When Does a Self-Serving Antitrust Authority Act in Society's Best Interests?[*]

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Abstract

If an antitrust authority chooses policies to maximize the number of successfully prosecuted cartels, when do those policies also serve to minimize the number of cartels that form? When the detection and prosecution of cartels is inherently difficult, we find that an antitrust authority's policies minimize the number of cartels, as is socially desirable. But when the detection and prosecution of cartels is not difficult, an antitrust authority is not aggressive enough in that it prosecutes too few cartel cases.

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1 Introduction

When it comes to cartels, the socially desirable objectives of an antitrust authority (AA) are desistance - causing existing cartels to shut down - and deterrence - preventing cartels from forming. Desistance is achieved by discovering and successfully prosecuting cartels. The act of desistance along with the critical role of penalization serve the goal of deterrence. Putting aside heterogeneity in cartels, one simple characterization of what we want an AA to do is to implement policies that minimize the number of cartelized industries, which I will refer to as the cartel rate.

While we want an AA to minimize the cartel rate, do their interests coincide with such an objective? Whatever manner in which a member of an AA is rewarded - internal promotion, bonuses, status, personal satisfaction, career advancement - it is reasonable to suppose that broadly defined compensation is tied to some observable measure of performance. Unfortunately, the cartel rate is not observed. We can document how many suspected cartels there are and how many are convicted, but that doesn't tell us how many cartels are active. As an AA can presumably only be rewarded based on *observable* measures of performance, its actions should not be intended to minimize the unobservable cartel rate. The behavior of an AA could depend on how it impacts the number of cases pursued, how many cases are won, how much in fines is collected, the size of industry revenue impacted, and other observable variables, but not the cartel rate.

Due to the unobservability of the cartel rate, there is then a possible incongruity between what society wants an AA to do and what an AA actually does. In this paper we explore this issue. We assume the performance of an AA is measured by the number of successfully prosecuted cartels and investigate to what extent the actions of an AA coincide with those that minimize the number of active cartels.

2 Model

In our simple model, it is implicitly assumed that cartels are homogeneous so all that matters is the cartel rate which is the fraction of industries that are cartelized. Denoted $C(\sigma)$, the cartel rate is assumed to depend on σ which is the probability that a cartel is caught, prosecuted, and convicted; in other words, σ is the probability that a cartel will be penalized. It is natural to assume that the higher is the probability that prospective cartels attach to paying penalties, the fewer cartels there are.[1]

Assumption 1 $C(\sigma) : [0,1] \rightarrow [0,1]$ is a twice differentiable decreasing function.

$\sigma = q \times r \times s$ where q is the probability that a cartel is discovered, r is the fraction of discovered cartels that the antitrust authority (AA) prosecutes, and s is the probability that the AA gets a conviction for a cartel it is prosecuting. The initial

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[1] The relationship between the probability of paying penalties and the cartel rate is endogenized in Harrington and Chang (2008) by modelling the equlibrium formation of cartels. Here we take it as exogenous in order to explore other issues.

discovery of a cartel is presumed to be exogenous and to come from customers, uninvolved employees, the accidental discovery of evidence through a proposed merger, and so forth; $q \in (0,1]$ is then a parameter. What the AA controls is how many cases to take on which is the fraction of reported cases that the AA chooses to prosecute, r. We will also refer to r as the AA's enforcement policy. Finally, of those cases discovered and prosecuted, the likelihood of the AA being successful, s, depends on the AA's caseload. Implicit is that the AA has a limited amount of resources so that a bigger caseload means fewer resources per case and, therefore, a lower probability of winning any case. Let $s = p(R)$, where $R = qrC(qrs)$ is the mass of cases handled by the AA, and we assume p is decreasing.

Assumption 2 $p(R) : [0,1] \to [0,1]$ is a twice differentiable, decreasing, and weakly concave function.

