The Impact of a Corporate Leniency Program on Antitrust Enforcement and Cartelization*

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Abstract

To explore the efficacy of a corporate leniency program, a Markov process is constructed which models the stochastic formation and demise of cartels. Cartels are born when given the opportunity and market conditions are right, while cartels die because of internal collapse or they are caught and convicted by the antitrust authority. The likelihood that a cartel, once identified, is convicted depends inversely on the caseload of the antitrust authority due to an implicit resource constraint. The authority also chooses an enforcement policy in terms of the fraction of non-leniency cases that it prosecutes. Using numerical analysis, the impact of a leniency program on the steady-state cartel rate is investigated. Holding the enforcement policy of the antitrust authority fixed, a leniency program lowers the frequency of cartels. However, the additional caseload provided by the leniency program induces the antitrust authority to prosecute a smaller fraction of cartel cases identified outside of the program. Because of this less aggressive enforcement policy, it is possible that the cartel rate is higher when there is a leniency program.

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1 Introduction

One of the major innovations in competition policy in the last few decades is the 1993 revision of the Corporate Leniency Program of the U.S. Department of Justice's Antitrust Division. This program gives a member of a cartel the opportunity to avoid government penalties if it is the first to provide evidence (subject to certain conditions). In a story that has been told many times by DOJ officials, this revision converted a moribund program into a prolific generator of leniency applications, and the information provided by those applications was instrumental in securing the convictions of other cartel members. Similar leniency programs have been put in place in dozens of countries throughout the world.

Starting with the pioneering paper of Motta and Polo (2003), there has been a burgeoning academic literature investigating the operation and implications of leniency programs. How do they work? What features are most effective? Is full leniency optimal? Should partial leniency be awarded to later applicants?[1] While this research has allowed us to better understand leniency programs, the answers provided are limited in an important way because a critical element of the environment is held fixed when evaluating the impact of a leniency program. It is assumed that the introduction of a leniency program does not affect the probability that a cartel is caught and convicted outside of the leniency program.[2] But is that a reasonable assumption?

In recent years the European Commission has been overwhelmed with amnesty applications which leads one to wonder how many resources are left to prosecute cases that did not arise from the leniency program. If cases involving the leniency program are much easier to prosecute - due to the presence of an informant - the antitrust authority may prefer to pursue those cases and avoid the more challenging non-leniency cases. This issue is directly pertinent to evaluating the efficacy of a leniency program. A firm will come forward under the leniency program if it feels the chances of it being successfully prosecuted are sufficiently high when it does not apply for leniency. But, as we've just suggested, the likelihood of successful prosecution depends on the allocation of resources by the antitrust authority, and that allocation could well depend on how many leniency cases there are. In short, the institution of a leniency program affects the probability that a cartel is caught through other means but the probability of catching a cartel through other means directly determines a firm's decision whether to apply for leniency. If a cartel member believes it has no chance being caught then it has no reason to apply for leniency. That a leniency program and general enforcement policy interact is a point made in Friederiszick and Maier-Rigaud (2008) in the context of discussing how active the European Commission should be in pursuing *ex officio* cases given the purported success of the leniency program. This is exactly the issue that motivates this research project.

The objective of this paper is to take a more comprehensive approach to assess-

[1] For a review of some of this research, see Spagnolo (2008).

[2] An exception is Motta and Polo (2003) which we discuss later.

ing the implications of a leniency program. This we do by extending the model of Harrington and Chang (2008). In that paper, a population of industries is modelled, each of which decides whether to form a cartel. The opportunity to form a cartel is stochastic and, once that opportunity presents itself, a cartel forms if and only if it is incentive compatible. At the same time that a cartel is stochastically born, it can stochastically die because it is hit by market conditions that cause internal collapse or because it is successfully prosecuted by the authorities. After its death, a cartel can stochastically reconstitute itself in the future. We have then constructed a Markov process over the cartel status of a population of industries and thereby endogenized the frequency of cartels.

