest savings is equal to the transactions cost — implicitly ignoring option value. Given infrequent adjustment, however, it is optimal for borrowers to refinance at a smaller differential than they otherwise would — making a net present value rule a near-optimal rule of thumb.

The paper proceeds as follows: Section 2 reviews the literature on refinancing; Sections 3 through 5 derive a simple, analytically tractable model of refinancing under the assumption that consumers only have an opportunity to refinance at some hazard rate. We show that the standard option-pricing model of refinancing, with continuous opportunities to refinance, is a special case of this model; Section 6 shows that we can calibrate our model to match the pattern of refinancing observed in the

data; Section 7 documents the four empirical puzzles described above, and shows the model can match them; and Section 8 concludes.

2. LITERATURE REVIEW

Refinancing has long been of interest to both practitioners and researchers interested in the valuation of mortgage-backed securities and researchers interested in consumer choice. Dickinson and Heuson (1994) and Kau and Keenan (1995) provide extensive surveys.

The theoretical work on borrowers' optimal choice to refinance starts with Dunn and McConnell (1981a, 1981b), who develop a continuous-time option-value model of prepayment, later extended by Dunn and Spatt (1986), Chen and Ling (1989) and Follain, Scott and Yang (1992). Henderson and Van Order (1987) endogenize the decision to default. These papers implicitly solve for the optimal refinancing differentials as solutions to partial differential equations, which are evaluated numerically. The optimal differentials they find typically lie between 100 and 250 basis points for a range of parameter values; Follain, Scott and Yang (1992) characterize the differentials they derive as implying "that the commonly used 'rule of thumb'- refinance if the interest rate declines by 200 basis points- is a fair approximation."

It soon became apparent that borrower behavior deviated in significant ways from the predictions of these models. Participants in the mortgage-backed securities industry had long noticed that some consumers did not refinance even after very large drops in mortgage rates. The failure of this group to exercise in-the-money options led them to be labeled "woodheads." Prepayment generally seemed to be characterized by "burnout," in which, after a rate drop, the fraction of refinancers declined over time; those who refinanced at or close to the optimal differential were labeled "ruthless." Some borrowers exhibited the opposite problem: they refinanced even when rates had risen. These discrepancies were picked up in estimates of the hazard

rates of default (Green and Shoven, 1986; 1988 Special Issue of the Journal of Real Estate Literature; Schwartz and Torous 1989,1992, 1993; Giliberto and Thibodeau, 1989; Richard and Roll, 1989).

To resolve these puzzle, authors added additional complexity to the option-pricing model. Archer and Ling (1993) add heterogeneity in transactions costs. Stanton (1995) adds both heterogeneity in transactions costs and an exogenous probability of prepayment. Downing, Stanton and Wallace (2001) allow variations in housing price to affect prepayment. In hazard-rate estimates of prepayment, Archer, Ling and McGill(1996), Hakim (1997), LaCour-Little (1999) and Bennett, Peach and Peristiani (2000, 2001) find that refinancing depends on borrower-specific characteristics.

More recently, Hurst (1999) and Hurst and Stafford (2002) have argued that refinancing at higher rates is a consequence of consumption-smoothing: borrowers are willing to refinance at higher rates because by “cashing out” some of their housing equity, they are able to cover other expenses.

3. MODEL

We present a continuous-time model of the mortgage refinancing decision. The current section introduces notation, the next section summarizes the key results, and Section 5 provides the formal arguments.

The real interest rate, r , and inflation rate, π , are Ito Processes:

$$dr = \sigma_r dz_r \tag{1}$$

$$d\pi = \sigma_\pi dz_\pi, \tag{2}$$

where dz represents Brownian increments, and $cov(dz_r, dz_\pi) = \sigma_{r\pi} dt$.

A mortgage has a nominal interest rate $i = r + \pi$, which is fixed at the time the mortgage is issued. Our analysis hinges on the gap between the current nominal

interest rate, $i' = r' + \pi'$, and the mortgage interest rate, $i = r + \pi$. Let x represent the difference between the current nominal interest rate and mortgage interest: $x \equiv i' - i$.

This implies that

$$dx = \sqrt{\sigma_r^2 + \sigma_\pi^2 + 2\sigma_{r\pi}} dz \quad (3)$$

$$= \sigma dz, \quad (4)$$

where $\sigma \equiv \sqrt{\sigma_r^2 + \sigma_\pi^2 + 2\sigma_{r\pi}}$.

For analytic tractability (i.e., to eliminate a state variable), we assume that mortgage payments are structured so that the real value of the mortgage remains constant, implying that

$$\text{mortgage payment} = (r + \pi - \pi')M = (i - \pi')M. \quad (5)$$

Mortgage holder can refinance their mortgage at real cost $C(M)$. Refinancing costs are not tax deductible, so the before-tax cost is the actual cost multiplied by $\frac{1}{1-\tau}$, where τ is the marginal tax rate of the household holding the mortgage. We assume that refinancing opportunities only arise from time-to-time. With hazard rate ξ a distracted mortgage holder has an opportunity to refinance (e.g., an undistracted free weekend).

Mortgage contracts terminate for exogenous reasons, including fixed terms (e.g., 30-year mortgages) or exogenous separation from a residence (e.g., death or a move). We capture *all* of these effects with a single stationary mechanism. We assume that the mortgage is repaid with hazard rate λ .

We assume that mortgage holders pick the policy that minimizes the NPV of their real interest payments, applying a personal discount rate, ρ . We also assume that mortgage holders are risk neutral.

Summing up these considerations, the consumer *minimizes* the expected value of her after-tax, real mortgage payments. Let value function $V(r, r', \pi, \pi')$, represent this value function.

4. OUR MAIN RESULT.

The following theorem characterizes the equilibrium threshold rule and the equilibrium value functions. The threshold rule is written with respect to the variable x , which represents the after-tax difference between the current nominal interest rate, i' , and the nominal interest rate of the mortgage, i .