The probability of a conviction depends on the caseload, $s = p(R)$, and the caseload depends on the number of cartels, $R = qrC(qrs)$, which depends on the probability of conviction. Thus, the *equilibrium* probability of conviction is a fixed point:

$$s^* = p(qrC(qrs^*)). \tag{1}$$

Define $\psi(s) \equiv p(qrC(qrs))$. First note that $\psi(s)$ is an increasing function:

$$\psi'(s) = (qr)^2 p'(qrC(qrs))\, C'(qrs) > 0.$$

Next note that: $\psi(0) > 0, \psi(1) < 1$. Hence, by the continuity of ψ, there exists an interior fixed point. We will further assume that this fixed point is unique.

Assumption 3 There exists unique $s^* \in (0,1)$ such that $s^* = p(qrC(qrs^*))$.

To present sufficient conditions for Assumption 3 to hold, note that

$$\psi''(s) = (qr)^3 \left[qrp''(qrC(qrs))\left(C'(qrs)\right)^2 + p'(qrC(qrs))\, C''(qrs)\right].$$

If $\psi''(s) \leq 0$ then ψ is a contraction mapping and thus has a unique fixed point. If $C(\sigma)$ is linear then, along with Assumptions 1 and 2, $\psi''(s) \leq 0$. If $p(R)$ is strictly concave and $C(\sigma)$ is not too concave then again $\psi''(s) \leq 0$. We will sometimes use the expression $\sigma^*(r) \equiv qrs^*(r)$ for the equilibrium probability that a cartel pays penalties, given enforcement policy r.

A social planner (SP) is assumed to choose the enforcement policy, r, that minimizes the cartel rate, which is equivalent to maximizing the probability of paying penalties:

$$\min_{r \in [0,1]} C(qrs^*(r)) \Leftrightarrow \max_{r \in [0,1]} qrs^*(r).$$

Denote r^{sp} as the social planner's optimal enforcement policy:

$$r^{sp} \in \arg\min C(qrs^*(r)).$$

Under Assumptions 1-3, r^{sp} exists.

The AA is rewarded according to some observable measure of performance. As the cartel rate is unobserved, the number of successful cases is used as the performance measure. The AA is assumed to choose an enforcement policy that maximizes the number of convicted cartels.

$$\max_{r\in[0,1]} qrs^*(r)\, C(qrs^*(r)).$$

Denote r^{aa} as the AA's optimal enforcement policy:

$$r^{aa} \in \arg\max qrs^*(r)\, C(qrs^*(r)),$$

which exists by our assumptions. Defining $\Delta \equiv \{qrs^*(r) : r \in [0,1]\}$, we can also cast the AA's problem as: $\max_{\sigma\in\Delta} \sigma C(\sigma)$. It'll be useful to assume that the AA's objective function is strictly quasi-concave.

Assumption 4 $\sigma C(\sigma)$ is strictly quasi-concave in σ.

Finally, Assumption 5 is made, which will hold if q is sufficiently small or p and C are not too sensitive.

Assumption 5 $p'(qrC(qrs))(qr)^2 C'(qrs) < 1$, $\forall (r,s) \in [0,1]^2$.

3 Results

The AA is assumed to choose an enforcement policy to maximize the number of successfully prosecuted cases, while the socially optimal policy is one that minimizes the cartel rate. In this section, we show when these two distinct objectives lead to the same policy. As stated in Theorem 1, if the SP's optimal enforcement policy is not to prosecute all cases then the AA's optimal enforcement policy coincides with the socially optimal policy. Thus, the policy that maximizes the number of successful cases also serves to minimize the cartel rate. The proof is in the appendix.

Theorem 1 *If* $r^{sp} \in (0,1)$ *then* $r^{aa} = r^{sp}$.