The current paper modifies the model of Harrington and Chang (2008) in two significant ways. First, a corporate leniency program is introduced so that a cartel member always has the option of going to the authorities and receiving a reduction in fines. Second, we allow for an implicit resource constraint on the antitrust authority in that the more cases it pursues, the lower is the probability of gaining a conviction for a case which lacks an informant through the leniency program. Furthermore, we model the antitrust authority's decision regarding how many non-leniency cases to pursue.

With this model, there are several questions we want to start to address. When can we expect a leniency program to be successful in the sense that it reduces the frequency of cartels? If cartels are heterogeneous, does a leniency program have a differential effect across cartels and, if so, what are the implications for the cartel rate? How does a leniency program affect the marginal productivity of enforcement expenditure? Does the introduction of a leniency program mean that the antitrust authority's budget should be expanded or contracted? And how do these answers depend on the objective we assign to the antitrust authority? While not all of these questions are addressed here, the framework that is developed can be used to do so.

To summarize our main findings, first we show that, holding fixed the enforcement policy of the antitrust authority (that is, the fraction of non-leniency cases it takes on), a leniency program has the desired effect of reducing the cartel rate. Second, when it is allowed to adjust its enforcement policy, the antitrust authority becomes less aggressive in pursuing cases which do not involve an applicant to the leniency program. Third, the introduction of a leniency program can either lower or raise the frequency of cartels. There are many parameter configurations for which the antitrust authority sufficiently reduces its enforcement with respect to non-leniency cases that the fraction of industries that are cartelized is actually higher after the introduction of a leniency program. This higher cartel rate is driven by the differential impact of the leniency program on heterogeneous cartels. Industries that produce innately unstable cartels are made worse off after a leniency program in that cartels in those industries no longer form. Industries that produce innately stable cartels are made better off after a leniency program in that cartels continue to form and have longer duration. A leniency program then results in fewer cartels forming but those that do form last longer.

2 Model

2.1 Industry Environment

Firm behavior is modelled using a modification of a Prisoners' Dilemma formulation. Firms simultaneously decide whether to *collude* or *compete*. Prior to making that choice, firms observe a stochastic realization of the market's profitability that is summarized by the variable $\pi \geq 0$.[3] If all firms choose *collude* then each firm earns π, while if all choose *compete* then each earns $\alpha\pi$ where $\alpha \in [0, 1)$. $1 - \alpha$ then measures the competitiveness of the non-collusive environment. π has a continuously differentiable cdf $H : [\underline{\pi}, \overline{\pi}] \to [0, 1]$ where $0 < \underline{\pi} < \overline{\pi}$. $h(\cdot)$ denotes the associated density function and let $\mu \equiv \int \pi h(\pi)\, d\pi$ denote its finite mean. If all other firms choose *collude*, the profit a firm earns by deviating - choosing *compete* - is $\eta\pi$ where $\eta > 1$. This information is summarized in the table below. Note that the Bertrand price game is represented by $(\alpha, \eta) = (0, n)$ where n is the number of firms. The Cournot quantity game with linear demand and cost functions in which firms collude at the joint profit maximum is represented as $(\alpha, \eta) = \left(\frac{4n}{(n+1)^2}, \frac{(n+1)^2}{4n}\right)$.[4]

Own action	All other firms' action	Own profit
collude	*collude*	π
compete	*collude*	$\eta\pi$
compete	*compete*	$\alpha\pi$

Firms interact in an infinite horizon setting where $\delta \in (0, 1)$ is the common discount factor. It is not a repeated game because, as explained later, each industry is in one of two states: it can be a cartel or not. If firms are a cartel then they have the option of colluding, though whether collusion actually occurs depends on it being incentive compatible. More specifically, if firms are cartelized then they simultaneously choose between *collude* and *compete*, and, at the same time, whether or not to apply to the corporate leniency program. Details on the description of the leniency program are provided later. If it is incentive compatible for all firms to choose *collude* then each earns π. If instead a firm prefers *compete* when all other firms choose *collude* then collusion is not incentive compatible (that is, it is not part of the subgame perfect equilibrium for the infinite horizon game) and each firm earns $\alpha\pi$. When firms are not a cartel then each firm earns $\alpha\pi$ as, according to equilibrium, they all choose *compete*.