Theorem 1. *Refinance when*

$$x \equiv i' - i < x^*,$$

and a refinancing opportunity arises, where x^* solves

$$[\psi(\rho + \lambda + \xi) - \phi(\rho + \lambda)] \left[x^* + (\rho + \lambda) \frac{C(M)}{M} \right] = \xi (1 - e^{-\psi x^*}), \quad (6)$$

with

$$\psi = \frac{-\sqrt{2(\rho + \lambda)}}{\sigma} \quad (7)$$

$$\phi = \frac{\sqrt{2(\rho + \lambda + \xi)}}{\sigma}. \quad (8)$$

When $x > x^*$ the value function is

$$V^R(r, r', \pi, \pi') = Re^{\psi x} + \frac{(i - \pi' + \lambda) M}{\rho + \lambda}. \quad (9)$$

When $x \leq x^*$ the value function is

$$V^L(r, r', \pi, \pi') = Le^{\phi x} + \frac{(i - \pi' + \lambda) M}{\rho + \lambda}. \quad (10)$$

The constants are given by

$$R = (e^{\psi x^*} - 1)^{-1} \left(\frac{x^* M}{\rho + \lambda} + C(M) \right) \quad (11)$$

$$L = \frac{1}{\phi} \left[\psi R - \frac{\xi M e^{-\psi x^*}}{(\rho + \lambda + \xi)(\rho + \lambda)} \right]. \quad (12)$$

Finally, the option value of being able to refinance is $Re^{\psi x}$ when $x > x^*$ and $Le^{\phi x}$ when $x \leq x^*$.

This theorem implies that x^* is the refinancing threshold for a household with a zero marginal tax rate.

Our analysis refers to some additional threshold values. We begin by defining the threshold value at which the gains from refinancing are exactly offset by the costs of refinancing assuming that the household will never refinance again. We refer to this as the NPV threshold.

Definition 2. *The NPV threshold, x^{NPV} , is defined as*

$$-\frac{x^{NPV} M}{\rho + \lambda} = C(M). \quad (13)$$

Intuitively, the NPV threshold is the point at which the expected after-tax interest payments saved from refinancing, $-\frac{xM}{\rho+\lambda}$, exactly offset the cost of refinancing, $C(M)$.

As one would expect, as the hazard rate of mortgage refinancing opportunities goes to zero, the optimal threshold, x^* , converges to the NPV threshold.

Corollary 3. *As $\xi \rightarrow 0$, x^* converges to the NPV threshold.*

Finally, as the hazard rate of mortgage refinancing opportunities goes to infinity, the optimal threshold, x^* , converges to the optimal refinancing threshold in a model in which refinancing opportunities are continuously available.

Corollary 4. *As $\xi \rightarrow \infty$, x^* converges to the optimal refinancing threshold in the model with continuous refinancing.*

Intuitively, if refinancing opportunities arise arbitrarily often, you should act as if you always have the chance to refinance.

5. DETAILS OF THE ARGUMENT FOR THE MAIN THEOREM

We derive Theorem 1 in this section. The theorem characterizes an equilibrium threshold rule and a value function. The value function has two regions of interest: a refinancing region and a no-refinance region. In the refinancing region, the agent refinances if an opportunity to refinance stochastically arises. In the no-refinancing region, the agent does not refinance, even if a refinancing opportunity occurs.

In general, the value functions for this problem have five state variables: r, π, r', π', M . In this section, we show that the value function can be *decomposed* into two components that can drastically simplified, reducing the most difficult analysis to a problem with one state variable:

$$\begin{aligned} x &= i' - i \\ &= r' + \pi' - r - \pi. \end{aligned}$$

The value function in the refinance region can be reduced to a second order ordinary differential equation (ODE). Likewise, the value function in the no-refinance region can be reduced to a second order ODE. Hence, the general solution can be expressed as a problem with five unknowns: two constants in the refinance ODE, two constants in the no-refinance ODE, and one free boundary, x^* .

To solve for these five unknowns we need to identify five boundary conditions. We exploit value matching at point $x = x^*$, where the refinance region borders the no-refinance region. We exploit smooth pasting at the same boundary. We exploit a value matching constraint that links the value function the instant before refinancing at $x = x^*$ and the instant after refinancing (when as a result of refinancing $x = 0$). Finally, we derive asymptotic boundary properties for $x \rightarrow \infty$ and $x \rightarrow -\infty$.

With these five boundary conditions in hand, we can solve for the value functions and the optimal threshold rule.

Now that the broad thrust of the proof has been explained, some readers may wish to jump immediately to our calibration results in section 6. The remainder of this section develops the detailed proof of Theorem 1.

We begin by decomposing the total value function, V , into two components: Z , the value function without the option to refinance, and W , the value of the option to refinance.

Definition 5. $V(r, r', \pi, \pi')$ is the expected value of mortgage payments.

Definition 6. $Z(r, r', \pi, \pi')$ is the expected value of mortgage payments if you can not refinance.

It's easy to show that

$$Z(r, r', \pi, \pi') = \frac{(i - \pi' + \lambda) M}{\rho + \lambda}. \quad (14)$$

The expected value of mortgage payments (without refinancing) is given by the expected real interest payments $(i - \pi') M$ plus expected real principle payments, λM , divided by the effective discount rate, $\rho + \lambda$.

We now define the the option value of refinancing as the difference between V and Z .

Definition 7. $W(r, r', \pi, \pi')$ is the expected value of the right to refinance a mortgage, so

$$W(r, r', \pi, \pi') \equiv V(r, r', \pi, \pi') - Z(r, r', \pi, \pi'). \quad (15)$$

Characterizing W will be an intermediate goal of our analysis. We begin by showing that W satisfies four regularity properties.