Let us provide some intuition as why Theorem 1 is true. Suppose the SP has an interior solution, $r^{sp} \in (0,1)$. As the SP is choosing r to maximize the probability a cartel pays penalties, $qrs^*(r)$, an interior optimum means that

$$\frac{\partial \sigma^*(r)}{\partial r} = q\left[s^*(r^{sp}) + r^{sp}\left(\frac{\partial s^*(r^{sp})}{\partial r}\right)\right] = 0$$

and, therefore,

$$\frac{\partial s^*(r^{sp})}{\partial r} < 0.$$

Thus, the conviction rate is declining in the enforcement policy. Since $s = p(qrC(qrs))$ then as r increases, there are two forces affecting the conviction rate. Holding the cartel rate C fixed, more aggressive enforcement (that is, higher r) means a bigger caseload qrC and a lower conviction rate. Secondly, more aggressive enforcement lowers the cartel rate $C(qrs)$ which reduces the caseload and thus raises the conviction rate. The former effect obviously dominates when $\partial s^*/\partial r < 0$. Thus, when the SP has an interior solution, an increase in the enforcement rate raises the caseload, $qrC(qrs)$, holding s fixed. Now consider the marginal effect of enforcement on the AA's objective:

$$\begin{aligned}\frac{\partial\sigma^*(r)\,C(\sigma^*(r))}{\partial r} &= \left(\frac{\partial\sigma^*}{\partial r}\right)\left[C(\sigma^*)+\sigma^* C'(\sigma^*)\right] \qquad (2)\\ &= q\left[s^* + r\left(\frac{\partial s^*}{\partial r}\right)\right]\left[C(\sigma^*)+\sigma^* C'(\sigma^*)\right].\end{aligned}$$

Holding s fixed, the caseload $qrC(qrs)$ is increasing in r if and only if the number of successfully prosecuted cartels $qrsC(qrs)$ is increasing in r. The latter condition can be stated as

$$C(\sigma^*)+\sigma^* C'(\sigma^*) > 0,$$

and, using (2), we have

$$sign\left\{\frac{\partial\sigma^*(r)\,C(\sigma^*(r))}{\partial r}\right\} = sign\left\{q\left[s^* + r\left(\frac{\partial s^*}{\partial r}\right)\right]\right\} = sign\left\{\frac{\partial\sigma^*(r)}{\partial r}\right\}.$$

Hence, what maximizes $\sigma^*(r)\,C(\sigma^*(r))$ also maximizes $\sigma^*(r)$ and, therefore, minimizes $C(\sigma^*(r))$.

That the conviction rate is declining in the enforcement policy means the caseload is increasing in the enforcement policy. Since the number of successfully prosecuted cartels is just the caseload multiplied by the conviction rate then, holding the conviction rate fixed, the number of successfully prosecuted cartels is increasing in the enforcement rate. This implies that the AA wants to increase σ since $C(\sigma)$ is increasing in σ. Of course, the SP wants to increase σ since its objective is to maximize σ. Hence, both the SP and AA choose an enforcement policy that maximizes σ.

By Theorem 1, there are three possible cases. First, minimizing the cartel rate requires not prosecuting all cases and, by Theorem 1, this means that the AA chooses the socially optimal policy: $r^{sp} = r^{aa} \in (0,1)$. Second, it is socially optimal to prosecute all cases and the AA does so: $r^{sp} = r^{aa} = 1$. And, third, it is socially optimal to prosecute all cases and the AA does not do so: $r^{aa} < r^{sp} = 1$. In the remainder of this section, sufficient conditions are provided for each of these cases to occur.

Starting with the first case, a sufficient condition for $r^{sp} \in (0,1)$ is

$$\left[\frac{\partial\sigma^*(r)}{\partial r}\right]_{r=1} < 0 \Rightarrow s^*(1) + \left(\frac{\partial s^*(1)}{\partial r}\right) < 0.$$

Substituting the expression for $\partial s^*(1)/\partial r$ (a derivation of which is in the Appendix), we have:

$$s^*(1) + \frac{qp'(qC(qs^*(1)))\left[C(qs^*(1)) + qs^*(1)\,C'(qs^*(1))\right]}{1 - p'(qC(qs^*(1)))\,q^2 C'(qs^*(1))} < 0$$

If we let $s^*(1) \to 0$, this expression converges to

$$\frac{qp'(qC(0))\,C(0)}{1 - p'(qC(0))\,q^2C'(0)} < 0$$

which does indeed hold because $p'(qC(0)) < 0$ and $C(0) > 0$. Thus, if the probability of conviction is sufficiently low when all cartel cases are prosecuted ($s^*(1) \simeq 0$) then the policy that minimizes the cartel rate involves prosecuting some but not all cases, and this policy is also chosen by the AA, even though its interest lies in maximizing the number of convicted cartels.