At the end of the period, there is the random event whereby the antitrust authority (AA) may pursue an investigation; this can only occur if firms colluded in the current or previous period (that is, they entered the period in the cartel state) and no firm applied for leniency.[5] Let $\sigma \in [0, 1)$ denote the probability that firms are discovered,

[3] The informational setting is as in Rotemberg and Saloner (1986).

[4] We have only specified a firm's profit when all firms choose *compete*, all firms choose *collude*, and it chooses *compete* and all others firm choose *collude*. We must also assume that *compete* strictly dominates *collude* for the stage game. It is unnecessary to provide any further specification.

[5] To allow it to depend on collusion farther back in time would require introducing another state

prosecuted, and convicted (below, we will endogenize σ though, from the perspective of an individual industry, it is fixed). In that event, each firm incurs a penalty of $\frac{F}{1-\delta}$ (so that F is the per-period penalty).

It is desirable to allow F to depend on the extent of collusion. Given there is only one level of collusion in the model, the "extent of collusion" necessarily refers to the number of periods that firms had colluded. A proper accounting of that effect would require that each cartel have a state variable which is the length of collusion; this would seriously complicate the analysis. As an approximation, we instead assume that the penalty is proportional to the average increase in profit from being cartelized (rather than the realized increase in profit). If Y denotes the expected per period profit from being in the "cartel state" then $F = \gamma (Y - \alpha\mu)$ where $\gamma > 0$. This avoids the need for state variables but still allows the penalty to be sensitive to the (average) extent of collusion.[6]

In addition to being discovered by the authorities, a cartel can be uncovered because one of its members comes forth under the corporate leniency policy. Suppose a cartel is in place. If a single firm applies for leniency then all firms are convicted for sure and the firm that applied receives a per period penalty of θF where $\theta \in [0, 1]$, while the other cartel members each pay F. If all firms simultaneously apply for leniency then each firm pays a penalty of ωF where $\omega \in (\theta, 1)$. For example, if only one firm can receive leniency and each firm has an equal probability of being first in the door then $\omega = \frac{n-1+\theta}{n}$ when there are n cartel members. It is sufficient for the ensuing analysis that we specify the leniency program when either one firm applies or all firms apply. Also, leniency is not awarded to firms that apply after another firm has done so.

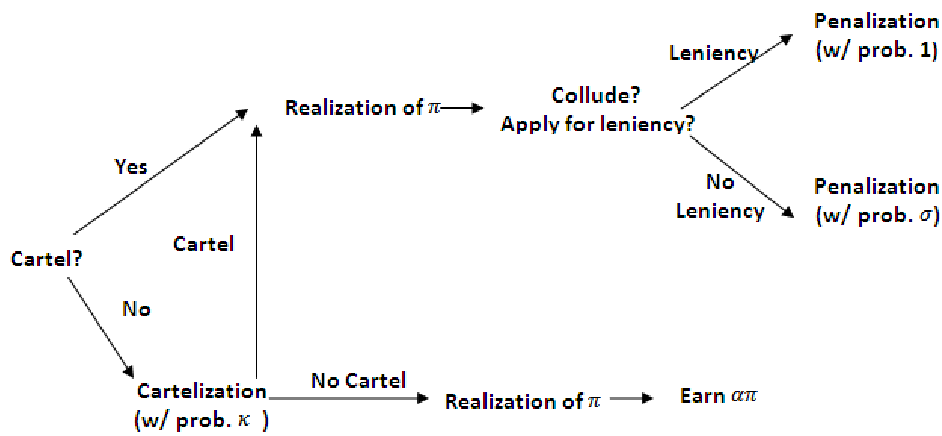
From the perspective of firms, antitrust policy is summarized by the four-tuple $(\sigma, \gamma, \theta, \omega)$ which are, respectively, the probability of paying penalties (in the absence of any firm using the leniency program), the penalty multiple, the leniency parameter when only one firm applies (where $1 - \theta$ is the proportion of fines waived), and the leniency parameter when all firms apply (where $1 - \omega$ is the proportion of fines waived).

