Modelling the Birth and Death of Cartels with an Application to Evaluating Antitrust Policy*

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Abstract

One of the primary challenges to measuring the impact of antitrust policy on collusion is that the cartel population is unobservable; we observe only the population of *discovered* cartels. To address this challenge, a model of cartel creation and dissolution is developed to endogenously derive the populations of cartels and discovered cartels. It is then shown how one can infer the impact of antitrust policy on the population of cartels by measuring its impact on the population of discovered cartels. In particular, the change in the distribution on the *duration* of discovered cartels could be informative in assessing whether a new antitrust policy is reducing the latent rate of cartels.

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1 Introduction

The Antitrust Division of the U.S. Department of Justice instituted a corporate leniency program in 1978 that allows all government penalties to be waived for the first member of a cartel who comes forward and cooperates fully. After 15 years of being largely ignored by price-fixers, the program was substantially revised in 1993.[1] The revision made three significant changes: i) amnesty is automatic if there is no pre-existing investigation; ii) amnesty may still be available even when an investigation is underway; and iii) all officers, directors, and employees who cooperate are protected from criminal prosecution. The response in terms of the number of leniency applications was dramatic as it rose from about one application per year to about two per *month*. Though this redesign clearly made the program more attractive to firms, what we are ultimately interested in is its impact on cartel formation. Has the 1993 revision of the corporate leniency program reduced the frequency of cartels in the economy?

A similar event took place in the European Union. In response to the impact of the 1993 revision on the number of leniency applications, the EU instituted a leniency program in 1996 (which it later revised in 2002). A simple tracking of the number of convicted cartels reveals that it went from around 2.5 per year during 1990-95 to about 4.75 per year during 1996-2003 (Brenner, 2006). Of course, this rise in the rate of convictions could be due to many factors other than the leniency program, such as changes in the economy due to increased European integration and more resources put into prosecuting cartels. But even if one were to effectively control for these other factors and concluded that the institution of a leniency program caused a rise in the number of convicted cartels, what does this imply about the program's impact on the number of cartels? Is a rise in convictions a signal of fewer or more cartels?

In both of these examples, a new policy has been put in place to fight cartels and an assessment of its efficacy means understanding how it has affected the rate of cartels in the economy. The fundamental obstacle is that we do not observe the population of cartels. Due to their illegality, cartels hide themselves; we observe only the population of *discovered* cartels. To see the difficulties that can arise, consider an antitrust policy that impacts both the rate of cartel formation and the rate at which cartels are discovered. By both raising the discovery rate and reducing the number of cartels, the effect on the number of convictions is generally ambiguous. Hence, there could be little effect on the number of convictions even though the policy is working as intended. Alternatively, the lack of change in the number of convictions could reflect the ineffectiveness of the new policy. How are we to judge the efficacy of a policy when the intended variable to be affected - the cartel rate - is not observed?

The approach of this paper is to develop a model that endogenizes the population of cartels and the population of discovered cartels and then identifies how these two populations are related. What observable change in the population of discovered cartels is informative as to what is happening with the frequency of cartels? To address this question, a population of heterogeneous industries is considered and the birth

[1] For a survey on leniency programs, see Spagnolo (2006).

and death process for cartels is modelled. Industries are given stochastic opportunities to form a cartel and do so if it is incentive compatible. Because of random market conditions, a cartel may persist or perish because it is no longer incentive compatible to collude; they may also be discovered by the antitrust authorities. Cartel formation and demise is then a stochastic process and I characterize its stationary distribution to derive a rate of cartelization for this population of industries. The main finding is that though the change in the rate of discovered cartels is not a useful proxy for the change in the rate of cartels (in response to at least some types of policies), the change in the *duration* of discovered cartels can be informative as to the change in the rate of cartels.

Though this project is focused on providing methods for evaluating policies designed to fight cartels, it makes a broader contribution to the general theoretical literature on collusion by modelling the birth and death process of cartels. Despite its immensity and richness, the theory of collusion focuses on a single industry in describing what conditions are conducive to collusion and what types of patterns in prices and quantities are associated with collusion. Though there is research that endogenizes cartel formation - such as Selten (1973) - no previous work models the birth and death of cartels and thus are incapable of addressing the questions motivating this study. This paper shows how one can model the stochastic process determining the population of cartels and use it to generate a rate of cartelization for an economy. Though the model is developed using the simple Prisoners' Dilemma, I believe it can be extended to allow for richer oligopoly models though it may require numerical analysis to characterize the stationary equilibrium.

Finally, it is worth noting that the measurement problem motivating this paper is not unique to price-fixing; it arises as well for other forms of criminal activity including tax evasion, extortion, blackmail, and kidnapping. These types of crimes are common in that they are often not reported; either because the victim doesn't know they are being victimized (such as with price-fixing and tax evasion) or the victim doesn't have an incentive to report (due to the threat of some punishment). This creates the challenge of measuring the rate of criminal activity and assessing the impact of a policy on the crime rate. One approach to measuring the crime rate is to engage in random sampling. While that may work with tax evasion, it would be difficult to randomly sample industries for collusion or small businesses for extortion. There is then a fundamental challenge in measuring the latent rate of criminal activity with data based on a non-random sampling.

2 Model

The objective is to construct a model in which some industries collude and some do not, some cartels collapse and some do not, some cartels are caught and some are not. As I am going to consider a population of industries, the model of each industry is kept simple by using a Prisoners' Dilemma formulation. For each industry, there is a stochastic realization of a market's profitability which is summarized by the variable $\pi \geq 0$. If firms are colluding then each firm earns π and, if not colluding, then each earns $\alpha\pi$ where $\alpha \in [0,1)$. π has a continuously differentiable cdf $H : [\underline{\pi}, \overline{\pi}] \rightarrow [0,1]$ where $0 < \underline{\pi} < \overline{\pi}$ and $\overline{\pi}$ may be finite or infinite. $h(\cdot)$ denotes the associated density function. Let $\mu \equiv \int \pi H'(\pi)\, d\pi$ denote its finite mean. As in Rotemberg and Saloner (1986), π is observed prior to firms deciding how to behave. If all other firms are colluding, the profit a firm earns by deviating is $\eta\pi$ where $\eta > 1$. Let $\delta \in (0,1)$ denote the common discount factor. Note that the Bertrand price game is represented by $(\alpha, \eta) = (0, n)$ where n is the number of firms. The Cournot quantity game with linear demand and cost functions in which firms collude at the joint profit maximum can be represented as $(\alpha, \eta) = \left(\frac{4n}{(n+1)^2}, \frac{(n+1)^2}{4n}\right)$.

At the start of each period, an industry is either cartelized or not. If it was cartelized at the end of the previous period then it is currently cartelized. If was not cartelized at the end of the previous period then with probability $\kappa \in (0,1)$ it has an opportunity to do so.[2] Given the realization of π, if firms either are not cartelized or if collusion is not incentive compatible then each firm earns $\alpha\pi$. If there is a cartel and collusion is incentive compatible then each earns π.

At the end of the period, there is the random event whereby the antitrust authority may pursue an investigation; this can only occur if firms colluded in the current or previous period.[3] Let $\sigma \in [0,1)$ denote the probability that firms are discovered and convicted. In that case, each firm incurs a penalty of $\frac{F}{1-\delta}$ (so that F is the per-period penalty). It is desirable to allow F to depend on the extent of collusion. Given there is only one collusive price in the model, the "extent of collusion" necessarily refers to the number of periods that firms had colluded. A proper accounting of that effect would require that each cartel have a state variable which is the length of collusion; this would be a serious complication of the model. As an approximation, I instead assume that the penalty is proportional to the average increase in profit from being cartelized (rather than the realized increase in profit). If Y denotes the expected per period profit from being in the "cartel state" then $F = \gamma(Y - \alpha\mu)$ where $\gamma > 0$. This avoids the need for state variables but still allows the penalty to be sensitive to the (average) extent of collusion. In sum, the antitrust policy parameters are (σ, γ) which are, respectively, the probability of discovery and conviction and the penalty multiple.

Whenever a cartel is shutdown - whether due to internal collapse or having been successfully prosecuted - the industry may re-cartelize in the future. Specifically, it has an opportunity to do so with probability κ in each period that it is not currently colluding. Alternatively, one could imagine having two distinct probabilities - one to reconstitute collusion after a firm cheated (the probability of moving from the punishment to the cooperative phase) and another to reform the cartel after having been convicted. For purposes of parsimony and tractability, those two probabilities

[2] That $\kappa < 1$ may be because cartelization requires having a set of managers willing to break the law or that feel they can communicate and trust each other or an opportunity arises to communicate without much risk of being caught.

[3] To allow it to depend on collusion farther back in time would require introducing a state variable that would unnecessarily complicate the analysis. Having it depend on collusion in the previous period will simplify some of the expressions and, furthermore, it seems quite reasonable that detection can occur, to a limited extent, after the fact.

are assumed to be the same.

The modelling of collusion here differs from how it is typically done. The standard approach presumes that firms are always coordinating; even when they are competing, it is a coordinated punishment in response to suspected cheating. In a perfect monitoring setting such as Rotemberg and Saloner (1986), it may require adjusting the collusive price so as to maintain cartel stability. Or, in an imperfect monitoring setting, it may require periodic shifts to distinctly lower prices. But, at all times, firms are coordinating their behavior. I do not believe that is always a reasonable representation of reality. There are many well-documented episodes in which a cartel truly collapses in the sense that coordination stops and what emerges from the ashes is competition. Of course, coordination may start up again but it need not be immediate and the prospect of it re-starting may be reasonably viewed as uncertain from the perspective of firms.

In modelling a population of industries, it is compelling to allow industries to vary in terms of cartel stability. For this purpose, I assume industries may differ in terms of the parameter η. If one takes this assumption literally, it can be motivated by heterogeneity in the elasticity of firm demand or the number of firms (as with the Bertrand price game). Our intent is not to be literal but rather to think of this as a parsimonious way in which to encompass industry heterogeneity. Let the cdf on industry types be represented by the continuously differentiable function $G : \left[\underline{\eta}, \overline{\eta}\right] \rightarrow [0,1]$ where $1 < \underline{\eta} < \overline{\eta}$. $g(\cdot)$ denotes the associated density function. Alternatively, one could have heterogeneity with respect to δ and I suspect results would hold. The appeal of η is that it is a parameter which influences the frequency of collusion but does not directly affect the value of the firm's profit stream since, in equilibrium, firms do not cheat; this makes for an easier analysis.

3 Equilibrium Cartel Formation

Characterization of the stationary distribution on cartels is a three-step procedure. First, the equilibrium conditions for a cartel to be stable in a type-η industry are characterized in this section. This result is then used in the ensuing section where the stationary distribution on the sub-population of type-η industries is first characterized and then that result is integrated over all values of η to derive the aggregate stationary distribution.

3.1 Existence of an Equilibrium

A collusive strategy for a type-η industry entails colluding when π is sufficiently low and not colluding otherwise. The logic is as in Rotemberg and Saloner (1986). When π is high, the incentive to deviate is strong since a firm increases current profit by $(\eta - 1)\pi$. Given π is *iid* then the future payoff is independent of the current realization of π. Since the payoff to cheating is increasing in π while the future payoff is independent of π, the incentive compatibility of collusion is more problematic when π is higher. Depending on the parameter values, it is possible that collusion is not incentive compatible for any $\pi \in [\underline{\pi}, \overline{\pi}]$, in which case those industries never cartelize. Similarly, it may be the case that collusion is incentive compatible for all $\pi \in [\underline{\pi}, \overline{\pi}]$, in which case such cartels are never subject to internal collapse and are only shut down by the authorities.