Next consider: $r^{aa} = r^{sp} = 1$. A sufficient condition for $r^{sp} = 1$ is

$$\frac{\partial \sigma^*(r)}{\partial r} = q\left[s^*(r) + r\left(\frac{\partial s^*(r)}{\partial r}\right)\right] > 0,\ \forall r. \tag{3}$$

(3) holds if $\partial s^*/\partial r$ is sufficiently small relative to $s^*(r)$. Since

$$\frac{\partial s^*}{\partial r} = \frac{qp'(qrC(\sigma^*))\left[C(\sigma^*) + \sigma^* C'(\sigma^*)\right]}{1 - p'(qrC(\sigma^*))\,(qr)^2\,C'(\sigma^*)} \tag{4}$$

then

$$\lim_{q \to 0} \frac{\partial s^*}{\partial r} = 0,$$

and, furthermore,

$$\lim_{q \to 0} s^*(r) = p(0) > 0.$$

Thus, (3) holds when q is sufficiently small, from which we conclude:

$$\lim_{q \to 0} r^{sp} = 1.$$

Next consider the AA's optimal policy. A sufficient condition for $r^{aa} = 1$ is

$$\frac{\partial \sigma^*(r)\,C(\sigma^*(r))}{\partial r} = \left(\frac{\partial \sigma^*(r)}{\partial r}\right)\left[1 - 2\sigma^*(r)\right] > 0,\ \forall r. \tag{5}$$

As we just showed that $\partial \sigma^*(r)/\partial r > 0$ when q is sufficiently small, (5) holds when

$$1 - 2\sigma^*(r) > 0,\ \forall r. \tag{6}$$

Given that

$$\lim_{q \to 0} \sigma^*(r) = 0,$$

it follows from (6) that (5) is true, and therefore

$$\lim_{q \to 0} r^{aa} = 1.$$

To conclude, if detection is weak - that is, q is sufficiently low - then an AA implements the socially optimal policy of prosecuting all cartel cases.

Finally, consider case 3 so that the AA is not sufficiently aggressive: $0 < r^{aa} < r^{sp} = 1$. We already stated that a sufficient condition for $r^{sp} = 1$ is (3), which holds when $\partial s^*(r)/\partial r$ is sufficiently small. By (4), $\partial s^*(r)/\partial r$ is close to zero when $p'(R)$ is close to zero. Thus, if the probability of conviction is not too sensitive to the caseload then prosecuting all cases minimizes the cartel rate, $r^{sp} = 1$. A necessary and sufficient condition for $r^{aa} < 1$ is

$$\left[\frac{\partial \sigma^*(r) C(\sigma^*(r))}{\partial r}\right]_{r=1} = \left(\frac{\partial \sigma^*(1)}{\partial r}\right)[1 - 2\sigma^*(1)] < 0. \tag{7}$$

Since we already have that $\partial \sigma^*(1)/\partial r > 0 \forall r$, (7) holds if

$$\sigma^*(1) = qs^*(1) > \frac{1}{2},$$

which is true when q and $s^*(1)$ are sufficiently close to one.

Summing up case 3, if it is not difficult to detect ($q \simeq 1$) and convict ($s^*(1) \simeq 1$) cartels and the AA is relatively unconstrained in terms of prosecutorial resources - so that $p'(R) \simeq 0$ and thus the probability of conviction is not very sensitive to the caseload - then the policy that minimizes the cartel rate is to prosecute all cases. However, an AA which is interested in maximizing the number of convicted cartels will instead choose *not* to prosecute all cases. The AA may prosecute fewer cases than is socially optimal in order to reduce deterrence and raise the cartel rate. From the perspective of the AA, an enforcement policy can be too aggressive in that it is so effective in terms of deterrence that the ensuing decline in the cartel rate results in fewer convicted cartels.