Next, let us describe how an industry's cartel status evolves. Suppose it enters the period cartelized. The industry will exit the period still a cartel if: 1) all firms chose *collude* (which requires that collusion be incentive compatible); 2) no firm applied for leniency; and 3) the AA did not discover and convict the firms of collusion. Otherwise, the cartel collapses and they revert to the "no cartel" state. If instead the industry entered the period in the "no cartel" state then with probability $\kappa \in (0, 1)$ firms cartelize. For that cartel to still be around at the end of the period, conditions (1)-(3) above must be satisfied. Note that whenever a cartel is shutdown - whether due to internal collapse, applying to the leniency program, or having been successfully prosecuted - the industry may re-cartelize in the future. Specifically, it

variable that would unnecessarily complicate the analysis. Having it depend on collusion in the previous period will simplify some of the expressions and, furthermore, it seems quite reasonable that detection can occur, to a limited extent, after the fact.

[6] A more standard assumption in the literature is to assume F is fixed which is certainly simpler but less realistic than our specification.

has an opportunity to do so with probability κ in each period that it is not currently colluding.[7] The timing of events is summarized in the figure below.



In modelling a population of industries, it is compelling to allow industries to vary in terms of cartel stability. For this purpose, industries are assumed to differ in terms of the parameter η. If one takes this assumption literally, it can be motivated by heterogeneity in the elasticity of firm demand or the number of firms (as with the Bertrand price game). Our intent is not to be literal but rather to think of this as a parsimonious way in which to encompass industry heterogeneity. Let the cdf on industry types be represented by the continuously differentiable function $G : [\underline{\eta}, \overline{\eta}] \rightarrow [0, 1]$ where $1 < \underline{\eta} < \overline{\eta}$. $g(\cdot)$ denotes the associated density function. The appeal of η is that it is a parameter which influences the frequency of collusion but does not directly affect the value of the firm's profit stream since, in equilibrium, firms do not cheat; this property makes for an easier analysis.

2.2 Antitrust Enforcement Technology

In Harrington and Chang (2008), we explored the preceding model though without the presence of a leniency program (or, alternatively, the preceding model with $\theta = 1$). The main innovation of this paper is to allow for a leniency program *and* endogenize σ by modelling the resource constraint faced by the AA and the optimal behavior of the AA. Here, we describe how the caseload affects the probability of gaining a conviction. In Section 3.4, the objective of the AA is discussed.

σ is the probability that a cartel pays penalties when no member has applied for leniency. It is the compounding of three events: 1) the cartel is discovered by the AA; 2) the AA decides to prosecute the cartel; and 3) the AA is successful in its prosecution. The initial discovery of a cartel is presumed to be exogenous and

[7] Alternatively, one could imagine having two distinct probabilities - one to reconstitute collusion after a firm cheated (the probability of moving from the punishment to the cooperative phase) and another to reform the cartel after having been convicted. For purposes of parsimony, those two probabilities are assumed to be the same.

to come from customers, uninvolved employees, the accidental discovery of evidence through a proposed merger, and so forth. q denotes the probability of discovery and is a parameter throughout the paper.[8] What the AA controls is how many cases to take on which is represented by r which is the fraction of reported cases that the AA chooses to prosecute. Finally, of those cases discovered and prosecuted, the AA gets a conviction in a fraction s of them where s will depend on the AA's caseload.