Lemma 8. *Replication.*

$$W(r, r', \pi, \pi') = W(r + \Delta, r' + \Delta, \pi, \pi') \quad (16)$$

$$= W(r, r', \pi + \Delta, \pi' + \Delta) \quad (17)$$

$$= W(r, r' + \Delta, \pi + \Delta, \pi') \quad (18)$$

$$= W(r + \Delta, r', \pi, \pi' + \Delta). \quad (19)$$

Proof. Consider an agent in state $(r + \Delta, r' + \Delta, \pi, \pi')$. Let this agent replicate the refinancing strategy of an agent in state (r, r', π, π') . In other words, refinance after every sequence of *innovations* in the Ito processes that would make the agent who started at (r, r', π, π') refinance. So the agent in state $(r + \Delta, r' + \Delta, \pi, \pi')$ will generate refinancing choices valued at $V(r, r', \pi, \pi') + \frac{\Delta M}{\rho + \lambda}$. Hence,

$$V(r + \Delta, r' + \Delta, \pi, \pi') \leq V(r, r', \pi, \pi') + \frac{\Delta M}{\rho + \lambda}.$$

Likewise, we have

$$V(r, r', \pi, \pi') \leq V(r + \Delta, r' + \Delta, \pi, \pi') - \frac{\Delta M}{\rho + \lambda}.$$

Combining these two inequalities, and substituting equations (14) and (15), yields equation 16.

We now repeat this type of argument for other cases. By replication,

$$\begin{aligned} V(r, r', \pi + \Delta, \pi' + \Delta) &\leq V(r, r', \pi, \pi') \\ V(r, r', \pi, \pi') &\leq V(r, r', \pi + \Delta, \pi' + \Delta). \end{aligned}$$

Combining these two inequalities, we have equation 17.

By replication,

$$\begin{aligned} V(r, r' + \Delta, \pi + \Delta, \pi') &\leq V(r, r', \pi, \pi') + \frac{\Delta M}{\rho + \lambda} \\ V(r, r', \pi, \pi') &\leq V(r + \Delta, r', \pi, \pi' + \Delta) - \frac{\Delta M}{\rho + \lambda}. \end{aligned}$$

This argument is subtle. Before refinancing, the perturbed agent pays Δ more (the inflation rate at which the perturbed agent borrowed is $\pi + \Delta$ rather than π). After refinancing, the perturbed agent pays Δ more (the real interest rate at which the perturbed agent refinances is $r' + \Delta$ rather than r'). Combining the two inequalities, we have equation 18.

By replication,

$$\begin{aligned} V(r + \Delta, r', \pi, \pi' + \Delta) &\leq V(r, r', \pi, \pi') \\ V(r, r', \pi, \pi') &\leq V(r + \Delta, r', \pi, \pi' + \Delta). \end{aligned}$$

This argument is also subtle. Before refinancing, the perturbed agent pays Δ more (the real interest at which the perturbed agent borrowed is $r + \Delta$ rather than r) and Δ less (the current inflation rate for the perturbed agent is $\pi' + \Delta$ rather than π'). These two effects are perfectly offsetting. After refinancing, the perturbed agent pays Δ more (the inflation rate at which the perturbed agent refinances is $\pi' + \Delta$ rather than π') and Δ less (the current inflation rate for the perturbed agent is $\pi' + \Delta$

rather than π'). Combining the two inequalities, we have equation 19. ■

Lemma 9. *Abusing notation, we can rewrite the option value function, $W(r, r', \pi, \pi')$, simply as $W(x)$.*

Proof. Follows from Lemma 8. ■

This critical Lemma reduces a four-state value function to a single-state value function. This simplification will dramatically simplify our analysis and enable us to generate closed form solutions.

We now derive the five boundary conditions.

Lemma 10. *The boundary conditions for W are given by*

$$\begin{aligned} W^L(x^*) &= W^R(x^*) \\ W^{L'}(x^*) &= W^{R'}(x^*) \\ C + W^R(0) &= W^L(x^*) - \frac{x^*M}{\rho + \lambda} \\ \lim_{x^* \rightarrow -\infty} W^{L'}(x^*, r') &= \frac{\xi M}{(\rho + \lambda)(\rho + \lambda + \xi)} \\ \lim_{x^* \rightarrow +\infty} W^R(x^*, r') &= 0 \end{aligned}$$

Proof. We derive these from the boundary conditions on V . The value matching and smooth pasting conditions at *refinancing boundary* x^* are:

$$\begin{aligned} V^L(x^*, r') &= V^R(x^*, r'). \\ V_x^L(x^*, r') &= V_x^R(x^*, r'). \end{aligned}$$

Substitution implies

$$\begin{aligned} W^L(x^*) &= W^R(x^*) \\ W^{L'}(x^*) &= W^{R'}(x^*). \end{aligned}$$

The optimality condition is

$$C(M) + V^R(0, r') = V^L(x^*, r').$$

Substitution implies

$$C(M) + W^R(0) - W^L(x^*) + \frac{x^* M}{\rho + \lambda} = 0.$$

The asymptotic boundary conditions (for W) are:

$$\begin{aligned} \lim_{x^* \rightarrow -\infty} W^{L'}(x^*, r') &= \frac{\xi M}{(\rho + \lambda)(\rho + \lambda + \xi)} \\ \lim_{x^* \rightarrow +\infty} W^R(x^*, r') &= 0 \end{aligned} \tag{20}$$

To derive equation 20 we note the following facts. As $x^* \rightarrow -\infty$, the agent will almost surely refinance at the next opportunity. Hence, the value function becomes linear in x (and the second order terms in Ito's Lemma drop away). This implies that as x goes to $-\infty$,

$$\rho V^L = (-x + r') M + \xi (C + V^R(0, r') - V^L) + \lambda(M - V^L)$$

Collecting terms yields

$$V^L = \frac{(-x + r') M + \xi (C + V^R(0, r')) + \lambda M}{\rho + \lambda + \xi}$$

This implies that

$$V_x^L = \frac{-M}{\rho + \lambda + \xi}$$

Since we also know that

$$V^L = W + \frac{(-x + r' + \lambda) M}{\rho + \lambda + \xi}$$

Differentiating both sides with respect to x yields

$$V_x^L = W^{L'} + \frac{-M}{\rho + \lambda}$$

Combining our two expressions, yields,

$$W^{L'} + \frac{-M}{\rho + \lambda} = \frac{-M}{\rho + \lambda + \xi}$$

which implies that

$$W^{L'} = \frac{\xi M}{(\rho + \lambda)(\rho + \lambda + \xi)}.$$

■

We are now ready to prove the main theorem.