To summarize these three cases, if conditions are tough for fighting cartels - either detection is difficult or conviction is difficult - then the AA implements the policy which minimizes the cartel rate. If conditions are sufficiently conducive to fighting cartels then enforcement by the AA is less aggressive than is socially optimal. Finally, let us highlight that the AA never over-prosecutes; the fraction of cases that it prosecutes is always weakly less than that which minimizes the cartel rate.

Corollary 2 $r^{aa} \leq r^{sp}$.

4 Concluding Remarks

When it comes to fighting cartels, previous research has typically modelled an antitrust authority as a social welfare-maximizer when deriving its behavior; examples include Besanko and Spulber (1989), Motta and Polo (2003), and Harrington (2008). While the specification of that objective is fine for identifying what is the desired policy, it is not necessarily appropriate for describing the actual policy choices of an antitrust authority. In this paper, a more reasonable alternative is considered which is that an antitrust authority acts to maximize the number of convicted cartels, which is an *observable* measure of performance.

Though maximizing the number of successfully prosecuted cartels is distinct from minimizing the number of cartels, conditions are derived whereby the resulting behavior is equivalent. In particular, if the socially optimal policy is not to prosecute all suspected cartels then a self-serving antitrust authority will choose it. However, if the socially optimal policy is to prosecute all suspected cartels then an antitrust authority which is interested in maximizing the number of convicted cartels may choose not to prosecute all cartel cases.

Reflective of the possible policy challenges that may arise when an antitrust authority is not driven to maximize social welfare is Chang and Harrington (2008). In that paper, the implications of an antitrust authority that maximizes the number of convicted cartels are explored with respect to evaluating the impact of a corporate leniency program. Contrary to the current model, the effect of antitrust policy on the cartel rate is endogenized through an equilibrium process of cartel birth and death. In that model, the enforcement policy is the fraction of non-leniency cases that the antitrust authority pursues. When enforcement is chosen to minimize the cartel rate, a leniency program is shown to lower the cartel rate. However, this need not be the case when the antitrust authority is motivated by the number of convicted cartels. The additional caseload provided by the leniency program induces the antitrust authority to prosecute a smaller fraction of cartel cases identified outside of the program and, because of this less aggressive enforcement policy, it is possible that the cartel rate is *higher* when there is a leniency program.

Understanding the behavior of an antitrust authority and then designing policy in light of any behavioral biases are large issues, while this is a small paper that barely scratches the surface. Clearly, more attention to these matters is warranted.

5 Appendix

Prior to proving Theorem 1, two preliminary results will prove useful. Totally differentiating (1), an explicit expression for how the enforcement policy affects the probability of a conviction can be derived:

$$\begin{aligned}
\frac{\partial s^*}{\partial r} &= qp'\left(qrC\left(qrs^*\right)\right)\left\{C\left(qrs^*\right)+qrC'\left(qrs^*\right)\left[s^*+r\left(\frac{\partial s^*}{\partial r}\right)\right]\right\} \\
\frac{\partial s^*}{\partial r} &= \frac{qp'\left(qrC\left(qrs^*\right)\right)\left[C\left(qrs^*\right)+qrs^*C'\left(qrs^*\right)\right]}{1-p'\left(qrC\left(qrs^*\right)\right)\left(qr\right)^2C'\left(qrs^*\right)}
\end{aligned}$$

$$\frac{\partial s^*}{\partial r}=\frac{qp'\left(qrC\left(\sigma^*\right)\right)\left[C\left(\sigma^*\right)+\sigma^*C'\left(\sigma^*\right)\right]}{1-p'\left(qrC\left(\sigma^*\right)\right)\left(qr\right)^2C'\left(\sigma^*\right)}. \tag{8}$$

The next result derives a sufficient condition for $\sigma^*\left(r\right)\equiv qrs^*\left(r\right)$, which is what the SP is maximizing, to be strictly quasi-concave in r.