The AA is then faced with a resource constraint: the more cases it takes on, the fewer resources are applied to each case and the lower is the probability of winning any individual case. More specifically, we assume

$$s = p(\lambda L + R) \text{ where } \lambda \in [0,1].$$

L is the number (or mass) of leniency cases, R is the number of non-leniency cases, and s is the probability of winning one of the R cases, where it is assumed leniency cases are won for sure. $\lambda \in [0,1]$ seems natural as leniency cases ought to take up fewer resources than those cases lacking an informant. p is assumed to be a decreasing function so that a bigger caseload means a lower probability of winning a non-leniency case. In sum, the probability that a cartel pays penalties is

$$\sigma = q \times r \times s = q \times r \times p(\lambda L + R).$$

It is endogenous as the AA chooses r and, in addition, s is determined by the number of leniency and non-leniency cases which depends on the number of cartels.

We chose a fairly flexible functional form for how the caseload relates to the probability of a conviction:

$$p(\lambda L + R) = \frac{\tau}{\xi + v(\lambda L + R)^{\rho}}, \text{ where } v > 0, \rho \geq 1, \tau \in (0,1], \xi \geq \tau.$$

Note that

$$p(0) = \frac{\tau}{\xi}, \lim_{\lambda L + R \to +\infty} p(\lambda L + R) = 0.$$

As long as $\rho > 1$, $p(\lambda L + R)$ is initially concave and then convex with an inflection point at

$$\lambda L + R = \left(\frac{\xi(\rho - 1)}{v(\rho + 1)}\right)^{1/\rho}.$$

For all values of ξ and v considered in this paper, the inflection point is increasing in ρ, which means that the range over which it is concave is larger when ρ is higher.

Before moving on, it needs to be noted that Motta and Polo (2003) do allow for optimal enforcement expenditure by modelling a trade-off between monitoring and prosecution. They endow an antitrust authority with a fixed amount of resources that can be allocated between finding suspected episodes of collusion and prosecuting the cases that are found or, in the language of our model, between raising q and lowering s (assuming $r = 1$). However, they do not consider a population of industries and do not solve for the steady-state frequency of cartels.

[8] One could imagine endogenizing q by allowing the AA to invest resources in screening for cartels. Such an activity is discussed in Harrington (2007) and the European Commission has developed a procedure for screening, see Friederiszick and Maier-Rigaud (2008).

3 Equilibrium Cartel Rate

In this section, we describe the equilibrium frequency with which industries are cartelized. Prior to getting into the details, let us provide a brief overview.

1. Taking as given σ (the per period probability that a cartel pays penalties), we first solve for equilibrium collusive behavior. For a type-η industry, this entails solving for the set of market conditions (values for π) such that collusion is incentive compatible.

2. With step 1 completed, we can then define the Markov process on cartel creation and destruction and solve for the stationary distribution of industries in terms of their cartel status, for each industry type η. By aggregating over all industry types, the equilibrium cartel rate, $C(\sigma)$, is derived, given σ.

3. Next we derive the equilibrium conviction rate, s^*. The probability of the AA gaining a conviction, $p(\lambda L + R)$, depends on the mass of leniency cases, L, and the mass of non-leniency cases, R. Note that L and R are both influenced by how many cartels there are, $C(\sigma)$. s^* is then a fixed point: $s^* = p(\lambda L(C(qrs^*)) + R(C(qrs^*)))$, where recall that $\sigma = qrs$. In other words, the probability that firms assign to being caught, prosecuted, and convicted determines the cartel rate, and the cartel rate determines the number of cases handled by the AA and thus the probability that they are able to get a conviction on a case.

4. The final step is to specify an objective for the AA and then solve for the optimal value for r, which is the fraction of non-leniency cases that it pursues.

3.1 Cartel Formation and Collusive Value

A collusive strategy for a type-η industry entails colluding when π is sufficiently low and not colluding otherwise. The logic is as in Rotemberg and Saloner (1986). When π is high, the incentive to deviate is strong because a firm increases current profit by $(\eta - 1)\pi$. At the same time, the future payoff is independent of the current realization of π, given that π is *iid*. Since the payoff to cheating is increasing in π while the future payoff is independent of π, the incentive compatibility of collusion is more problematic when π is higher.