Suppose firms are able to collude for at least some realizations of π. Let W^o and Y^o denote the payoff when the industry is not cartelized and is cartelized, respectively. If not cartelized then, with probability κ, firms have an opportunity to cartelize with resulting payoff Y^o. With probability $1-\kappa$, firms do not have such an opportunity and continue to compete. In that case, each firm earns current expected profit of $\alpha\mu$ and a future value of W^o. Thus, the payoff when not colluding is defined recursively by:

$$W^o = (1-\kappa)(\alpha\mu + \delta W^o) + \kappa Y^o. \tag{1}$$

It'll be easier to work with re-scaled payoffs, so define:

$$W \equiv (1-\delta) W^o,\ Y \equiv (1-\delta) Y^o$$

Multiplying both sides of (1) by $1-\delta$ and re-arranging yields:

$$\begin{aligned} (1-\delta) W^o &= (1-\kappa)[(1-\delta)\alpha\mu + \delta(1-\delta) W^o] + \kappa(1-\delta) Y^o \Leftrightarrow \\ W &= (1-\kappa)[(1-\delta)\alpha\mu + \delta W] + \kappa Y \Leftrightarrow \end{aligned}$$

$$W = \frac{(1-\kappa)(1-\delta)\alpha\mu + \kappa Y}{1-\delta(1-\kappa)} \tag{2}$$

Also note that the incremental value to being in the cartelized state is:

$$Y - W = Y - \left(\frac{(1-\kappa)(1-\delta)\alpha\mu - \kappa Y}{1-\delta(1-\kappa)}\right) = \frac{(1-\kappa)(1-\delta)(Y-\alpha\mu)}{1-\delta(1-\kappa)}. \tag{3}$$

Suppose firms are cartelized and π is realized. The incentive compatibility constraint (ICC) is:

$$\begin{aligned} (1-\delta)\pi + \delta[(1-\sigma)Y + \sigma(W-F)] &\geq (1-\delta)\eta\pi + \delta(W-\sigma F) \Leftrightarrow \\ \delta(1-\sigma)(Y-W) &\geq (1-\delta)(\eta-1)\pi \Leftrightarrow \end{aligned}$$

$$\pi \leq \frac{\delta(1-\sigma)(Y-W)}{(1-\delta)(\eta-1)} \left(\equiv \phi(Y,\eta)\right). \tag{4}$$

Collusion is stable when the profit realization is sufficiently low. Given all other firms collude, a firm earns π by also colluding and has an expected continuation payoff of $(1-\sigma)Y + \sigma(W-F)$ since with probability σ it is caught and convicted in which case the industry shifts to the non-cartel state and each firm pays a penalty of F.[4] Note that the expected penalty does not impact the ICC because it is unaffected by whether a firm cheats or colludes. The presumption is that discovery depends only on whether firms attempted to coordinate this period and not on whether such coordination was successful (that is, a firm doesn't believe its act of cheating alters the

[4] I will later substitute $\gamma(Y-\alpha\mu)$ for F.

likelihood of paying penalties in the current period). This is clearly a simplification and indeed there have been analyses modelling how the realized price path influences the likelihood of detection and feeds back to impact the optimal cartel price path (Harrington, 2004, 2005; Harrington and Chen, 2005).

The expected (rescaled) payoff to being cartelized can now be recursively defined. Given an industry is cartelized and given a profit realization $\pi \leq \phi(Y, \eta)$, each firm earns profit of π and the cartel is caught with probability σ in which case each receives the future non-collusive payoff W less the penalty F; and if not caught each earns the future collusive value Y. If instead $\phi(Y, \eta) < \pi$ then the cartel collapses so each firm earns $\alpha\pi$ and the future value is W less expected penalties.[5] The following equation then defines the implied collusive value, $\psi(Y)$, when firms perceive it to be Y.

$$\begin{aligned} \psi(Y) &= \int_{\underline{\pi}}^{\frac{\delta(1-\sigma)(Y-W)}{(1-\delta)(\eta-1)}} \left[(1-\delta)\pi + \delta Y - \delta\sigma(Y-W)\right] h(\pi)\, d\pi \\ &\quad + \int_{\frac{\delta(1-\sigma)(Y-W)}{(1-\delta)(\eta-1)}}^{\overline{\pi}} \left[(1-\delta)\alpha\pi + \delta W\right] h(\pi)\, d\pi - \delta\sigma F. \end{aligned} \tag{5}$$

To derive an expression in only one unknown, Y, substitute for $(Y - W)$ using (3) and for W using (2). Also, replace F with $\gamma(Y - \alpha\mu)$.

$$\begin{aligned} \psi(Y) &= \int_{\underline{\pi}}^{\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}} \left[(1-\delta)\pi + \delta Y - \left(\frac{\delta\sigma(1-\kappa)(1-\delta)(Y-\alpha\mu)}{1-\delta(1-\kappa)}\right)\right] h(\pi)\, d\pi \\ &\quad + \int_{\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}}^{\overline{\pi}} \left[(1-\delta)\alpha\pi + \delta\left(\frac{(1-\kappa)(1-\delta)\alpha\mu + \kappa Y}{1-\delta(1-\kappa)}\right)\right] h(\pi)\, d\pi - \delta\sigma\gamma(Y-\alpha\mu). \end{aligned} \tag{6}$$

A fixed point to ψ is an equilibrium value for Y. That is, given an anticipated collusive value Y, the resulting equilibrium behavior results in that same value when firms are cartelized.

As an initial step to exploring the set of fixed points, first note that $\psi(\alpha\mu) = \alpha\mu$. Hence, one fixed point to ψ is the degenerate solution without collusion. It is straightforward to show $\psi(\mu) \leq \mu$ and, if $\sigma > 0$ and $\gamma > 0$, then $\psi(\mu) < \mu$. The issue is whether there is another fixed point - in which case $Y > \alpha\mu$ - and thus firms are cartelizing with positive probability. Towards establishing when that is the case, Theorem 1 will prove useful. Proofs are in the appendix.

Theorem 1 *If* $\frac{\kappa}{1-\delta(1-\kappa)} > \sigma\gamma$ *then: i)* $\psi : [\alpha\mu, \mu] \rightarrow [\alpha\mu, \mu]$; *ii)* $\psi'(Y) \geq 0,\ \forall Y \in [\alpha\mu, \mu)$; *and iii) if* $\phi(Y, \eta) \in (\underline{\pi}, \overline{\pi})$ *then* $\psi'(Y) > 0$.

Thus, if the probability of discovery and conviction, σ, and/or the penalty multiple, γ, are sufficiently low then ψ maps $[\alpha\mu, \mu]$ into itself and is an increasing function

[5] One might expect the probability of being caught to be lower if firms are not currently colluding (though did collude in the previous period). This I consider a reasonable approximation in that the difference in probability is likely to be small and making this assumption simplifies the analysis.

of Y. From hereon, assume:

$$\frac{\kappa}{1-\delta(1-\kappa)} > \sigma\gamma.$$

Define Y^* to be the maximal fixed point of ψ:

$$Y^*(\eta) \equiv \max\{Y \in [\alpha\mu, \mu] : \psi(Y;\eta) = Y\}. \tag{7}$$

A collusive solution is said to exist when $Y^* > \alpha\mu$. The next result shows that, when the probability of having to pay penalties is sufficiently low, a cartel forms with positive probability when η is sufficiently low. Otherwise, a cartel never forms.

Theorem 2 *If σ is sufficiently close to zero then $\exists\widehat{\eta} > 1$, such that*

$$Y^*(\eta) \begin{cases} \in (\alpha\mu, \mu] & \text{if } \eta \in (1, \widehat{\eta}] \\ = \alpha\mu & \text{if } \eta > \widehat{\eta} \end{cases}$$

From hereon, assume σ is sufficiently low and $\underline{\eta}$ is sufficiently close to one so that a cartel forms for sufficiently low values of η (that is, $\widehat{\eta} > \underline{\eta}$) and, in addition, does not form for sufficiently high values of η (that is, $\overline{\eta}$ is sufficiently high so that $\widehat{\eta} < \overline{\eta}$).

If there are multiple interior solutions to (6), I make the usual selection that firms achieve the equilibrium with the highest value, which is $Y^*(\eta)$. Given $Y^*(\eta)$, define $\phi^*(\eta)$ as the maximum profit realization such that the cartel is stable:

$$\phi^*(\eta) \equiv \frac{\delta(1-\sigma)(1-\kappa)[Y^*(\eta) - \alpha\mu]}{[1-\delta(1-\kappa)](\eta-1)}. \tag{8}$$

$\phi^*(\eta)$ is a measure of cartel stability since the cartel is stable iff $\pi \leq \phi^*(\eta)$. This can be seen more clearly by noting that the probability a cartel survives in any period is $(1-\sigma) H(\phi^*(\eta))$ which is non-decreasing in $\phi^*(\eta)$.

3.2 Comparative Statics

Theorem 3 provides some intuitive comparative statics regarding the value to cartelizing, $Y^*(\eta)$, and the measure of cartel stability, $\phi^*(\eta)$.

Theorem 3 *$Y^*(\eta)$ is non-increasing in σ, γ, and η. If $Y^*(\eta) \in (\alpha\mu, \mu)$ then $Y^*(\eta)$ is decreasing in σ and γ. If $\phi^*(\eta) \in (\underline{\pi}, \overline{\pi})$ then $Y^*(\eta)$ is decreasing in η and $\phi^*(\eta)$ is decreasing in σ, γ, η, and α.*

When antitrust policy is made tougher - as reflected in a higher value for σ or γ - the value to forming a cartel, $Y^*(\eta)$, is reduced. There is both a direct effect and an indirect effect underlying this result. The direct effect is that the expected penalty is higher - by making conviction more likely or increasing the penalty multiple - and this lowers the collusive value, holding firm behavior fixed. This higher expected penalty induces an indirect (behavioral) effect as it reduces the range of profit realizations for which the cartel is stable (that is, $\phi^*(\eta)$ is decreasing in σ and γ). Therefore, firms expect a shorter duration from forming a cartel and this also serves to reduce the collusive value. Hence, both the direct and indirect effects work to reduce the value to being cartelized and the stability of the cartel. With regards to the effect of the industry type η, only the indirect effect is operative. There is a greater payoff to cheating for an industry with a higher value for η and this translates into a less stable cartel and a lower value to being cartelized. A final parameter of interest is the non-collusive profit rate which is controlled by α. As α rises, the expected incremental profit gain from colluding, $(1-\alpha)\mu$, is reduced and this weakens the incentive to form a cartel and thereby reduces cartel stability.[6]

Recall from Theorem 2 that if η is sufficiently high then an industry will not cartelize for any profit state; only industries for which $\eta \leq \widehat{\eta}$ can potentially cartelize. The next result shows that when antitrust policy is tougher or the non-collusive profit is higher, there is a smaller set of industries that are able to cartelize. The intuition is the same as with Theorem 3.

Theorem 4 *$\widehat{\eta}$ is decreasing in σ, γ, and α.*

4 Stationary Distribution on Cartels

Using the preceding analysis, the stochastic process by which cartels are born and die (either through internal collapse or being caught) is characterized in this section. The random events driving this process are: i) the opportunity to cartelize; ii) the profit conditions; and iii) detection by the antitrust authorities. I initially characterize the stationary distribution for the subset of type-η industries. The stationary distribution for the entire population of industries is then characterized by integrating the distributions for the type-η industries.