Lemma 3 *If* $C\left(qrs^*\left(r\right)\right)+qrs^*\left(r\right)C'\left(qrs^*\left(r\right)\right)>0\ \forall r\in[0,r']$ *then* $rs^*\left(r\right)$ *is strictly quasi-concave for* $r\in[0,r']$.

Proof. Using (8), first note that:

$$\begin{aligned}
\frac{\partial qrs^{*}\left(r\right)}{\partial r} &= q\left[s^{*}\left(r\right)+r\left(\frac{\partial s^{*}\left(r\right)}{\partial r}\right)\right]=q\left[s^{*}+r\left(\frac{qp'\left(qrC\left(\sigma^{*}\right)\right)\left[C\left(\sigma^{*}\right)+\sigma^{*}C'\left(\sigma^{*}\right)\right]}{1-p'\left(qrC\left(\sigma^{*}\right)\right)\left(qr\right)^{2}C'\left(\sigma^{*}\right)}\right)\right] \\
\frac{\partial qrs^{*}\left(r\right)}{\partial r} &= q\left[\frac{s^{*}+qrp'\left(qrC\left(\sigma^{*}\right)\right)C\left(\sigma^{*}\right)}{1-p'\left(qrC\left(\sigma^{*}\right)\right)\left(qr\right)^{2}C'\left(\sigma^{*}\right)}\right]
\end{aligned}$$

By the preceding expression and Assumption 5,

$$sign\left\{\frac{\partial qrs^{*}\left(r\right)}{\partial r}\right\}=sign\left\{s^{*}+qrp'\left(qrC\left(\sigma^{*}\right)\right)C\left(\sigma^{*}\right)\right\},$$

$qrs^{*}\left(r\right)$ is strictly quasi-concave over $[0,r']$ if either: i) $s^{*}+qrp'\left(qrC\left(\sigma^{*}\right)\right)C\left(\sigma^{*}\right)>0$, $\forall r\in[0,r']$; or ii) $\exists r''\in(0,r')$ such that $s^{*}+qrp'\left(qrC\left(\sigma^{*}\right)\right)C\left(\sigma^{*}\right)\gtreqqless 0$, as $r\lesseqqgtr r''$, $\forall r\in[0,r']$. Evaluating $s^{*}+qrp'\left(qrC\left(\sigma^{*}\right)\right)C\left(\sigma^{*}\right)$ at $r=0$, we know that it is positive. Thus, for (i) or (ii) to hold, it is sufficient that $s^{*}+qrp'\left(qrC\left(\sigma^{*}\right)\right)C\left(\sigma^{*}\right)$ is monotonic in r. After some manipulation, we derive

$$\begin{aligned}
&\frac{\partial\left[s^{*}+qrp'\left(qrC\left(\sigma^{*}\right)\right)C\left(\sigma^{*}\right)\right]}{\partial r} \\
= &\frac{\partial s^{*}\left(r\right)}{\partial r}+qp'\left(qrC\left(\sigma^{*}\right)\right)C\left(\sigma^{*}\right)+q^{2}rp'\left(qrC\left(\sigma^{*}\right)\right)C'\left(\sigma^{*}\right)\left[s^{*}\left(r\right)+r\left(\frac{\partial s^{*}\left(r\right)}{\partial r}\right)\right] \\
&+q^{2}rp''\left(qrC\left(\sigma^{*}\right)\right)C\left(\sigma^{*}\right)\left[C\left(\sigma^{*}\right)+qrC'\left(\sigma^{*}\right)\left(s^{*}\left(r\right)+r\left(\frac{\partial s^{*}\left(r\right)}{\partial r}\right)\right)\right] \\
= &\; q\left[C\left(\sigma^{*}\right)+\sigma^{*}C'\left(\sigma^{*}\right)\right]\left\{\left(\frac{p'\left(qrC\left(\sigma^{*}\right)\right)}{1-p'\left(qrC\left(\sigma^{*}\right)\right)\left(qr\right)^{2}C'\left(\sigma^{*}\right)}\right)\times\right. \\
&\left[1+\left(qr\right)^{2}p'\left(qrC\left(\sigma^{*}\right)\right)C'\left(\sigma^{*}\right)+\left(qr\right)^{3}p''\left(qrC\left(\sigma^{*}\right)\right)C\left(\sigma^{*}\right)C'\left(\sigma^{*}\right)\right] \\
&\left.+p'\left(qrC\left(\sigma^{*}\right)\right)+qrp''\left(qrC\left(\sigma^{*}\right)\right)C\left(\sigma^{*}\right)\right\}
\end{aligned}$$