Suppose firms are able to collude for at least some realizations of π, and let W^o and Y^o denote the payoff when the industry is not cartelized and is cartelized, respectively. If not cartelized then, with probability κ, firms have an opportunity to cartelize with resulting payoff Y^o. With probability $1 - \kappa$, firms do not have such an opportunity and continue to compete. In that case, each firm earns current expected profit of $\alpha\mu$ and a future value of W^o. Thus, the payoff when not colluding is defined recursively by:

$$W^o = (1 - \kappa)(\alpha\mu + \delta W^o) + \kappa Y^o. \tag{1}$$

As it'll be easier to work with re-scaled payoffs, define:

$$W \equiv (1-\delta) W^o, \ Y \equiv (1-\delta) Y^o.$$

Multiplying both sides of (1) by $1-\delta$ and re-arranging yields:

$$\begin{aligned} (1-\delta) W^o &= (1-\kappa) [(1-\delta) \alpha\mu + \delta (1-\delta) W^o] + \kappa (1-\delta) Y^o \Leftrightarrow \\ W &= (1-\kappa) [(1-\delta) \alpha\mu + \delta W] + \kappa Y \Leftrightarrow \end{aligned}$$

$$W = \frac{(1-\kappa)(1-\delta)\alpha\mu + \kappa Y}{1-\delta(1-\kappa)}$$

Also note that the incremental value to being in the cartelized state is:

$$Y - W = Y - \left(\frac{(1-\kappa)(1-\delta)\alpha\mu - \kappa Y}{1-\delta(1-\kappa)} \right) = \frac{(1-\kappa)(1-\delta)(Y-\alpha\mu)}{1-\delta(1-\kappa)}. \qquad (2)$$

Suppose firms are cartelized and π is realized. When a firm decides whether to collude or cheat, it decides at the same time whether to apply for leniency. If it decides to collude, it is clearly not optimal to apply for leniency since the cartel is going to be shut down by the authorities in which case the firm ought to maximize current profit by cheating. The more relevant issue is whether it should apply for leniency if it decides to cheat. The incentive compatibility constraint (ICC) is:

$$(1-\delta)\pi + \delta[(1-\sigma) Y + \sigma (W - \gamma (Y-\alpha\mu))] \geq (1-\delta)\eta\pi + \delta[W - \min\{\sigma,\theta\}\gamma(Y-\alpha\mu)]. \qquad (3)$$

Examining the lhs expression, if it colludes then it earns current profit of π (given all other firms are colluding). With probability $1-\sigma$, the cartel is not shut down by the AA and, given the industry is in the cartel state, the future payoff is Y. With probability σ, the cartel is caught and convicted by the AA - which means a penalty of $\gamma(Y-\alpha\mu)$ - and since the industry is no longer cartelized, the future payoff is W. Turning to the rhs expression, the current profit from cheating is $\eta\pi$. Since this defection causes the cartel to collapse, the future payoff is W. There is still a chance of being caught and convicted. A deviating firm will apply for leniency iff the penalty from doing so is less than the expected penalty from not doing so (and recall that the other firms are colluding and thus not apply for leniency): $\theta\gamma(Y-\alpha\mu) < \sigma\gamma(Y-\alpha\mu)$ or $\theta < \sigma$. Given optimal use of the leniency program, the deviating firm's expected penalty is then $\min\{\sigma,\theta\}\gamma(Y-\alpha\mu)$. Re-arranging (3) and using (2), the ICC can be presented as:

$$\pi \leq \frac{\left(\frac{\delta(1-\sigma)(1-\kappa)(1-\delta)(Y-\alpha\mu)}{1-\delta(1-\kappa)}\right) - \delta[\sigma - \min\{\sigma,\theta\}]\gamma(Y-\alpha\mu)}{(1-\delta)(\eta-1)} \equiv \phi(Y,\sigma,\eta). \qquad (4)$$

Collusion is incentive compatible iff the current market condition is sufficiently low.