Section 3 characterized behavior for a specific industry. There it was shown that if a type-η industry was not cartelized at the end of the previous period then it'll cartelize and collude in the current period with probability $\kappa H(\phi^*(\eta))$. If a type-η industry was cartelized at the end of the previous period then it'll still be cartelized at the end of this period with probability $(1-\sigma)H(\phi^*(\eta))$. Suppose there is a continuum of type-η industries with independent realizations of the stochastic events each period. The task is to characterize the stationary distribution with regards to the frequency and duration of cartels.

Let $\beta(l;\eta)$ denote the proportion of type-η industries with cartels of length $l \in \{0,1,2,\ldots\}$. $l=0$ means firms are not cartelized so that $1-\beta(0;\eta)$ is the fraction of cartels among type-η industries. To reduce the notational burden, η will often be suppressed. The stationary distribution is defined by the following set of equations:

$$\begin{aligned} \beta(0;\eta) &= \beta(0;\eta)\left[(1-\kappa)+\kappa(1-H(\phi^*))+\kappa\sigma H(\phi^*)\right] \\ &\quad +\left[1-\beta(0;\eta)\right]\left[(1-H(\phi^*))+\sigma H(\phi^*)\right] \end{aligned} \tag{9}$$

[6] The collusive value can be shown to be increasing in the non-collusive profit rate (which is controlled by α) but that is not very informative because the collusive value is also based on periods during which firms earn the non-collusive profit. More informative is that the incremental expected gain from colluding, $Y^*(\eta;\alpha)-\alpha\mu$, can be shown to be decreasing in α. It is the latter property that results in $\phi^*(\eta;\alpha)$ being decreasing in α.

$$\beta(1;\eta) = \beta(0;\eta)\,\kappa(1-\sigma)\,H(\phi^*) \tag{10}$$

$$\beta(l;\eta) = \beta(l-1;\eta)(1-\sigma)\,H(\phi^*),\, l \in \{2,3,\ldots\} \tag{11}$$

Note that η enters through ϕ^*. Considering the rhs of (9), a fraction $\beta(0;\eta)$ of type-η industries were not cartelized in the previous period. Out of those industries, a fraction $1-\kappa$ will not have the opportunity to cartelize and thus will not collude in the current period, while a fraction $\kappa(1-H(\phi^*))$ will have the opportunity but, due to a high profit realization, find it is not incentive compatible to collude. Of the industries that were colluding in the previous period, which have mass $1-\beta(0;\eta)$, a fraction $1-H(\phi^*)$ will collapse for internal reasons and a fraction $\sigma H(\phi^*)$ will instead be caught by the authorities and thus shutdown. Turning to (10), an industry can go from competing to colluding by being presented with the opportunity to cartelize, which occurs with probability κ, and having a sufficiently low profit realization, which occurs with probability $H(\phi^*)$. Finally, an existing cartel continues to collude - which means if it is of length $l-1$ then its length grows to l - if the profit realization is sufficiently low and it is not discovered and convicted; the joint probability of that event is $(1-\sigma)\,H(\phi^*)$ which gives us (11).

Solving (9) for $\beta(0;\eta)$:

$$\beta(0;\eta) = \frac{1-(1-\sigma)\,H(\phi^*)}{1-(1-\kappa)(1-\sigma)\,H(\phi^*)}. \tag{12}$$

In the steady-state, the fraction of cartels among type-η industries is then:

$$1-\beta(0;\eta) = \frac{\kappa(1-\sigma)\,H(\phi^*)}{1-(1-\kappa)(1-\sigma)\,H(\phi^*)}. \tag{13}$$

Next note that:

$$\beta(l;\eta) = \beta(0;\eta)\,\kappa\,[H(\phi)(1-\sigma)]^l,\ l \geq 1. \tag{14}$$

A mass $\beta(0;\eta)\,\kappa$ of industries get the chance to form a cartel and a fraction of $[H(\phi)(1-\sigma)]^l$ will still be cartelized l periods later. Using (12), we can substitute for $\beta(0;\eta)$ in (14):

$$\beta(l;\eta) = \frac{[1-(1-\sigma)\,H(\phi^*)]\,\kappa\,[H(\phi^*)(1-\sigma)]^l}{1-(1-\kappa)(1-\sigma)\,H(\phi^*)},\ l \geq 1.$$

Next define the probability distribution over cartel length among cartels, $\{f(l;\eta)\}_{l=1}^{\infty}$.

$$\begin{aligned}
f(l;\eta) &\equiv \frac{\beta(l;\eta)}{1-\beta(0;\eta)},\ l \geq 1 \\
&= \left(\frac{[1-(1-\sigma)\,H(\phi^*)]\,\kappa\,[H(\phi^*)(1-\sigma)]^l}{1-(1-\kappa)(1-\sigma)\,H(\phi^*)}\right)\left(\frac{\kappa(1-\sigma)\,H(\phi^*)}{1-(1-\kappa)(1-\sigma)\,H(\phi^*)}\right)^{-1} \\
&= [1-(1-\sigma)\,H(\phi^*)]\,[(1-\sigma)\,H(\phi^*)]^{l-1} \\
&= [(1-\sigma)\,H(\phi^*)]^{l-1} - [(1-\sigma)\,H(\phi^*)]^l
\end{aligned}$$

The average cartel length is then:

$$\sum_{l=1}^{\infty} lf\left(l;\eta\right) = \frac{1}{1-\left(1-\sigma\right)H\left(\phi^*\right)}.$$

Finally, I turn to the entire population of industries by integrating the type-η distribution over types, $\eta \in \left[\underline{\eta}, \overline{\eta}\right]$. $\left\{\widetilde{\beta}\left(l\right)\right\}_{l=0}^{\infty}$ denotes the stationary distribution on cartel length where

$$\widetilde{\beta}\left(l\right) = \int_{\underline{\eta}}^{\overline{\eta}} \beta\left(l;\eta\right) g\left(\eta\right) d\eta,\ \ l = 0, 1, 2, \ldots$$

Of particular relevance is the frequency with which industries are cartelized:

$$\begin{aligned} 1-\widetilde{\beta}\left(0\right) &= \int_{\underline{\eta}}^{\widehat{\eta}} \left[1-\beta\left(0;\eta\right)\right] g\left(\eta\right) d\eta \\ &= \int_{\underline{\eta}}^{\widehat{\eta}} \left[\frac{\kappa\left(1-\sigma\right)H\left(\phi^*\left(\eta\right)\right)}{1-\left(1-\kappa\right)\left(1-\sigma\right)H\left(\phi^*\left(\eta\right)\right)}\right] g\left(\eta\right) d\eta, \end{aligned}$$

and the frequency of discovered cartels:

$$\sigma\left[1-\widetilde{\beta}\left(0\right)\right] = \sigma\int_{\underline{\eta}}^{\widehat{\eta}} \left[\frac{\kappa\left(1-\sigma\right)H\left(\phi^*\left(\eta\right)\right)}{1-\left(1-\kappa\right)\left(1-\sigma\right)H\left(\phi^*\left(\eta\right)\right)}\right] g\left(\eta\right) d\eta.$$

5 Evaluating the Impact of Antitrust Policy

In this section, the two main questions motivating this study are tackled. First, what is the effect of antitrust policy on the cartel rate? Second, to what extent can one infer the impact of antitrust policy on the cartel rate from observing its impact on the discovered cartel rate?

The next result shows that when penalties for collusion are increased, there are both fewer cartels and fewer discovered cartels according to the stationary distribution. This is not surprising since more severe penalties result in less stable cartels - as $\phi^*\left(\eta\right)$ is decreasing in γ by Theorem 3 - which means that a given industry is in the cartelized state a smaller fraction of the time. Furthermore, by Theorem 4, $\widehat{\eta}$ is decreasing in γ which means that a smaller fraction of industries are ever able to cartelize.

Theorem 5 *The rate of cartel formation,* $1-\widetilde{\beta}\left(0\right)$, *and the rate of discovered cartels,* $\sigma\left[1-\widetilde{\beta}\left(0\right)\right]$, *are decreasing in the severity of penalties,* γ.

Let us now use these results to infer the impact of a new policy on the unobserved rate of cartels by observing the rate of discovered cartels. (This exercise is straightforward but I bother to go through it in preparation for a later discussion.) Suppose there is a change in the penalty policy which corresponds to a change in γ from γ'

to γ'' but there is uncertainty as to the impact of the policy. It is possible that the policy has lowered the rate of cartel formation which, by our previous results, occurs if $\gamma'' > \gamma'$. It is also possible that the policy might have some perverse consequences so that $\gamma'' < \gamma'$ and the cartel rate has risen. Or the policy is totally ineffective so that $\gamma'' = \gamma'$.[7] Ultimately, the interest is not *per se* in γ'' but rather in how the new policy has affected the cartel rate.

To infer the policy impact on the cartel rate from the discovered cartel rate, consider the ratio of the new to the old steady-state discovered cartel rate:

$$\frac{\sigma\left[1-\widetilde{\beta}\left(0;\gamma''\right)\right]}{\sigma\left[1-\widetilde{\beta}\left(0;\gamma'\right)\right]}=\frac{1-\widetilde{\beta}\left(0;\gamma''\right)}{1-\widetilde{\beta}\left(0;\gamma'\right)}.$$

It is then obvious that the rate of cartels rises (falls) if and only if the rate of discovered cartels rises (falls). One can then at least conclude whether or not the policy change has had a favorable impact on the number of cartels by observing the number of discovered cartels. Theorem 5 provides the basis for using the change in the rate of the discovered cartels to infer something about the change in the rate of cartels.

Next consider the effect of a policy change that is designed to affect the probability of discovery and conviction. While a tougher enforcement policy - as reflected in a higher value for σ - reduces the cartel rate, its impact on the rate of discovered cartels is ambiguous.

Theorem 6 *The rate of cartel formation,* $1-\widetilde{\beta}\left(0\right)$, *is decreasing in* σ *but the effect of* σ *on the rate of discovered cartels,* $\sigma\left[1-\widetilde{\beta}\left(0\right)\right]$, *is ambiguous (that is, its sign depends on parameter values).*

The proof of Theorem 6 shows that the cartel rate is decreasing in σ and we'll argue here that σ has an ambiguous effect on the rate of discovered cartels. Raising σ from σ' to σ'' impacts the rate of discovered cartels by an amount equal to:

$$\sigma''\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')}\left[1-\beta\left(0;\eta,\sigma''\right)\right]g\left(\eta\right)d\eta-\sigma'\int_{\underline{\eta}}^{\widehat{\eta}(\sigma')}\left[1-\beta\left(0;\eta,\sigma'\right)\right]g\left(\eta\right)d\eta.$$

This can be re-arranged so that the frequency of discovered cartels goes up if and only if (iff):

$$\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')}\left[\sigma''\left(1-\beta\left(0;\eta,\sigma''\right)\right)-\sigma'\left(1-\beta\left(0;\eta,\sigma'\right)\right)\right]g\left(\eta\right)d\eta>\sigma'\int_{\widehat{\eta}(\sigma'')}^{\widehat{\eta}(\sigma')}\left[1-\beta\left(0;\eta,\sigma'\right)\right]g\left(\eta\right)d\eta. \tag{15}$$

The rhs term is the reduction in discovered cartels because the marginally stable cartels no longer form (or, if they did form, they now collapse) and thus are not there to be caught. The lhs is the change in the rate of discovered cartels among

[7] Cyrenne (1999), Harrington (2004), and Chen and Harrington (2005) show how antitrust policy can have the perverse effect of making collusion more stable.

those cartels that continue to form in spite of the higher chance of paying penalties. Depending on σ' and σ'', (15) could either hold or not hold so that a rise in the probability of detection could either raise or lower the number of discovered cartels.