Proof. Let x^* represent the refinancing threshold. When a refinancing opportunity arises and $x < x^*$ then refinance.

It's now convenient to rewrite the value function as a function of x and r' . In an abuse of notation we write

$$V(x, r') = W(x) + \frac{(-x + r' + \lambda) M}{\rho + \lambda}$$

Using Ito's Lemma, we can derive a continuous time Bellman Equation in the no-refinance region (for $x > x^*$),

$$\rho V^R = (-x + r') M + W^{R''} \frac{\sigma^2}{2} + \lambda [M - V^R]. \quad (21)$$

Substituting for V yields

$$\rho \left(W^R + \frac{(-x + r' + \lambda) M}{\rho + \lambda} \right) = (-x + r') M + W^{R''} \frac{\sigma^2}{2} + \lambda \left[M - W^R - \frac{(-x + r' + \lambda) M}{\rho + \lambda} \right].$$

This simplifies to,

$$\rho W^R = W^{R''} \frac{\sigma^2}{2} - \lambda W^R. \quad (22)$$

We now repeat this analysis in the refinance region, where the continuous time Bellman Equation is

$$\rho V^L = (-x + r') M + W^{L''} \frac{\sigma^2}{2} + \lambda [M - V] + \xi [C + V^R(0, r') - V^L(x, r')]. \quad (23)$$

Substituting for V yields,

$$\begin{aligned} \rho \left(W^L + \frac{(-x + r' + \lambda) M}{\rho + \lambda} \right) = & (-x + r') M + W^{L''} \frac{\sigma^2}{2} + \lambda \left[M - W^L - \frac{(-x + r' + \lambda) M}{\rho + \lambda} \right] + \\ & \xi \left[C + W(0) + \frac{(-0 + r' + \lambda) M}{\rho + \lambda} - W^L(x) - \frac{(-x + r' + \lambda) M}{\rho + \lambda} \right]. \end{aligned}$$

This simplifies to,

$$\rho W^L = W^{L''} \frac{\sigma^2}{2} - \lambda W^L + \xi \left[C(M) + W(0) - W^L(x) + \frac{xM}{\rho + \lambda} \right]. \quad (24)$$

Note that we have now reduced our analysis to a pair of second-order differential equations in W — equations 22 and 24. The original value function V has been eliminated from the analysis, as has the variable r' .

The option value function $W^R(x)$ has a solution of the form $W^R(x) = Re^{\psi x}$, with

exponent

$$\psi = \frac{-\sqrt{2(\rho + \lambda)}}{\sigma}.$$

The remaining two parameters, R and x^* , solve the system of equations derived from the value matching and smooth pasting conditions.

$$Re^{\psi x^*} = R + \frac{x^* M}{\rho + \lambda} + C \quad (25)$$

$$\psi Re^{\psi x^*} = W^{L'}(x^*). \quad (26)$$

Now we turn to the option value function in the action region. Our Bellman Equation is,

$$\rho W^L = W^{L''} \frac{\sigma^2}{2} - \lambda W^L + \xi \left[W^R(0) + \frac{xM}{\rho + \lambda} + C - W^L \right]. \quad (27)$$

The remaining unused boundary conditions at the *refinancing boundary* (x^*) are:

$$W^L = W^R$$

$$W^{L'} = W^{R'}$$

$$\lim_{x \rightarrow -\infty} W_x^L(x) = \frac{\xi M}{(\xi + \rho + \lambda)(\rho + \lambda)} \quad (28)$$

The reduced equation of the option value function $W^L(x)$ has a solution of the form $Le^{\phi x}$, with exponents given by

$$\rho = \phi^2 \frac{\sigma^2}{2} - \lambda - \xi.$$

$$\phi = \frac{\sqrt{2(\rho + \lambda + \xi)}}{\sigma}.$$

The equation has a particular solution of $(A + Bx)$, where

$$\rho(A + Bx) = -\lambda(A + Bx) + \xi \left[W^R(0) + \frac{xM}{\rho + \lambda} + C - (A + Bx) \right].$$

$$A = \frac{\xi [W^R(0) + C]}{\rho + \lambda + \xi}.$$

$$B = \frac{\xi M}{(\rho + \lambda + \xi)(\rho + \lambda)}.$$

Note correspondence between this last equation and equation ??.

So the general solution to the Bellman equation will be

$$W(x) = \frac{\xi [W^R(0) + C]}{\rho + \lambda + \xi} + \frac{\xi Mx}{(\rho + \lambda + \xi)(\rho + \lambda)} + Le^{\phi x}$$

The three free parameters, L , R , and x^* , solve the system of equations derived from the value matching and smooth pasting conditions.

$$Re^{\psi x^*} = R + \frac{x^* M}{\rho + \lambda} + C \quad (29)$$

$$\psi Re^{\psi x^*} = \frac{\xi M}{(\rho + \lambda + \xi)(\rho + \lambda)} + \phi Le^{\psi x^*} \quad (30)$$

$$Re^{\psi x^*} = \frac{\xi [W^R(0) + C]}{\rho + \lambda + \xi} + \frac{\xi Mx^*}{(\rho + \lambda + \xi)(\rho + \lambda)} + Le^{\phi x^*} \quad (31)$$

Our goal is to solve for x^* , so we focus on

$$R = (e^{\psi x^*} - 1)^{-1} \left(\frac{x^* M}{\rho + \lambda} + C \right) \quad (32)$$