Since $p'<0, 1-p'\left(qr\right)^{2}C'>0, C'<0$, and $p''\leq 0$ then

$$sign\left\{\frac{\partial\left[s^{*}+qrp'\left(qrC\left(\sigma^{*}\right)\right)C\left(\sigma^{*}\right)\right]}{\partial r}\right\}=-sign\left\{C\left(\sigma^{*}\right)+\sigma^{*}C'\left(\sigma^{*}\right)\right\},$$

which proves the lemma. ■

Proof of Theorem 1. If $r^{sp}\in(0,1)$ then there exists at least one local maximum. Let $\widetilde{r}$ be the lowest local maximum so that: $s^{*}+r\left(\frac{\partial s^{*}}{\partial r}\right)\geqq 0\ \forall r\leqq\widetilde{r}$, and, at $r=\widetilde{r}$,

$$s^{*}+r\left(\frac{\partial s^{*}}{\partial r}\right)=s^{*}+r\left(\frac{qp'\left(qrC\left(\sigma^{*}\right)\right)\left[C\left(\sigma^{*}\right)+\sigma^{*}C'\left(\sigma^{*}\right)\right]}{1-p'\left(qrC\left(\sigma^{*}\right)\right)\left(qr\right)^{2}C'\left(\sigma^{*}\right)}\right)=0.$$

This implies

$$C\left(q\widetilde{r}s^{*}\left(\widetilde{r}\right)\right)+q\widetilde{r}s^{*}\left(\widetilde{r}\right)C'\left(q\widetilde{r}s^{*}\left(\widetilde{r}\right)\right)>0. \tag{9}$$

Since $qrs^{*}\left(r\right)$ is non-decreasing in r $\forall r\leq\widetilde{r}$, it follows from the strict quasi-concavity of $\sigma C\left(\sigma\right)$ in σ and (9) that

$$C\left(qrs^{*}\left(r\right)\right)+qrs^{*}\left(r\right)C'\left(qrs^{*}\left(r\right)\right)>0,\ \forall r\in\left[0,\widetilde{r}\right]. \tag{10}$$

By the continuity of $qrs^*(r)$ with respect to r and of $C(\sigma)$ and $C'(\sigma)$ with respect to σ, it follows from (10) that $\exists\varepsilon > 0$ such that

$$C(qrs^*(r)) + qrs^*(r)\,C'(qrs^*(r)) > 0,\ \forall r \in [0, \widetilde{r} + \varepsilon]\,. \tag{11}$$

By Lemma 3, it follows from (11) that $qrs^*(r)$ is strictly quasi-concave in r, for $r \in [0, \widetilde{r} + \varepsilon]$. Hence, $s^* + r\left(\frac{\partial s^*}{\partial r}\right) = 0$ for $r = \widetilde{r}$ implies

$$s^* + r\left(\frac{\partial s^*}{\partial r}\right) < 0,\ \forall r \in (\widetilde{r}, \widetilde{r} + \varepsilon)\,.$$

We next want to show that $qrs^*(r)$ is strictly quasi-concave in r, for $r \in [0,1]$. Suppose not, in which case $\exists r' > \widetilde{r}$ and $\varepsilon > 0$ such that

$$s^* + r\left(\frac{\partial s^*}{\partial r}\right) \leq 0\ \forall r \in \left(\widetilde{r}, r'\right)$$

and

$$s^* + r\left(\frac{\partial s^*}{\partial r}\right) \geq 0,\ \forall r \in \left[r', r' + \varepsilon\right)\,.$$