To show that both events are possible, let us hold $\sigma''(>0)$ fixed and make the initial probability of conviction sufficiently small ($\sigma' \simeq 0$). As $\sigma' \to 0$, (15) holds as the rhs goes to zero and the lhs is bounded above zero. Not surprisingly, the frequency of discovered cartels rises when the probability of conviction is raised from an initial level close to zero. Now suppose instead σ' is held fixed and is sufficiently small so that and $\widehat{\eta}(\sigma') > \underline{\eta}$; hence, some cartels form. Consider what happens when σ'' goes to one. Recall

$$\begin{aligned}\psi(Y) &= \int_{\underline{\pi}}^{\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}} \left[(1-\delta)\pi + \delta Y - \left(\frac{\delta\sigma(1-\kappa)(1-\delta)(Y-\alpha\mu)}{1-\delta(1-\kappa)}\right)\right] h(\pi)\, d\pi \\ &+ \int_{\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}}^{\overline{\pi}} \left[(1-\delta)\alpha\pi + \delta\left(\frac{(1-\kappa)(1-\delta)\alpha\mu + \kappa Y}{1-\delta(1-\kappa)}\right)\right] h(\pi)\, d\pi - \delta\sigma\gamma(Y-\alpha\mu).\end{aligned}$$

For $Y - \alpha\mu > 0$,

$$\lim_{\sigma\to 1} \frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)} = 0$$

which implies

$$\begin{aligned}\lim_{\sigma\to 1}\psi(Y) &= \int_{\underline{\pi}}^{\overline{\pi}} \left[(1-\delta)\alpha\pi + \delta\left(\frac{(1-\kappa)(1-\delta)\alpha\mu + \kappa Y}{1-\delta(1-\kappa)}\right)\right] h(\pi)\, d\pi - \delta\gamma(Y-\alpha\mu) \\ &= (1-\delta)\alpha\mu + \delta\left(\frac{(1-\kappa)(1-\delta)\alpha\mu + \kappa Y}{1-\delta(1-\kappa)}\right) - \delta\gamma(Y-\alpha\mu).\end{aligned}$$

If $\gamma = 0$ then

$$\begin{aligned}\lim_{\sigma\to 1}\psi(Y) &= (1-\delta)\alpha\mu + \delta\left(\frac{(1-\kappa)(1-\delta)\alpha\mu + \kappa Y}{1-\delta(1-\kappa)}\right) \\ &= \frac{(1-\delta)\alpha\mu + \delta\kappa Y}{1-\delta(1-\kappa)} \in (\alpha\mu, Y).\end{aligned}$$

Since $\psi(Y)$ is decreasing in γ then: if $Y > \alpha\mu$ then $\lim_{\sigma\to 1}\psi(Y) < Y$. This implies $Y^*(\eta) = \alpha\mu\ \forall\eta > 1$ as $\sigma \to 1$. Hence, $\lim_{\sigma\to 1}\widehat{\eta}(\sigma) = 1$ so the cartel rate is zero when σ'' is sufficiently close to one. Since $\beta(0;\eta,\sigma'') = 1\ \forall\eta > 1$, (15) does not hold since the rhs is positive and the lhs is zero.

In sum, suppose $\sigma'' > \sigma'$ so that the policy change makes detection and conviction more likely. This will reduce the number of cartels but its effect on the number of discovered cartels is unclear. If σ' is sufficiently close to zero that the policy change will raise the number of discovered cartels. If instead σ'' is sufficiently close to one then the policy change will reduce the number of discovered cartels.

From this result, it is easy to see that it is problematic to infer the effect of a policy change on the latent cartel rate by measuring what has happened to the observed rate of discovered cartels. Suppose a policy change has caused the probability of detection and conviction to change from σ' to σ'' but the values for σ' and σ'' are unknown. The task is to infer whether $\sigma'' > \sigma'$ (which would imply an efficacious policy and thus fewer cartels), $\sigma'' = \sigma'$ (an ineffective policy and thus no change in the number of cartels), or $\sigma'' < \sigma'$ (a perverse policy and thus more cartels). If, in fact, $\sigma'' > \sigma'$ then, depending on the values for σ' and σ'', the previous analysis tells us that the number of discovered cartels could go up or down. This is similarly the case when $\sigma'' < \sigma'$. Without additional information, one cannot infer from the direction of change in the number of discovered cartels, whether the policy change has raised or lowered the cartel rate. It is then difficult to assess the efficacy of a policy that is intended to make detection and conviction more likely.[8]

However, using the stationary distribution, one can derive an observable and unambiguous implication of a more effective detection policy by considering, not the number of discovered cartels, but rather the *duration* of discovered cartels.

The stationary distribution on cartel length, conditional on being cartelized ($l \geq 1$), is:

$$\widetilde{f}(l) \equiv \frac{\int_{\underline{\eta}}^{\widehat{\eta}} \beta(l;\eta) g(\eta) d\eta}{\int_{\underline{\eta}}^{\widehat{\eta}} [1-\beta(0;\eta)] g(\eta) d\eta}.$$

Perform the following steps:

$$\begin{aligned}
\widetilde{f}(l) &= \int_{\underline{\eta}}^{\widehat{\eta}(\sigma)} \left[\frac{\beta(l;\eta) g(\eta) d\eta}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma)} [1-\beta(0;\eta')] g(\eta') d\eta'} \right] \\
&= \int_{\underline{\eta}}^{\widehat{\eta}(\sigma)} \left(\frac{\beta(l;\eta,\sigma)}{1-\beta(0;\eta,\sigma)} \right) \left[\frac{(1-\beta(0;\eta,\sigma)) g(\eta)}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma)} [1-\beta(0;\eta',\sigma)] g(\eta') d\eta'} \right] d\eta \\
&= \int_{\underline{\eta}}^{\widehat{\eta}(\sigma)} f(l;\eta) \left[\frac{(1-\beta(0;\eta,\sigma)) g(\eta)}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma)} [1-\beta(0;\eta',\sigma)] g(\eta') d\eta'} \right] d\eta
\end{aligned}$$

$\widetilde{f}(l)$ is then a weighted average of $f(l;\eta)$ where the weight assigned to $f(l;\eta)$ is the fraction of all cartels that are of type η.

In considering the impact of raising σ from σ' to σ'' on $\left\{\widetilde{f}(l)\right\}_{l=1}^{\infty}$, I will break it apart into short-run and long-run effects. The short-run effect on the distribution over the duration of discovered cartels is from the immediate collapse of some cartels upon the institution of a more aggressive detection and conviction policy. The long-run effect is the change in the distribution on duration as it converges to the new stationary distribution. This can be made more concrete. Since $\widehat{\eta}$ is decreasing in σ, cartels for which $\eta \in (\widehat{\eta}(\sigma''), \widehat{\eta}(\sigma')]$ are no longer stable (for any profit realizations) after σ is raised and thereby immediately collapse. Thus, the policy change induces

[8] Or, alternatively, a policy that involves a change in multiple parameters. For example, one interpretation of the institution of a leniency program is that it both raises σ and lowers γ.

an immediate shift in the mass of cartels from

$$\int_{\underline{\eta}}^{\widehat{\eta}(\sigma')} \left[1 - \beta\left(0; \eta, \sigma'\right)\right] g(\eta)\, d\eta$$

to

$$\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')} \left[1 - \beta\left(0; \eta, \sigma'\right)\right] g(\eta)\, d\eta.$$

The distribution on discovered (and, for that matter, undiscovered) cartel duration shifts, in the short-run, from

$$\widetilde{f}\left(l; \sigma'\right) \equiv \int_{\underline{\eta}}^{\widehat{\eta}(\sigma')} f\left(l; \eta, \sigma'\right) \left[\frac{(1 - \beta(0; \eta, \sigma'))\, g(\eta)}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma')} [1 - \beta(0; \eta', \sigma')]\, g(\eta')\, d\eta'}\right] d\eta$$

to

$$\overline{f}\left(l; \sigma', \sigma''\right) \equiv \int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')} f\left(l; \eta, \sigma'\right) \left[\frac{(1 - \beta(0; \eta, \sigma'))\, g(\eta)}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')} [1 - \beta(0; \eta', \sigma')]\, g(\eta')\, d\eta'}\right] d\eta.$$

Notice that the relative weight on $f(l; \eta, \sigma')$, for $\eta \in [\underline{\eta}, \widehat{\eta}(\sigma'')]$, is unchanged.

Theorem 7 *If $\sigma'' > \sigma'$ then $\overline{f}(l; \sigma', \sigma'')$ first-order stochastically dominates (FOSD) $\widetilde{f}(l; \sigma')$. In other words, in response to a more aggressive detection and conviction policy, the duration of discovered cartels increases (in terms of FOSD) in the short-run.*

To sum, a rise in σ causes the immediate collapse of the least stable cartels (due to $\widehat{\eta}$ being decreasing in σ). This means the surviving cartels are those with lower η and thus longer duration. Since this is the pool from which one draws discovered cartels, the average duration of discovered cartels rises in the short-run in response to a more aggressive detection and conviction policy.

The transition from the short-run to the (new) long-run involves the distribution on cartel length shifting from

$$\overline{f}\left(l; \sigma', \sigma''\right) = \int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')} f\left(l; \eta, \sigma'\right) \left[\frac{(1 - \beta(0; \eta, \sigma'))\, g(\eta)}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')} [1 - \beta(0; \eta', \sigma')]\, g(\eta')\, d\eta'}\right] d\eta$$

to

$$\widetilde{f}\left(l; \sigma''\right) = \int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')} f\left(l; \eta, \sigma''\right) \left[\frac{(1 - \beta(0; \eta, \sigma''))\, g(\eta)}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')} [1 - \beta(0; \eta', \sigma'')]\, g(\eta')\, d\eta'}\right] d\eta.$$

The latter is the stationary distribution on cartel duration when $\sigma = \sigma''$.

Theorem 8 *If $\sigma'' > \sigma'$ then $\overline{f}(l; \sigma', \sigma'')$ FOSD $\widetilde{f}(l; \sigma'')$. In other words, in response to a more aggressive detection and conviction policy, the duration of discovered cartels increases (in terms of FOSD) as the industry goes from the short-run to the long-run.*

In response to a policy that alters the likelihood of detection and conviction, its effect on the rate of cartels can be inferred by observing the duration of discovered cartels in the short-run. If average cartel duration goes up (down) then the policy has caused σ to rise (fall) and thus we can conclude that it'll result in fewer (more) cartels forming in the new steady-state.

The empirical question is whether some change in antitrust policy - say the 1993 revision of the U.S. Corporate Leniency Program - translates into a substantive rise in σ or whether it is ineffective so that σ remains largely unaffected. If such a change is ineffective then there will be no observed effect on the distribution of cartel duration for discovered cartels, either in the short-run or long-run. However, if it is having the intended effect - that is, firms perceive σ as having risen - it can be measured by the impact on the distribution of cartel duration for discovered cartels. Discovered cartels should have longer duration in the short-run but shorter duration in the long-run. Note that these effects are monotonic in the size of the change in σ so the bigger is the change in σ, the bigger are the short-run and long-run effects.