The optimal refinancing differential solves the following non-linear equation

$$\psi \left(\frac{x^* M}{\rho + \lambda} + C \right) e^{\psi x^*} = \frac{\xi M (e^{\psi x^*} - 1)}{(\rho + \lambda + \xi)(\rho + \lambda)} + \phi \left[\left(\frac{x^* M}{\rho + \lambda} + C \right) e^{\psi x^*} - \frac{\xi \left[\frac{x^* M}{\rho + \lambda} + C e^{\psi x^*} \right]}{\rho + \lambda + \xi} - \frac{(e^{\psi x^*} - 1) \xi M x^*}{(\rho + \lambda + \xi)(\rho + \lambda)} \right]$$

where $\psi = \frac{-\sqrt{2(\rho + \lambda)}}{\sigma}$ and $\phi = \frac{\sqrt{2(\rho + \lambda + \xi)}}{\sigma}$. This expression simplifies to,

$$[\psi(\rho + \lambda + \xi) - \phi(\rho + \lambda)] \left[x^* + (\rho + \lambda) \frac{C(M)}{M} \right] = \xi (1 - e^{-\psi x^*}), \quad (33)$$

which completes the proof of the Theorem. ■

6. CALIBRATION

We begin by exploring the model's predictions for the optimal threshold value x^* . We solve equation (6) numerically with calibrated values of parameters λ , ρ , ξ , $C(M)$, and σ .

Recall that λ reflects exogenous mortgage repayment. In the real-world mortgages get repaid because they have a fixed, finite duration and because homeowners occasionally move for exogenous reasons. Our calibration of λ needs to capture both of these effects. We assume $\lambda = 0.05$ so that the expected time until *exogenous* future full repayment of the mortgage is $\frac{1}{\lambda}$ or 20 years.

Calibrating $\lambda = 0.05$ implies that the average homeowner repays fraction $[1 - \exp(-\lambda t)]$ of their real principal after t years. For example, if $\lambda = 0.05$ then after 20 years the *average* mortgage holder in our model will have repaid 63% of his real mortgage principal. This compares well to the repayment pattern in a real-world 30-year mortgage. For example, consider an economy with an inflation rate of 3% and a real interest rate of 3%, typical values during our sample period. In this environment, 70% of the *real*

principal of a 30-year mortgage will be repaid after 20 years. Appendix A provides the details of this analysis.

The 70% calculation in the previous paragraph applies to a household that simply repays a 30-year mortgage and never moves for exogenous reasons. Most real households move long before their 30-year mortgage is repaid. Hence, a realistic calibration of λ will lie *above* $\lambda = 0.05$, since λ does the double-duty of capturing exogenous repayment due to the finiteness of mortgage contracts and due to exogenous separation from a property (e.g., a death or a move).

We choose to calibrate the model conservatively (with $\lambda = 0.05$), because we want to bias our results *against* finding refinancing that is normatively “too early.” If a household has a “true” value of $\lambda = 0.05$ and we assume that the household has a higher λ value, then we may falsely find that the household refinances too early (since households with long horizons should refinance at relatively small interest differentials). To anticipate, we will find evidence for early refinancing, even though we calculate normative refinancing thresholds with a value of λ that is far below the “true” λ for almost all households.

We adopt a standard exponential discount rate with a discount rate of $\rho = 0.05$. We assume ξ takes on one of three values, 0, 1, or ∞ . When $\xi = 0$, the mortgage holder follows the NPV rule. When $\xi = 1$, the mortgage holder expects to have one refinancing “opportunity” per year. When $\xi = \infty$, the mortgage holder acts as if she is able to continuously refinance her mortgage. We use historical data on nominal mortgage interest rates to estimate the standard deviation of nominal interest rate movements: $\sigma = 0.0121$.²

We assume transactions costs of $C(M) = 0.01M + 2000$. The proportional point spread (0.01) was reported to us by our data provider: all of the mortgage

²For a household with a τ tax rate, σ should be scaled by $1 - \tau$.

refinancings in our dataset had approximately one point of proportional costs. Our data provider assures us that there is almost no variation in the point spread across households. The fixed cost (\$2000) reflects a range of fees including inspection costs, title insurance, lawyers fees, filing charges, and other transaction costs.³ Finally, note that refinancing points/fees are not tax deductible (in contrast to interest payments), which is reflected in our modelling set-up.⁴

6.1. Equilibrium refinancing thresholds.. The following table reports the optimal refinancing differentials calculated with our model for the calibration above (assuming a zero marginal tax rate).⁵

Optimal refinancing differentials ($-x^*$) in basis points

for a family with a zero marginal tax rate:

<u>Mortgage</u>	<u>$\xi = 0$</u>	<u>$\xi = 1$</u>	<u>$\xi = \infty$</u>
\$300,000	17	47	101
\$200,000	20	54	110
\$100,000	30	77	138
\$50,000	50	116	183

The first threshold column, $\xi = 0$, represents the threshold associated with the NPV rule. For example, a household that anticipates that they will probably never have another chance to refinance will follow an NPV rule if they encounter (a rare) chance to refinance. The last threshold column, $\xi = \infty$, represents the threshold associated with a household that has a continuous opportunity to refinance. This column reports optimal thresholds for households that are never distracted, so this column represents the classical continuous-time refinancing rule.

³See, e.g. Federal Reserve Board and Office of Thrift Supervision (1996).

⁴Points/fees are deductible for an “original” mortgage, although they are not tax-deductible for refinancings.

⁵To a first approximation, scale these thresholds by $(1 - \tau)^{-1}$ to generate optimal thresholds for arbitrary marginal tax rates.

The optimal refinancing threshold increases as mortgage size decreases, since interest savings from refinancing scale linearly with mortgage size but part of the refinancing cost is fixed. The optimal refinancing threshold increases with ξ since households with more opportunities to refinance wait until the right moment, whereas households with only occasional opportunities to refinance aren't quite as choosy. In essence, a highly distracted household recognizes that postponing refinancing may be very costly, since it may not soon have another opportunity to refinance. This mechanism, leads it to occasionally refinance at relatively low interest rate differentials.