By Assumption 5 and (8),

$$sign\left\{\frac{\partial s^*}{\partial r}\right\} = -sign\left\{C(\sigma^*) + \sigma^* C'(\sigma^*)\right\}. \tag{12}$$

From (12), a necessary condition for $s^* + r\left(\frac{\partial s^*}{\partial r}\right) \leq 0$ is that $C(\sigma) + \sigma C'(\sigma) > 0$. By the same argument as above, this implies $\exists\eta > 0$ such that

$$C(qrs^*(r)) + qrs^*(r)\,C'(qrs^*(r)) > 0,\ \forall r \in \left(r', r' + \eta\right),$$

and, furthermore,

$$C(qrs^*(r)) + qrs^*(r)\,C'(qrs^*(r)) > 0,\ \forall r \in \left[0, r' + \eta\right).$$

But then, by Lemma 3, $rs^*(r)$ is strictly quasi-concave in r, for $r \in [0, r' + \eta)$. However, this contradicts $rs^*(r)$ being increasing over $[0, \widetilde{r})$, decreasing over $(\widetilde{r}, r')$, and non-decreasing over $(r', r' + \eta)$. We conclude that the supposition that $qrs^*(r)$ is not strictly quasi-concave in r for $r \in [0,1]$ is false. Therefore, if $r^{sp} \in (0,1)$ then $qrs^*(r)$ is strictly quasi-concave in r for $r \in [0,1]$.

Consider the first-order condition for the AA:

$$\begin{aligned}
\frac{\partial \sigma^*(r)\,C(\sigma^*(r))}{\partial r} &= \left(\frac{\partial \sigma^*}{\partial r}\right)\left[C(\sigma^*) + \sigma^* C'(\sigma^*)\right] \\
\frac{\partial \sigma^*(r)\,C(\sigma^*(r))}{\partial r} &= q\left[s^* + r\left(\frac{\partial s^*}{\partial r}\right)\right]\left[C(\sigma^*) + \sigma^* C'(\sigma^*)\right]
\end{aligned}$$

$$\frac{\partial \sigma^*(r)\,C(\sigma^*(r))}{\partial r} = q\left[s^* + r\left(\frac{\partial s^*}{\partial r}\right)\right]\left[C(qrs^*(r)) + qrs^*(r)\,C'(qrs^*(r))\right]. \tag{13}$$

We've shown that

$$s^* + r\left(\frac{\partial s^*}{\partial r}\right) \gtreqqless 0 \text{ as } r \lesseqqgtr r^{sp}.$$

Since

$$s^* + r\left(\frac{\partial s^*}{\partial r}\right) = 0 \text{ at } r = r^{sp}$$

then

$$C\left(qrs^*\left(r\right)\right) + qrs^*\left(r\right)C'\left(qrs^*\left(r\right)\right) > 0 \text{ at } r = r^{sp},$$

which, by Assumption 4, implies

$$C\left(qrs^*\left(r\right)\right) + qrs^*\left(r\right)C'\left(qrs^*\left(r\right)\right) > 0 \ \forall\ r \leq r^{sp}. \tag{14}$$

In addition,

$$s^* + r\left(\frac{\partial s^*}{\partial r}\right) < 0 \text{ for } r > r^{sp}$$

implies

$$C\left(qrs^*\left(r\right)\right) + qrs^*\left(r\right)C'\left(qrs^*\left(r\right)\right) > 0 \text{ for } r > r^{sp}. \tag{15}$$

Combining (14) and (15), we have

$$C\left(qrs^*\left(r\right)\right) + qrs^*\left(r\right)C'\left(qrs^*\left(r\right)\right) > 0,\ \forall r \in [0,1]\,. \tag{16}$$

It follows from (13) and (16) that:

$$\frac{\partial \sigma^* C\left(\sigma^*\right)}{\partial r} \gtreqqless 0 \text{ as } r \lesseqqgtr r^{sp},$$

in which case

$$r^{sp} = \arg\max \sigma^* C\left(\sigma^*\right).$$

Therefore, $r^{aa} = r^{sp}$. ■

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