6 Concluding Remarks

In the last 15 years, major policy changes in the manner in which cartels are discovered, prosecuted, and penalized have occurred in the United States, European Union, and the rest of the world. Though there are many intermediate measures of the impact of these policies - most notably the number of convictions, the number of leniency applications, and the penalties imposed - the true measure of interest is the number of cartels in existence. Success and failure of a policy is ultimately concluded according to the extent to which cartels are deterred from forming. To our knowledge, this paper is the first to attempt to infer the impact of policy on the latent cartel rate using the observed population of discovered cartels. Though the results are tentative in light of the simplicity of our model, it does show that theory may be able to shed light on the impact of policy innovations and inject some needed substance into the policy debate.

There are many possible extensions of this framework. In markets lacking significant entry barriers, an important constraint on collusion is the prospect of either entry or expansion by small non-cartel members. Though the threat of entry was not a constraint for most of the markets controlled by the vitamins cartel, it was in the case of vitamin C where expansion by Chinese suppliers eventually disrupted collusion. One extension of our framework is to allow for industry heterogeneity with respect to entry barriers and then endogenize the effect of cartel formation on the number of firms.

Formally modelling the leniency program would be a particularly valuable extension in that other work has shown how it can impact incentive compatibility constraints and thereby cartel formation and duration (see the survey by Spagnolo, 2006). Leniency programs differ throughout the world and it is important to try to measure the impact of these differences. The availability of partial leniency to more than one firm is provided in many venues such as the EU, while the U.S. has only full leniency to the first firm though has informal methods of partial leniency through plea bargaining. Penalties vary greatly internationally. Most countries limit penalties to government fines which are often calculated in ways unrelated to the profits created by collusion. Though present in only a few countries, private customer damages are seriously being considered in the EU at present. The U.S. along with a few other countries like Canada, Norway, and most recently the United Kingdom, have criminalized price-fixing so that offenders can be placed in jail. Furthermore, there have been significant legislative changes over time; both maximal fines and prison sentences increased in the U.S. in the last few years. The type of framework presented in this paper may be modifiable to derive ways in which to measure the impact of the penalty structure on the cartel rate.

Currently, there is an active policy debate in many countries as to the design of anti-cartel policies. It is absolutely vital that economists play a role in that debate in order to ensure that sound policies are implemented and those policies that are implemented are properly evaluated as to their impact. This is an admittedly a difficult exercise but the alternative is to allow the debate to be dominated by casual and potentially misleading measures such as the number of convictions. Indeed, one would hope that a highly successful policy would ultimately be measured by the absence of convictions because fewer cartels are forming. But then we're always left with distinguishing such an absence from simply an ineffective policy; that is, there are still plenty of cartels, we're just not catching and convicting them. It is the importance of making that distinction that motivates this line of work and it is my hope that this paper will encourage others to venture into this arena and make more progress on this important economic and policy issue.

7 Appendix

Proof of Theorem 1. To ensure that $\psi : [\alpha\mu, \mu] \to [\alpha\mu, \mu]$, it is sufficient to show that $\psi'(Y) \geq 0, \forall Y$ since $\psi(\alpha\mu) = \alpha\mu$ and $\psi(\mu) \leq \mu$. Recall that

$$\begin{aligned}\psi(Y) &= \int_{\underline{\pi}}^{\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}} [(1-\delta)\pi + \delta Y - \delta\sigma(Y-W)]\, h(\pi)\, d\pi \\ &\quad + \int_{\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}}^{\overline{\pi}} [(1-\delta)\alpha\pi + \delta W]\, h(\pi)\, d\pi - \delta\sigma F.\end{aligned}$$

If

$$\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)} < \underline{\pi}$$

then

$$\psi(Y) = \int_{\underline{\pi}}^{\overline{\pi}} [(1-\delta)\alpha\pi + \delta W]\, h(\pi)\, d\pi - \delta\sigma F$$

and thus $\psi'(Y) = 0$. If

$$\overline{\pi} < \frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}$$

then

$$\psi(Y) = \int_{\underline{\pi}}^{\overline{\pi}} [(1-\delta)\pi + \delta Y - \delta\sigma(Y-W)]\, h(\pi)\, d\pi - \delta\sigma F$$

and thus $\psi'(Y) > 0$.

Let us now suppose

$$\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)} \in (\underline{\pi}, \overline{\pi}),$$

then:

$$\begin{aligned}\psi'(Y) &= \left[(1-\delta)\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right) + \delta Y - \delta\sigma\left(\frac{(1-\kappa)(1-\delta)(Y-\alpha\mu)}{1-\delta(1-\kappa)}\right)\right] \times \\ &\quad h\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right)\left(\frac{\delta(1-\sigma)(1-\kappa)}{[1-\delta(1-\kappa)](\eta-1)}\right) \\ &\quad + \int_{\underline{\pi}}^{\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}} \left[\delta - \delta\sigma\left(\frac{(1-\delta)(1-\kappa)}{1-\delta(1-\kappa)}\right)\right] h(\pi)\, d\pi \\ &\quad - \left[(1-\delta)\alpha\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right) + \delta\left(\frac{(1-\kappa)(1-\delta)\alpha\mu + \kappa Y}{1-\delta(1-\kappa)}\right)\right] \times \\ &\quad h\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right)\left(\frac{\delta(1-\sigma)(1-\kappa)}{[1-\delta(1-\kappa)](\eta-1)}\right) \\ &\quad + \int_{\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}}^{\overline{\pi}} \left(\frac{\delta\kappa}{1-\delta(1-\kappa)}\right) h(\pi)\, d\pi - \delta\sigma\gamma.\end{aligned}$$

Simplifying this expression yields:

$$\begin{aligned}
\psi'(Y) &= h\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right)\left(\frac{\delta(1-\sigma)(1-\kappa)}{[1-\delta(1-\kappa)](\eta-1)}\right)^2 \times \\
& (1-\delta)(\eta-\alpha)(Y-\alpha\mu) \\
& +\delta\left(\frac{(1-\delta)(1-\kappa)(1-\sigma)}{1-\delta(1-\kappa)}\right) H\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right) \\
& +\left(\frac{\delta\kappa}{1-\delta(1-\kappa)}\right) - \delta\sigma\gamma.
\end{aligned}$$

If

$$\frac{\kappa}{1-\delta(1-\kappa)} > \sigma\gamma$$

then $\psi'(Y) > 0$. Note that

$$\psi'(\alpha\mu) = \left(\frac{\delta\kappa}{1-\delta(1-\kappa)}\right) - \delta\sigma\gamma \in (0,1).$$

■

Proof of Theorem 2. Let us first prove that a collusive solution exists for η close to 1. First note that if $Y-\alpha\mu>0$ then

$$\lim_{\eta\to 1}\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)} = +\infty.$$

Hence, $\lim_{\eta\to 1}\phi(Y,\eta) > \overline{\pi}$ which implies:

$$\lim_{\eta\to 1}\psi(Y) = \int_{\underline{\pi}}^{\overline{\pi}}\left[(1-\delta)\pi+\delta Y-\left(\frac{\delta\sigma(1-\kappa)(1-\delta)(Y-\alpha\mu)}{1-\delta(1-\kappa)}\right)\right]h(\pi)\,d\pi - \delta\sigma\gamma(Y-\alpha\mu).$$

Next note:

$$\lim_{\sigma\to 0}\lim_{\eta\to 1}\psi(Y) = \int_{\underline{\pi}}^{\overline{\pi}}[(1-\delta)\pi+\delta Y]\,h(\pi)\,d\pi,$$

where the order of limits doesn't matter. Since $(1-\delta)\mu+\delta Y > Y$ iff $Y<\mu$ then: if $Y\in(\alpha\mu,\mu)$ then $\lim_{\sigma\to 0}\lim_{\eta\to 1}\psi(Y) > Y$. Thus, if $Y<\mu$ then $\psi(Y)>Y$ for η sufficiently close to one and σ sufficiently close to zero. By continuity, $\exists Y\in(\alpha\mu,\mu]$ such that $\psi(Y)=Y$. We then have: If σ is sufficiently close to zero then $\exists\widehat{\eta}>1$ such that $Y^*(\eta)\in(\alpha\mu,\mu]\ \forall\eta\in(1,\widehat{\eta}]$.

The next step is to prove that a collusive solution does not exist for large enough η. We want to show that

$$\psi(Y) < Y\ \forall Y > \alpha\mu$$

when $\eta\to+\infty$. The ICC is:

$$(1-\delta)\pi+\delta(1-\sigma)Y+\delta\sigma(W-F) \geq (1-\delta)\eta\pi+\delta W-\delta\sigma F \Leftrightarrow$$

$$\delta(1-\sigma)(Y-W) \geq (1-\delta)(\eta-1)\pi \tag{16}$$

which is decreasing in π and W and increasing in Y. Since $\pi \geq \underline{\pi}\,(>0)$, $W \geq \alpha\mu$, and $Y \leq \mu$ then a sufficient condition for (16) not to hold $\forall\pi$ is:

$$\begin{aligned} \delta(1-\sigma)(1-\alpha)\mu &< (1-\delta)(\eta-1)\underline{\pi} \Leftrightarrow \\ \frac{(1-\delta)\underline{\pi}+\delta(1-\sigma)(1-\alpha)\mu}{(1-\delta)\underline{\pi}} &< \eta. \end{aligned}$$

Thus, $\exists\eta'$ such that a cartel never forms when $\eta > \eta'$.

To sum, $\exists\widehat{\eta} > 1$ such that $Y^*(\eta) \in (\alpha\mu, \mu]$ $\forall\eta \in (1, \widehat{\eta}]$ and $\exists\eta' > \widehat{\eta}$ such that $\psi(Y;\eta') < Y\ \forall Y > \alpha\mu\ \forall\eta > \eta'$ and thus $Y^*(\eta) = \alpha\mu\ \forall\eta > \eta'$. It is shown in the proof of Theorem 3 that $\psi(Y)$ is decreasing in η. Hence, if $\psi(Y;\eta'') < Y\ \forall Y > \alpha\mu$ then $\psi(Y;\eta) < Y\ \forall Y > \alpha\mu\ \forall\eta > \eta''$. We conclude that $\exists\widehat{\eta} > 1$ such that $Y^*(\eta) \in (\alpha\mu, \mu]$ $\forall\eta \in (1, \widehat{\eta}]$ and $Y^*(\eta') = \alpha\mu\ \forall\eta > \widehat{\eta}$. ■

Proof of Theorem 3. If $Y^*(\eta) \in \{\alpha\mu, \mu\}$ then the maximal fixed point is a corner solution in which case, generically, marginal changes in parameters do not affect $Y^*(\eta)$. For the remainder of the proof suppose $Y^*(\eta) \in (\alpha\mu, \mu)$ and further suppose, initially, that $\phi \in (\underline{\pi}, \overline{\pi})$.