7. FOUR PREDICTIONS

7.1. Late refiners: Woodheads. Many distracted households will refinance “too late.” More formally, distracted consumers will finance at interest rate differentials that are often much larger than the differentials predicted by the benchmark of optimal refinancing with no distraction (i.e., the benchmark of $\xi = \infty$). The prediction of late refinancing follows immediately from our assumption that households only have an opportunity to refinance every so often. When households get unlucky, they won't have a chance to refinance until long after interest rates move in their favor.

The existence of delayed refinancing is already well-documented (add cites here). The mortgage market has many nicknames for the households that fail to refinance in the face of substantial incentives. Some analysts refer to such households as “woodheads.”

7.2. Early refiners: Opportunists. The model of distracted consumers also predicts that some households will refinance too *early*. This counterintuitive prediction follows from the fact that distracted consumers understand their own distraction. They reason, “I know that today's mortgage rates aren't at an optimal value for refinancing, but I should refinance now anyway since I may not have an opportunity

to refinance when mortgage rates improve a little bit more. A bird in the hand — when I have an opportunity to refinance — is worth two birds in the bush.” Advisors to distracted consumers may reason exactly the same way, recommending that distracted consumers refinance whenever it is profitable, instead of waiting until the optimal threshold has been reached. We return to this point in subsection 7.4.

These early refinancing effects are formally captured by the following proposition.

Proposition 11. *The optimal refinancing threshold with $0 < \xi < \infty$ is less than the optimal refinancing threshold as $\xi \rightarrow \infty$.*

$$|x_{0 < \xi < \infty}^*| < \lim_{\xi \rightarrow \infty} |x_{\xi}^*|.$$

Proof. Totally differentiate equation (6) to show that $\frac{dx^*}{d\xi} < 0$. The proposition follows, since $x^* < 0$. ■

See our calibration table in the previous section for a numerical example of this effect. In our calibrated model (with $\xi = 1$) the optimal refinancing thresholds are roughly half what they would be if consumers were completely undistracted.

We present new evidence that many consumers do refinance too early. To document this claim, we examine a unique panel data set from a large financial institution, hereafter referred to as “the Intermediary.” The dataset contains three kinds of information:

1. Data on initial characteristics of the mortgage, including the mortgage amount, the initial terms (APR), the term structure (duration and fixed vs. variable) and property location (at the zip code level).
2. Credit bureau information, notably quarterly FICO score updates.
3. Transaction information, including the loan-to-value ratio and the actual record of monthly payments.

We restrict analysis to 30-year fixed conventional mortgages (i.e., $\$50,000 < M < \$300,000$) that were owned by the Intermediary and then refinanced at the Intermediary. We restrict attention to these accounts since we do not observe refinancing outcomes when a loan is refinanced at another bank. We restrict attention to loans that were refinanced for an amount within ten percent of the remaining mortgage balance. We exclude accounts that exhibit default or bankruptcy *before* they refinance. Finally, we eliminate a handful of loans that were purportedly refinanced at negative differentials. This leaves 840 mortgage refinancings, observed from December 1992 to December 2001.

In this sample, the mortgages have a mean value of \$166,874, with a 25th percentile of \$100,109 and a 75th percentile of \$217,147. The original APR has a mean of 8.5%, with a 25th percentile of 6.75%, and a 75th percentile of 12.50%. The APR after refinancing has a mean of 5.875%, with a 25th percentile of 7.365% and a 75th percentile of 9.75%. Loan-to-value ratio (LTV) has a mean of 64.13% with a 25th percentile of 52.32% and a 75th percentile of 78.93%. LTV and credit scores are virtually unchanged after refinancing.

Figures 1 and 2 plot refinancing percentages at each differential and cumulative percentages by differential, respectively. Two kinds of seemingly anomalous behavior can be seen in this figure. First, over a third of borrowers refinance at rates significantly lower than the optimal interest rate differential. Second, a few borrowers refinance at very high differentials.

We directly calculate the fraction of households that refinance too early, computing an optimal threshold for each household based on the size of the household's mortgage. We find that 33.7% of refinancers do so at thresholds at least 10 basis points below the optimal threshold implied by the undistracted version of our model (i.e., with $\xi = \infty$). However, almost no households refinance at thresholds below the optimal threshold implied by the model with distraction, specifically with $\xi = 1$.

7.3. Early refinancing is a new phenomenon. Our model also makes predictions, about the low frequency patterns in early refinancing. Distracted consumers will refinance “too late” in environments with falling interest rates and distracted consumers will refinance “too early” when interest rates tend to vary in a tight range. Intuitively, distracted consumers are late to respond to a rapid fall in interest rates, since distracted consumers only have a chance to refinance every so often. Likewise, distracted consumers will refinance too early in static interest-rate environments, since distracted consumers will refinance even at relatively small interest rate differentials. These observations imply that late refinancing should have been common in the 1980’s, while early refinancing should have become common in the 1990’s.

To quantitatively evaluate these predictions, we simulate a population of consumers starting in 1970. We seed our simulation by giving each of these consumers a new mortgage in 1970. These consumers have the following parameter values: $\lambda = 0.1$, $\tau = 0.25$, $\xi = 1$, $\rho = 0.05$, $C(M) = (0.01)M + 2000$, $\sigma = 0.0121$, and $M = 200,000$. These consumers face interest rate realizations that match the actual mortgage interest rate data from 1970 to 2002. When a consumer drops out of the simulated sample (because of a λ -event), we introduce a replacement consumer with a new mortgage.

When analyzing our simulated data, we use the same definition for early refinancing as we use with our real data. Specifically, a household is coded as refinancing too early if the household refinances at an interest rate threshold that is at least 10 basis points below the interest rate threshold implied by an optimal refinancing calculation with $\lambda = 0.05$ and $\xi = \infty$.