To explore the effect of σ on $Y^*(\eta)$, consider:

$$\begin{aligned} \frac{\partial\psi(Y)}{\partial\sigma} = & -\Bigg[(1-\delta)\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right) \\ & + \frac{\delta[1-\delta(1-\kappa)]Y - \delta\sigma(1-\kappa)(1-\delta)(Y-\alpha\mu)}{1-\delta(1-\kappa)}\Bigg] \\ & h\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right)\left(\frac{\delta(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right) \\ & - \int_{\underline{\pi}}^{\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}} \left[\frac{\delta(1-\kappa)(1-\delta)(Y-\alpha\mu)}{1-\delta(1-\kappa)}\right] h(\pi)\,d\pi \\ & + \left[(1-\delta)\alpha\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right) + \left(\frac{\delta(1-\kappa)(1-\delta)\alpha\mu+\kappa Y}{1-\delta(1-\kappa)}\right)\right] \times \\ & h\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right)\left(\frac{\delta(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right) - \delta\gamma(Y-\alpha\mu) \end{aligned}$$

Simplifying yields:

$$\begin{aligned} \frac{\partial\psi(Y)}{\partial\sigma} = & -h\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right)\left(\frac{\delta(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right) \times \\ & \left[\frac{\delta(1-\kappa)(1-\delta)(1-\sigma)(Y-\alpha\mu)}{1-\delta(1-\kappa)}\right] \\ & - \int_{\underline{\pi}}^{\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}} \left[\frac{\delta(1-\kappa)(1-\delta)(Y-\alpha\mu)}{1-\delta(1-\kappa)}\right] h(\pi)\,d\pi - \delta\gamma(Y-\alpha\mu) < 0. \end{aligned} \tag{17}$$

Now consider increasing σ from σ' to σ''. By (17), this causes $\psi(Y)$ to shift down. Since $\psi(Y, \sigma') \leq Y$ as $Y \geq Y^*(\sigma')$ then $\psi(Y, \sigma'') < Y\ \forall Y \geq Y^*(\sigma')$ which implies

$Y^*(\sigma'') < Y^*(\sigma')$. The other comparative statics on Y^* will also use this method of showing how a change in a parameter affects $\psi(Y)$.

Next consider changing γ. Since

$$\frac{\partial \psi(Y)}{\partial \gamma} = -\delta\sigma(Y - \alpha\mu) < 0,$$

then $\psi(Y)$ is also decreasing in γ which implies Y^* is decreasing in γ.

If instead $\phi = \underline{\pi}$ then $Y^*(\eta) = \alpha\mu$ and thus $Y^*(\eta)$ is, generically, independent of σ and γ. If $\phi = \overline{\pi}$ then

$$\psi(Y) = \int_{\underline{\pi}}^{\overline{\pi}} \left[(1-\delta)\pi + \frac{\delta[1-\delta(1-\kappa)]Y - \delta\sigma(1-\kappa)(1-\delta)(Y-\alpha\mu)}{1-\delta(1-\kappa)}\right] h(\pi)\,d\pi - \delta\sigma\gamma(Y-\alpha\mu),$$

which is decreasing in σ and γ and, therefore, Y^* is decreasing in σ and γ.

Now consider the impact of changing η. Note that η only operates through ϕ since, in equilibrium, a firm never cheats. Hence, $Y^*(\eta)$ is independent of η when $\phi \notin (\underline{\pi}, \overline{\pi})$. Let us then suppose $\phi \in (\underline{\pi}, \overline{\pi})$ in which case:

$$\begin{aligned}
\frac{\partial \psi(Y)}{\partial \eta} &= -\left[(1-\delta)\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right) + \delta(1-\sigma)Y + \delta\sigma W\right] \times \\
&\quad h\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right)\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)^2}\right) \\
&\quad + \left[(1-\delta)\alpha\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right) + \delta W\right] \times \\
&\quad h\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right)\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)^2}\right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \psi(Y)}{\partial \eta} &= -h\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right)\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)^2}\right) \times \\
&\quad \left[(1-\delta)(1-\alpha)\left(\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}\right) + \delta(1-\sigma)(Y-W)\right],
\end{aligned}$$

which is negative. Hence, raising η lowers $\psi(Y)$ and thus lowers Y^*.

Turning to comparative statics for ϕ^*, first note that if $\phi^* > \underline{\pi}$ then $Y^* > \alpha\mu$ and

$$\phi^* = \frac{\delta(1-\sigma)(1-\kappa)(Y^*-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}$$

Consider $\sigma'' > \sigma'$:

$$\begin{aligned}
\phi^*(\sigma'') - \phi^*(\sigma') &= \frac{\delta(1-\sigma'')(1-\kappa)(Y^*(\sigma'')-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)} - \frac{\delta(1-\sigma')(1-\kappa)(Y^*(\sigma')-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)} \\
&= \left(\frac{\delta(1-\kappa)}{[1-\delta(1-\kappa)](\eta-1)}\right)[(1-\sigma'')(Y^*(\sigma'')-\alpha\mu) \\
&\quad -(1-\sigma')(Y^*(\sigma')-\alpha\mu)],
\end{aligned}$$

which is negative. Thus, $\phi^*(\sigma)$ is decreasing in σ.

Consider $\gamma'' > \gamma'$:

$$\begin{aligned}\phi^*(\gamma'') - \phi^*(\gamma') &= \frac{\delta(1-\sigma)(1-\kappa)(Y^*(\gamma'') - \alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)} - \frac{\delta(1-\sigma')(1-\kappa)(Y^*(\gamma') - \alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)} \\ &= \left(\frac{\delta(1-\sigma)(1-\kappa)}{[1-\delta(1-\kappa)](\eta-1)}\right)[Y^*(\gamma'') - Y^*(\gamma')] < 0,\end{aligned}$$

since, by Theorem 3, Y^* is decreasing in γ.

Consider $\eta'' > \eta'$:

$$\begin{aligned}\phi^*(\eta'') - \phi^*(\eta') &= \frac{\delta(1-\sigma)(1-\kappa)(Y^*(\eta'') - \alpha\mu)}{[1-\delta(1-\kappa)](\eta''-1)} - \frac{\delta(1-\sigma')(1-\kappa)(Y^*(\eta') - \alpha\mu)}{[1-\delta(1-\kappa)](\eta'-1)} \\ &= \left(\frac{\delta(1-\sigma)(1-\kappa)}{[1-\delta(1-\kappa)]}\right)\left[\left(\frac{Y^*(\eta'') - \alpha\mu}{\eta''-1}\right) - \left(\frac{Y^*(\eta') - \alpha\mu}{\eta'-1}\right)\right] < 0.\end{aligned}$$

since Y^* is decreasing in η (by Theorem 3), $Y^*(\eta') - \alpha\mu > Y^*(\eta'') - \alpha\mu > 0$, and $\eta'' - 1 > \eta' - 1$.

Finally, let us show that ϕ^* is decreasing in α when $\phi^* \in (\underline{\pi}, \overline{\pi})$. Recall that a fixed point is defined by:

$$\begin{aligned}Y &= \int_{\underline{\pi}}^{\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}} \left[(1-\delta)\pi + \delta Y - \left(\frac{\delta\sigma(1-\kappa)(1-\delta)(Y-\alpha\mu)}{1-\delta(1-\kappa)}\right)\right] h(\pi)\,d\pi \\ &+ \int_{\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{[1-\delta(1-\kappa)](\eta-1)}}^{\overline{\pi}} \left[(1-\delta)\alpha\pi + \delta\left(\frac{(1-\kappa)(1-\delta)\alpha\mu + \kappa Y}{1-\delta(1-\kappa)}\right)\right] h(\pi)\,d\pi - \delta\sigma\gamma(Y-\alpha\mu).\end{aligned}$$

Subtract $\alpha\mu$ from both sides and define $\Delta \equiv Y - \alpha\mu$ so that we now are looking for a fixed point in Δ :

$$\begin{aligned}\Delta &= \Phi(\Delta, \alpha) \equiv \int_{\underline{\pi}}^{\frac{\delta(1-\sigma)(1-\kappa)\Delta}{[1-\delta(1-\kappa)](\eta-1)}} \left[(1-\delta)\pi + \delta\Delta + \delta\alpha\mu - \left(\frac{\delta\sigma(1-\kappa)(1-\delta)\Delta}{1-\delta(1-\kappa)}\right)\right] h(\pi)\,d\pi \\ &+ \int_{\frac{\delta(1-\sigma)(1-\kappa)\Delta}{[1-\delta(1-\kappa)](\eta-1)}}^{\overline{\pi}} \left[(1-\delta)\alpha\pi + \delta\left(\frac{(1-\kappa)(1-\delta)\alpha\mu + \kappa\Delta}{1-\delta(1-\kappa)}\right) + \frac{\delta\kappa\alpha\mu}{1-\delta(1-\kappa)}\right] h(\pi)\,d\pi \\ &- \delta\sigma\gamma\Delta - \alpha\mu \\ &= \int_{\underline{\pi}}^{\frac{\delta(1-\sigma)(1-\kappa)\Delta}{[1-\delta(1-\kappa)](\eta-1)}} \left[(1-\delta)\pi + \delta\Delta - \left(\frac{\delta\sigma(1-\kappa)(1-\delta)\Delta}{1-\delta(1-\kappa)}\right)\right] h(\pi)\,d\pi \\ &+ \int_{\underline{\pi}}^{\frac{\delta(1-\sigma)(1-\kappa)\Delta}{[1-\delta(1-\kappa)](\eta-1)}} \delta\alpha\mu h(\pi)\,d\pi + \int_{\frac{\delta(1-\sigma)(1-\kappa)\Delta}{[1-\delta(1-\kappa)](\eta-1)}}^{\overline{\pi}} \left(\frac{\delta\kappa\alpha\mu}{1-\delta(1-\kappa)}\right) h(\pi)\,d\pi \\ &+ \int_{\frac{\delta(1-\sigma)(1-\kappa)\Delta}{[1-\delta(1-\kappa)](\eta-1)}}^{\overline{\pi}} \left[(1-\delta)\alpha\pi + \delta\left(\frac{(1-\kappa)(1-\delta)\alpha\mu + \kappa\Delta}{1-\delta(1-\kappa)}\right)\right] h(\pi)\,d\pi - \delta\sigma\gamma\Delta - \alpha\mu.\end{aligned}$$

Let $\Phi : [0, \mu] \to [0, \mu]$.

Let $\alpha = \alpha'$ and suppose $\phi^*(\eta;\alpha') \in (\underline{\pi}, \overline{\pi})$ which implies the maximal fixed point, $\Delta^*(\alpha')$, is interior and thus:

$$\Phi\left(\Delta, \alpha'\right) \leqq 0 \text{ as } \Delta \geqq \Delta^*\left(\alpha'\right).$$

If we then show that $\Phi(\Delta, \alpha)$ is decreasing in α, it follows that if $\alpha'' > \alpha'$ then

$$\Phi\left(\Delta, \alpha''\right) < 0 \;\forall\; \Delta \geqq \Delta^*\left(\alpha'\right),$$

and therefore

$$\Delta^*\left(\alpha''\right) < \Delta^*\left(\alpha'\right).$$

If $\phi^*(\eta;\alpha') \in (\underline{\pi}, \overline{\pi})$ then:

$$\begin{aligned}
\frac{\partial \Phi(\Delta,\alpha)}{\partial \alpha} &= \int_{\underline{\pi}}^{\phi} \delta\mu h(\pi)\, d\pi + \int_{\phi}^{\overline{\pi}} \left(\frac{\delta\kappa\mu}{1-\delta(1-\kappa)}\right) h(\pi)\, d\pi \\
&\quad + \int_{\phi}^{\overline{\pi}} \left[(1-\delta)\pi + \delta\left(\frac{(1-\kappa)(1-\delta)\mu}{1-\delta(1-\kappa)}\right)\right] h(\pi)\, d\pi - \mu \\
&= -\int_{\underline{\pi}}^{\phi} (1-\delta)\pi h(\pi)\, d\pi < 0.
\end{aligned}$$

We conclude that if $\phi^*(\eta;\alpha') \in (\underline{\pi}, \overline{\pi})$ then $\Delta^*(\alpha')$ is (locally) decreasing in α.

Next note that:

$$\phi^*(\eta;\alpha) = \frac{\delta(1-\sigma)(1-\kappa)\left[Y^*(\eta;\alpha) - \alpha\mu\right]}{\left[1-\delta(1-\kappa)\right](\eta-1)} = \frac{\delta(1-\sigma)(1-\kappa)\Delta^*(\alpha)}{\left[1-\delta(1-\kappa)\right](\eta-1)}.$$

Hence, $\phi^*(\eta;\alpha)$ is decreasing in α because $\Delta^*(\alpha)$ is decreasing in α. ■

Proof of Theorem 4. Recall that $\widehat{\eta}$ satisfies the property:

$$Y^*(\eta) \begin{cases} > \alpha\mu & \text{if } \eta \in \left[\underline{\eta}, \widehat{\eta}\right] \\ = \alpha\mu & \text{if } \eta > \widehat{\eta} \end{cases}$$

By the continuity of $\psi(Y,\eta)$ with respect to Y and η and that $\psi(Y,\eta)$ is decreasing in η (when $Y^*(\eta) > \alpha\mu$), it follows that:

$$\psi(Y, \widehat{\eta}) \leq Y, \forall Y \text{ and } \psi\left(Y^*(\widehat{\eta}), \widehat{\eta}\right) = Y^*(\widehat{\eta}).$$

With this property, let us now argue that $\widehat{\eta}$ is decreasing in σ and γ.

Consider raising σ from σ' to σ''. Since $\psi(Y,\eta)$ is decreasing in σ for $Y > \alpha\mu$ then

$$\psi\left(Y, \widehat{\eta}\left(\sigma'\right), \sigma''\right) < Y, \;\forall Y > \alpha\mu,$$

follows from $\psi(Y, \widehat{\eta}(\sigma'), \sigma') \leq Y, \forall Y$. Given that $\psi(Y,\eta)$ is decreasing in η for $Y > \alpha\mu$ then $\widehat{\eta}(\sigma'') < \widehat{\eta}(\sigma')$. A similar argument applies to changes in γ.

To show that $\widehat{\eta}$ is decreasing in α, recall from the definition of $\widehat{\eta}(\alpha)$ that:

$$\Phi(\Delta, \widehat{\eta}(\alpha), \alpha) \leq \Delta \;\forall \Delta.$$

Since $\Phi(\Delta, \widehat{\eta}(\alpha), \alpha)$ is decreasing in α (holding $\widehat{\eta}$ fixed) then: if $\alpha'' > \alpha'$ then

$$\Phi\left(\Delta, \widehat{\eta}\left(\alpha'\right), \alpha''\right) < \Delta \ \forall \Delta > 0.$$

This implies $\widehat{\eta}(\alpha'') < \widehat{\eta}(\alpha')$ if $\Phi(\Delta, \widehat{\eta}(\alpha'), \alpha'')$ is decreasing in η. Since $\Phi(\Delta, \eta, \alpha) = \psi(Y, \eta, \alpha) - \alpha\mu$ and we've already shown that $\psi(Y, \eta, \alpha)$ is decreasing in η then $\Phi(\Delta, \eta, \alpha)$ is decreasing in η. ■

Proof of Theorem 5. Let us first consider the impact of γ on the stationary rate of cartels for a type-η sub-population. Suppose that penalties are made more severe as reflected in γ being increased from γ' to γ''. As the cartel rate is

$$\frac{\kappa(1-\sigma) H(\phi^*(\gamma, \eta))}{1-(1-\kappa)(1-\sigma) H(\phi^*(\gamma, \eta))}$$

then the change in the cartel rate is

$$\frac{\kappa(1-\sigma) H(\phi^*(\gamma'', \eta))}{1-(1-\kappa)(1-\sigma) H(\phi^*(\gamma'', \eta))} - \frac{\kappa(1-\sigma) H(\phi^*(\gamma', \eta))}{1-(1-\kappa)(1-\sigma) H(\phi^*(\gamma', \eta))}.$$

Next note that

$$\begin{aligned} & sign\left\{\frac{\kappa(1-\sigma) H(\phi^*(\gamma'', \eta))}{1-(1-\kappa)(1-\sigma) H(\phi^*(\gamma'', \eta))} - \frac{\kappa(1-\sigma) H(\phi^*(\gamma', \eta))}{1-(1-\kappa)(1-\sigma) H(\phi^*(\gamma', \eta))}\right\} \\ = \ & sign\left\{\kappa(1-\sigma)\left[H\left(\phi^*\left(\gamma'', \eta\right)\right) - H\left(\phi^*\left(\gamma', \eta\right)\right)\right]\right\} < 0, \end{aligned}$$

as $\phi^*(\gamma'') < \phi^*(\gamma')$ by Theorem 3.

It is then straightforward to extend this to the entire population of cartels. Recall that the cartel rate is

$$\int_{\underline{\eta}}^{\widehat{\eta}(\gamma)} \left[\frac{\kappa(1-\sigma) H(\phi^*(\gamma, \eta))}{1-(1-\kappa)(1-\sigma) H(\phi^*(\gamma, \eta))}\right] g(\eta)\, d\eta$$

after integrating over all type-η industries. It has just been shown that the integrand is decreasing in γ and since $\widehat{\eta}(\gamma)$ is decreasing in γ by Theorem 4 then this expression is decreasing in γ. Since the rate of discovered cartels is proportional to the rate of cartels then the discovered cartel rate is also decreasing in γ. ■

Proof of Theorem 6. Suppose conviction is made more likely so that σ is raised from σ' to σ''. Using (13), the change in the rate of cartels is:

$$\frac{\kappa(1-\sigma'') H(\phi^*(\sigma'', \eta))}{1-(1-\kappa)(1-\sigma'') H(\phi^*(\sigma'', \eta))} - \frac{\kappa(1-\sigma') H(\phi^*(\sigma', \eta))}{1-(1-\kappa)(1-\sigma') H(\phi^*(\sigma', \eta))}.$$

The sign of that expression is the same as:

$$\begin{aligned} & sign\left\{\kappa\left(1-\sigma''\right) H\left(\phi^*\left(\sigma'', \eta\right)\right)\left[1-(1-\kappa)\left(1-\sigma'\right) H\left(\phi^*\left(\sigma', \eta\right)\right)\right] - \right. \\ & \left. \kappa\left(1-\sigma'\right) H\left(\phi^*\left(\sigma', \eta\right)\right)\left[1-(1-\kappa)\left(1-\sigma''\right) H\left(\phi^*\left(\sigma'', \eta\right)\right)\right]\right\} \\ = \ & sign\left\{\left(1-\sigma''\right) H\left(\phi^*\left(\sigma'', \eta\right)\right) - \left(1-\sigma'\right) H\left(\phi^*\left(\sigma', \eta\right)\right)\right\} < 0. \end{aligned}$$

This expression is negative because $\sigma'' > \sigma'$ implies $1-\sigma'' < 1-\sigma'$ and $H\left(\phi^*\left(\sigma'', \eta\right)\right) < H\left(\phi^*\left(\sigma', \eta\right)\right)$, since $\phi^*\left(\sigma'', \eta\right) < \phi^*\left(\sigma', \eta\right)$ by Theorem 3.

It is then straightforward to extend this to the entire population of cartels. Recall that the cartel rate is

$$\int_{\underline{\eta}}^{\widehat{\eta}(\sigma)} \left[\frac{\kappa\left(1-\sigma\right) H\left(\phi^*\left(\sigma, \eta\right)\right)}{1-\left(1-\kappa\right)\left(1-\sigma\right) H\left(\phi^*\left(\sigma, \eta\right)\right)}\right] dG\left(\eta\right)$$

after integrating over all type-η industries. It has just been shown that the integrand is decreasing in σ and since $\widehat{\eta}\left(\sigma\right)$ is decreasing in σ by Theorem 4 then this expression is decreasing in σ.

That the rate of discovered cartels can be either increasing or decreasing in σ - depending on the parameter values - is shown in the text. ■

To prove Theorem 7, the next lemma will be useful. It shows for a type-η subpopulation that a fall in σ (and γ) cause a first-order stochastic dominance shift in the distribution over the length of cartels and discovered cartels.

Lemma 9 *A decrease in σ or γ causes a first-order stochastic dominance (FOSD) shift in the stationary type-η distribution on the duration of cartels and the duration of discovered cartels.*

Proof of Lemma. Suppose σ is raised from σ' to σ'' and consider the impact of the proportion of cartels of length l.

$$\begin{aligned}
\frac{f\left(l;\eta,\sigma''\right)}{f\left(l;\eta,\sigma'\right)} &= \frac{H\left(\phi^*\left(\sigma''\right)\right)^{l-1}\left(1-\sigma''\right)^{l-1}\left[1-H\left(\phi^*\left(\sigma''\right)\right)\left(1-\sigma''\right)\right]}{H\left(\phi^*\left(\sigma'\right)\right)^{l-1}\left(1-\sigma'\right)^{l-1}\left[1-H\left(\phi^*\left(\sigma'\right)\right)\left(1-\sigma'\right)\right]} \\
&= \left(\frac{H\left(\phi^*\left(\sigma''\right)\right)\left(1-\sigma''\right)}{H\left(\phi^*\left(\sigma'\right)\right)\left(1-\sigma'\right)}\right)^{l-1}\left(\frac{1-H\left(\phi^*\left(\sigma''\right)\right)\left(1-\sigma''\right)}{1-H\left(\phi^*\left(\sigma'\right)\right)\left(1-\sigma'\right)}\right).
\end{aligned}$$

A more aggressive antitrust policy makes a duration of one period more likely:

$$\frac{f\left(1;\eta,\sigma''\right)}{f\left(1;\eta,\sigma'\right)} = \left(\frac{1-H\left(\phi^*\left(\sigma''\right)\right)\left(1-\sigma''\right)}{1-H\left(\phi^*\left(\sigma'\right)\right)\left(1-\sigma'\right)}\right) > 1 \Rightarrow f\left(1;\eta,\sigma''\right) > f\left(1;\eta,\sigma'\right).$$

Perform the following steps:

$$\begin{aligned}
\frac{f\left(l;\eta,\sigma''\right)}{f\left(l;\eta,\sigma'\right)} &= \left(\frac{H\left(\phi^*\left(\sigma''\right)\right)\left(1-\sigma''\right)}{H\left(\phi^*\left(\sigma'\right)\right)\left(1-\sigma'\right)}\right)^{l-1}\left(\frac{1-H\left(\phi^*\left(\sigma''\right)\right)\left(1-\sigma''\right)}{1-H\left(\phi^*\left(\sigma'\right)\right)\left(1-\sigma'\right)}\right) \\
\ln\left(\frac{f\left(l;\eta,\sigma''\right)}{f\left(l;\eta,\sigma'\right)}\right) &= \left(l-1\right)\ln\left(\frac{H\left(\phi^*\left(\sigma''\right)\right)\left(1-\sigma''\right)}{H\left(\phi^*\left(\sigma'\right)\right)\left(1-\sigma'\right)}\right) + \ln\left(\frac{1-H\left(\phi^*\left(\sigma''\right)\right)\left(1-\sigma''\right)}{1-H\left(\phi^*\left(\sigma'\right)\right)\left(1-\sigma'\right)}\right)
\end{aligned}$$

Since

$$\frac{\partial \ln\left(\frac{f(l;\eta,\sigma'')}{f(l;\eta,\sigma')}\right)}{\partial l} = \ln\left(\frac{H\left(\phi^*\left(\sigma''\right)\right)\left(1-\sigma''\right)}{H\left(\phi^*\left(\sigma'\right)\right)\left(1-\sigma'\right)}\right) < 0$$

and

$$\frac{\partial \ln\left(\frac{f(l;\eta,\sigma'')}{f(l;\eta,\sigma')}\right)}{\partial l} = \frac{\frac{\partial(f(l;\eta,\sigma'')/f(l;\eta,\sigma'))}{\partial l}}{f(l;\eta,\sigma'')/f(l;\eta,\sigma')}$$

then

$$\frac{\partial\left(\frac{f(l;\eta,\sigma'')}{f(l;\eta,\sigma')}\right)}{\partial l} < 0.$$

To summarize, $\frac{f(l;\eta,\sigma'')}{f(l;\eta,\sigma')} > 1$ at $l = 1$ and $\frac{f(l;\eta,\sigma'')}{f(l;\eta,\sigma')}$ is decreasing in l. Since $\sum_{l=1}^{\infty} f(l;\eta,\sigma) = 1$ then it cannot be the case that $\frac{f(l;\eta,\sigma'')}{f(l;\eta,\sigma')} > 1$ for some l and $\frac{f(l;\eta,\sigma'')}{f(l;\eta,\sigma')} \geq 1\ \forall l$. It follows that, generically,

$$\exists l^* \geq 1 \text{ such that } f\left(l;\eta,\sigma''\right) > (<)\, f\left(l;\eta,\sigma'\right) \text{ as } l \geq (<)\, l^*.$$

$f(l;\eta,\sigma')$ then first-order stochastically dominates (FOSD) $f(l;\eta,\sigma'')$.