Our simulations predict that early refinancing was relatively rare in the 1980’s. Only 15.2% of simulated refinancers refinance too early from 1980 to 1989. By contrast, 35.2% of simulated refinancers refinance too early from 1990 to 2002.

Though these simulation-based predictions are hard to evaluate, they correspond

well to what little is known about early refinancing. First, our own empirical calculations (cf previous subsection), imply that 33.7% of refinancers did so too early from December 1992 to December 2001, precisely matching our simulation prediction for that period (34.0%). Second, our simulations provide a suggestive explanation of why early refinancing has not been studied to date: our simulations imply that early refinancing only became common in the late 1990's. Before 1997, the simulated fraction of early refinancers always lies below 50% of all refinancers. From 1997-2002, the simulated fraction of early refinancers lies above 50% in every year except 1998 (when it is 43%).

7.4. Financial advisors recommend using the NPV rule. Households utilize many different sources of advice on mortgage refinancing: mortgage brokers, financial advisers, book, and websites. We examined recommendations from the last two sources and found an overwhelming tendency to recommend use of the NPV rule.

First, we sampled 19 personal finance books. We chose books that were on top-ten sales lists at Amazon and Barnes & Noble web sites. We also chose books that were on the personal finance shelves at Wordsworth and Barnes & Noble stores.

Of the 19 books, 15 provided a break-even calculation of some sort. Six provided this calculation as their only guideline, and made no comment about waiting for extra profit (or in one case, even discouraged waiting). Nine other books discussed the break even calculation in addition to other rules of thumb. For example, *Keys to Mortgage Financing and Refinancing* recommends that consumers "Compare the savings to the costs to find the amount of time before you break even . . . if the borrower plans to stay in the home at least [this long], it pays to refinance the loan."

For websites, we entered the words "mortgage refinancing advice" into Google (<http://www.google.com>) and examined the top ten sites which offered information on refinancing. Two of these sites provided a fixed interest-rate differential of two

percent. The other eight sites offered a refinancing calculator based on a break-even criterion.

At first glance, these NPV recommendations fly in the face of option-value theory. However, in a world of distracted consumers, NPV recommendations turn out to be approximately optimal. Intuitively, if agents do not have continuous opportunities to refinance, then they won't be able to fully exploit the option value of waiting, pushing them much closer to the NPV rule.

How suboptimal is the NPV rule? To evaluate the optimality of the NPV rule, we consider a distracted agent that starts life with state variable $x = 0$ (e.g., a new mortgage). We calculate the expected cost of using the *suboptimal* NPV rule instead of using the *optimal* refinancing rule implied by Theorem 1.

Proposition 12. *The Loss from using the suboptimal NPV rule instead of using the optimal rule is given by,*

$$\begin{aligned} \frac{Loss}{M} &= \left(\frac{\xi}{(\rho + \lambda) [(\psi - \phi)e^{\psi x^*}(\rho + \lambda + \xi) + \phi\xi]} \right) \\ &\quad + (1 - e^{\psi x^*})^{-1} \left(\frac{x^*}{\rho + \lambda} + C \right). \\ \frac{Loss}{M} &= \frac{\phi\xi \left(\frac{x^*}{\rho + \lambda} + \frac{C(M)}{M} \right)}{(\psi - \phi)(\rho + \lambda + \xi)e^{\psi x^*} + \phi\xi} \\ \lim_{\xi \rightarrow \infty} \frac{Loss}{M} &= \frac{\frac{x^*}{\rho + \lambda} + \frac{C(M)}{M}}{1 - e^{\psi x^*}} \end{aligned} \tag{34}$$

Using the same calibration assumptions that were used in section 6, we calculate the economic losses of using the NPV rule instead of the optimal rule.

Expected Value of Dollar Losses from Using the NPV Rule Instead of Using the Optimal Refinancing Rule with Distraction.

<u>Mortgage</u>	<u>$\xi = 0$</u>	<u>$\xi = 1/2$</u>	<u>$\xi = 1$</u>	<u>$\xi = 2$</u>	<u>$\xi = \infty$</u>
300,000	0	288	912	2,335	55,910
175,000	0	246	742	1,804	30,917
50,000	0	203	512	1,030	6,879

As a Percent of Mortgage Face Value

<u>Mortgage</u>	<u>$\xi = 0$</u>	<u>$\xi = 1/2$</u>	<u>$\xi = 1$</u>	<u>$\xi = 2$</u>	<u>$\xi = \infty$</u>
300,000	0	0.1%	0.3%	0.8%	18.6%
175,000	0	0.1%	0.4%	1.0%	17.7%
50,000	0	0.4%	1.0%	2.1%	13.8%

As these tables demonstrate, the NPV rule is nearly optimal when $\xi \leq 1$. For example, a household with a \$175,000 mortgage expects to lose only \$742 over the life of their mortgage because they use the *NPV* rule.

8. CONCLUSION

Choosing when to refinance a mortgage is one of the most important option-pricing problems people face. The standard option-pricing approach to refinancing when there are transactions costs implies there is an optimal interest-rate differential above which borrowers should refinance; for reasonable ranges of parameter values, this differential ranges between 100 and 250 basis points. It has long been recognized that borrower behavior does not match the predictions of the model. The previous literature has focused on one kind of departure: the failure of “woodheads” to refinance even when rates have dropped by enough that their option is well in the money.

We show that adding a single feature to the model- distraction- solves this empirical puzzle, and several others we newly identify. Distracted consumers by assumption only have a chance to refinance with some hazard rate. For some of them, this opportunity will never arise- and they will appear to be “woodheads.” If the opportunity

does arise, though, the consumers should refinance at differentials smaller than those if opportunities arose continuously- since waiting for the next (distant) opportunity is too costly. This explains both the rise of “early refinancing,” which we document using a unique data set from a large financial institution, and the prevalence of popular financial advice advocating the use of a net present value rule that ignores the option value. Finally, distracted consumers turn out to be late refinancers in an environment in which interest rates rapidly fall (such as the 1980s), but early refinancers in an environment in which interest rates fluctuate in a narrow band (such as the 1990s). This accounts for the relative amount of attention paid to late refinancing in the literature in the late 1980s and early 1990s, and the lack of evidence on early refinancing until now.

Distraction is also complementary to some other explanations for these puzzles, and for other puzzles. One could, for example, write down a model of consumption smoothing for distracted consumers; or allow the interest rate process to be mean-reverting. Doing so would incur the cost of less analytical tractability.

Finding distraction in this market also complements previous findings that distraction explains puzzles in the markets for equities and in price-setting.

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9. APPENDIX A: MORTGAGE REPAYMENT FOR A REAL-WORLD MORTGAGE.

Assume that a mortgage is characterized by a constant nominal payment, p , in an environment with inflation π and real interest rate r . The real value of the mortgage is given by the differential equation,

$$\dot{M} = -p \exp(-\pi t) + rM$$

with boundary conditions $M(0) = M_0$. The solution to this equation is

$$M(t) = \left(M_0 - \frac{p}{r + \pi} \right) \exp(rt) + \frac{p}{r + \pi} \exp(-\pi t).$$

If the mortgage is repaid over a horizon of T years, then

$$M(T) = 0 = \left(M_0 - \frac{p}{r + \pi} \right) \exp(rT) + \frac{p}{r + \pi} \exp(-\pi T),$$

implying that the nominal payment stream is given by,

$$\frac{p}{M_0} = \frac{r + \pi}{1 - \exp(-(r + \pi) T)}.$$

Combining these results, we can conclude that the fraction of the real value of the mortgage that remains after t years is given by,

$$\frac{M(t)}{M_0} = \left(1 - \frac{1}{1 - \exp(-(r + \pi) T)} \right) \exp(rt) + \frac{1}{1 - \exp(-(r + \pi) T)} \exp(-\pi t).$$

Figure 1
Percent Refinancing By Interest Differential

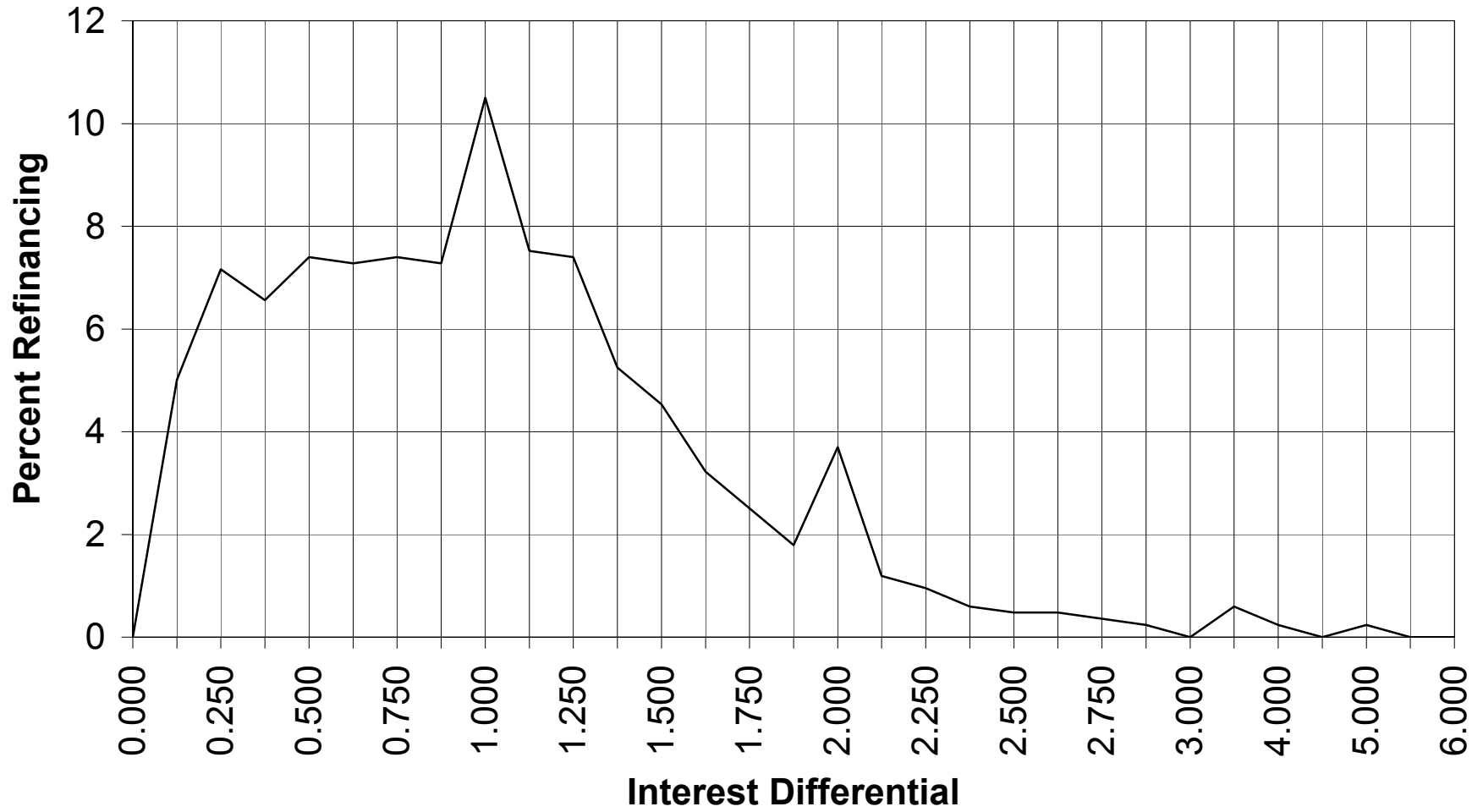


Figure 2
Cumulative Percent Refinancing By Interest Differential