A similar argument shows that lower penalties - as reflected in reducing γ from γ'' to γ' - results in a FOSD shift of the distribution of cartel duration.

$$\begin{aligned} \frac{f(l;\eta,\gamma'')}{f(l;\eta,\gamma')} &= \left(\frac{H(\phi^*(\gamma''))(1-\sigma)}{H(\phi^*(\gamma'))(1-\sigma)}\right)^{l-1}\left(\frac{1-H(\phi^*(\gamma''))(1-\sigma)}{1-H(\phi^*(\gamma'))(1-\sigma)}\right) \\ &= \left(\frac{H(\phi^*(\gamma''))}{H(\phi^*(\gamma'))}\right)^{l-1}\left(\frac{1-H(\phi^*(\gamma''))(1-\sigma)}{1-H(\phi^*(\gamma'))(1-\sigma)}\right). \end{aligned}$$

$$\frac{f(1;\eta,\gamma'')}{f(1;\eta,\gamma')} = \left(\frac{1-H(\phi^*(\gamma''))(1-\sigma)}{1-H(\phi^*(\gamma'))(1-\sigma)}\right) > 1 \Rightarrow f\left(1;\eta,\gamma''\right) > f\left(1;\eta,\gamma'\right).$$

$$sign\left\{\frac{\partial\left(\frac{f(l;\eta,\sigma'')}{f(l;\eta,\sigma')}\right)}{\partial l}\right\} = sign\left\{\frac{H(\phi^*(\gamma''))}{H(\phi^*(\gamma'))}\right\} < 0.$$

Hence,

$$\exists l^* \geq 1 \text{ such that } f\left(l;\eta,\gamma''\right) > (<)\, f\left(l;\eta,\gamma'\right) \text{ as } l \geq (<)\, l^*,$$

and, therefore, $f(l;\eta,\gamma')$ FOSD $f(l;\eta,\gamma'')$. ■

Proof of Theorem 7. We want to show that $\overline{f}(l;\sigma',\sigma'')$ FOSD $\widetilde{f}(l;\sigma')$. That is, for any l',

$$\begin{aligned} &\sum_{l=1}^{l'}\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')} f\left(l;\eta,\sigma'\right)\left[\frac{(1-\beta(0;\eta,\sigma'))\,g(\eta)\,d\eta}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')}[1-\beta(0;\eta',\sigma')]\,g(\eta')\,d\eta'}\right]d\eta \qquad (18) \\ < &\sum_{l=1}^{l'}\int_{\underline{\eta}}^{\widehat{\eta}(\sigma')} f\left(l;\eta,\sigma'\right)\left[\frac{(1-\beta(0;\eta,\sigma'))\,g(\eta)}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma')}[1-\beta(0;\eta',\sigma')]\,g(\eta')\,d\eta'}\right]d\eta \end{aligned}$$

Note that the term on the right of the inequality can be expressed as below.

$$\left(\frac{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')}\left[1-\beta\left(0;\eta',\sigma'\right)\right]g\left(\eta'\right)d\eta'}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma')}\left[1-\beta\left(0;\eta',\sigma'\right)\right]g\left(\eta'\right)d\eta'}\right)\times \tag{19}$$

$$\sum_{l=1}^{l'}\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')} f\left(l;\eta,\sigma'\right)\left[\frac{\left(1-\beta\left(0;\eta,\sigma'\right)\right)g\left(\eta\right)}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')}\left[1-\beta\left(0;\eta',\sigma'\right)\right]g\left(\eta'\right)d\eta'}\right]d\eta$$

$$+\left(\frac{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma')}\left[1-\beta\left(0;\eta',\sigma'\right)\right]g\left(\eta'\right)d\eta'-\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')}\left[1-\beta\left(0;\eta',\sigma'\right)\right]g\left(\eta'\right)d\eta'}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma')}\left[1-\beta\left(0;\eta',\sigma'\right)\right]g\left(\eta'\right)d\eta'}\right)\times$$

$$\sum_{l=1}^{l'}\int_{\widehat{\eta}(\sigma'')}^{\widehat{\eta}(\sigma')} f\left(l;\eta,\sigma'\right)\times$$

$$\left[\frac{\left(1-\beta\left(0;\eta,\sigma'\right)\right)g\left(\eta\right)}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma')}\left[1-\beta\left(0;\eta',\sigma'\right)\right]g\left(\eta'\right)d\eta'-\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')}\left[1-\beta\left(0;\eta',\sigma'\right)\right]g\left(\eta'\right)d\eta'}\right]d\eta$$

What we've done is to break apart the integration into $\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')}$ and $\int_{\widehat{\eta}(\sigma'')}^{\widehat{\eta}(\sigma')}$ and then multiplied the first term by

$$\frac{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')}\left[1-\beta\left(0;\eta',\sigma'\right)\right]g\left(\eta'\right)d\eta'}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')}\left[1-\beta\left(0;\eta',\sigma'\right)\right]g\left(\eta'\right)d\eta'}$$

and the second term by

$$\frac{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma')}\left[1-\beta\left(0;\eta',\sigma'\right)\right]g\left(\eta'\right)d\eta'-\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')}\left[1-\beta\left(0;\eta',\sigma'\right)\right]g\left(\eta'\right)d\eta'}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma')}\left[1-\beta\left(0;\eta',\sigma'\right)\right]g\left(\eta'\right)d\eta'-\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')}\left[1-\beta\left(0;\eta',\sigma'\right)\right]g\left(\eta'\right)d\eta'}.$$

Substitute (19) into (18), we then need to show:

$$\sum_{l=1}^{l'}\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')} f\left(l;\eta,\sigma'\right)\left[\frac{\left(1-\beta\left(0;\eta,\sigma'\right)\right)g\left(\eta\right)}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')}\left[1-\beta\left(0;\eta'\right)\right]g\left(\eta'\right)d\eta'}\right]d\eta \tag{20}$$

$$< \quad \upsilon\sum_{l=1}^{l'}\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')} f\left(l;\eta,\sigma'\right)\left[\frac{\left(1-\beta\left(0;\eta,\sigma'\right)\right)g\left(\eta\right)}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')}\left[1-\beta\left(0;\eta',\sigma'\right)\right]g\left(\eta'\right)d\eta'}\right]d\eta$$

$$+\left(1-\upsilon\right)\sum_{l=1}^{l'}\int_{\widehat{\eta}(\sigma'')}^{\widehat{\eta}(\sigma')} f\left(l;\eta,\sigma'\right)\times$$

$$\left[\frac{\left(1-\beta\left(0;\eta,\sigma'\right)\right)g\left(\eta\right)}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma')}\left[1-\beta\left(0;\eta',\sigma'\right)\right]g\left(\eta'\right)d\eta'-\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')}\left[1-\beta\left(0;\eta',\sigma'\right)\right]g\left(\eta'\right)d\eta'}\right]d\eta$$

where

$$\upsilon \equiv \frac{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')} \left[1-\beta\left(0;\eta',\sigma'\right)\right] g\left(\eta'\right) d\eta'}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma')} \left[1-\beta\left(0;\eta',\sigma'\right)\right] g\left(\eta'\right) d\eta'}.$$

Manipulating (20),

$$\begin{aligned} & (1-\upsilon) \sum_{l=1}^{l'} \int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')} f\left(l;\eta,\sigma'\right) \left[\frac{\left(1-\beta\left(0;\eta,\sigma'\right)\right) g\left(\eta\right)}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')} \left[1-\beta\left(0;\eta'\right)\right] g\left(\eta'\right) d\eta'}\right] d\eta \\ < \; & (1-\upsilon) \sum_{l=1}^{l'} \int_{\widehat{\eta}(\sigma'')}^{\widehat{\eta}(\sigma')} f\left(l;\eta,\sigma'\right) \times \\ & \left[\frac{\left(1-\beta\left(0;\eta,\sigma'\right)\right) g\left(\eta\right)}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma')} \left[1-\beta\left(0;\eta',\sigma'\right)\right] g\left(\eta'\right) d\eta' - \int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')} \left[1-\beta\left(0;\eta',\sigma'\right)\right] g\left(\eta'\right) d\eta'}\right] d\eta \Leftrightarrow \end{aligned}$$

$$\begin{aligned} & \sum_{l=1}^{l'} \int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')} f\left(l;\eta,\sigma'\right) \left[\frac{\left(1-\beta\left(0;\eta,\sigma'\right)\right) g\left(\eta\right)}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')} \left[1-\beta\left(0;\eta'\right)\right] g\left(\eta'\right) d\eta'}\right] d\eta \\ < \; & \sum_{l=1}^{l'} \int_{\widehat{\eta}(\sigma'')}^{\widehat{\eta}(\sigma')} f\left(l;\eta,\sigma'\right) \times \\ & \left[\frac{\left(1-\beta\left(0;\eta,\sigma'\right)\right) g\left(\eta\right)}{\int_{\underline{\eta}}^{\widehat{\eta}(\sigma')} \left[1-\beta\left(0;\eta',\sigma'\right)\right] g\left(\eta'\right) d\eta' - \int_{\underline{\eta}}^{\widehat{\eta}(\sigma'')} \left[1-\beta\left(0;\eta',\sigma'\right)\right] g\left(\eta'\right) d\eta'}\right] d\eta \end{aligned} \tag{21}$$

Recall that if $\eta'' > \eta'$ then $f\left(l;\eta',\sigma\right)$ FOSD $f\left(l;\eta'',\sigma\right)$ (Lemma 10). Hence, except for $\eta = \widehat{\eta}\left(\sigma''\right)$,

$$f\left(l;\eta',\sigma'\right) < f\left(l;\eta'',\sigma'\right), \; \forall \eta' \in \eta \in \left[\underline{\eta}, \widehat{\eta}\left(\sigma''\right)\right], \forall \eta'' \in \eta \left[\widehat{\eta}\left(\sigma''\right), \widehat{\eta}\left(\sigma'\right)\right].$$

It follows that (21) holds. ■

Proof of Theorem 8. Since $f\left(l;\eta,\sigma'\right)$ FOSD $f\left(l;\eta,\sigma''\right)$ $\forall \eta \in \left[\underline{\eta}, \widehat{\eta}\left(\sigma''\right)\right]$ (by Lemma 10) then the short-run distribution on cartel duration FOSD the long-run distribution. ■

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