In deriving an expression for the value to colluding, we need to discuss usage of the leniency program in equilibrium. Firms do not use it when market conditions result in the cartel being stable but may use it when the cartel collapses. As the continuation payoff is W regardless of whether leniency is used, a firm applies for leniency iff it reduces the expected penalty.[9] First note that an equilibrium either has no firms applying for leniency or all firms doing so because if at least one firm applies then another firm can lower its expected penalty from F to ωF by also doing so. This has the implication that it is is always an equilibrium for all firms to apply for leniency. Furthermore, it is the unique equilibrium when $\theta < \sigma$. To see why, suppose all firms were not to apply for leniency. A firm would then lower its penalty from σF to θF by applying. When instead $\sigma \leq \theta$, there is also an equilibrium in which no firm goes for leniency as to do so would increase its expected penalty from σF to θF. Using the selection criterion of Pareto dominance, we will assume that, upon internal collapse of the cartel, no firms apply when $\sigma \leq \theta$ and all firms apply when $\theta < \sigma$.

The expected payoff to being cartelized, $\psi(Y, \sigma, \eta)$, is then recursively defined by:

$$\psi(Y,\sigma,\eta) = \begin{cases} \int_{\underline{\pi}}^{\phi(Y,\sigma,\eta)} \{(1-\delta)\pi + \delta[(1-\sigma)Y + \sigma(W - \gamma(Y-\alpha\mu))]\} h(\pi)\, d\pi & \text{if } \sigma \leq \theta \\ + \int_{\phi(Y,\sigma,\eta)}^{\overline{\pi}} [(1-\delta)\alpha\pi + \delta W - \delta\sigma\gamma(Y-\alpha\mu)] h(\pi)\, d\pi & \\ \int_{\underline{\pi}}^{\phi(Y,\sigma,\eta)} \{(1-\delta)\pi + \delta[(1-\sigma)Y + \sigma(W - \gamma(Y-\alpha\mu))]\} h(\pi)\, d\pi & \text{if } \theta < \sigma \\ + \int_{\phi(Y,\sigma,\eta)}^{\overline{\pi}} [(1-\delta)\alpha\pi + \delta W - \delta\omega\gamma(Y-\alpha\mu)] h(\pi)\, d\pi & \end{cases}.$$

To understand this expression, first consider when $\sigma \leq \theta$, in which case leniency is never used. If $\pi \in [\underline{\pi}, \phi(Y,\sigma,\eta)]$ then collusion is incentive compatible; each firm earns current profit of π and an expected future payoff of $(1-\sigma)Y + \sigma(W - \gamma(Y-\alpha\mu))$. If instead $\pi \in (\phi(Y,\sigma,\eta), \overline{\pi}]$ then collusion is not incentive compatible, so each firm earns current profit of $\alpha\pi$ and an expected future payoff of $W - \sigma\gamma(Y-\alpha\mu)$. The expression when $\theta < \sigma$ differs only respect to when collusion breaks down in which case the future payoff is $W - \omega\gamma(Y-\alpha\mu)$ as all firms apply for leniency.

A fixed point to ψ is an equilibrium value for Y given σ. That is, given an anticipated future collusive value Y, the resulting equilibrium behavior - as represented by $\phi(Y,\sigma,\eta)$ - results in firms colluding for market states such that the value to being in a cartel is Y. We then want to solve:

$$Y^*(\sigma,\eta) = \psi(Y^*(\sigma,\eta), \sigma, \eta).$$

As an initial step to exploring the set of fixed points, first note that $\psi(\alpha\mu, \sigma, \eta) = \alpha\mu$. Hence, one fixed point to ψ is the degenerate solution without collusion. If there is a fixed point with collusion - that is, $Y > \alpha\mu$ - then we select the one with the highest Y.[10]

[9] It is important to remind the reader that, conditional on the cartel collapsing, the expected time until re-establishment of the cartel is the same whether or not the cartel is discovered by the authorities. That is probably not the case in practice. If, as one would expect, cartel re-formation takes longer when the cartel is discovered then firms will be less inclined to apply for leniency. Perhaps a future generation of this model can allow for this distinction.

[10] In Harrington and Chang (2008), sufficient conditions are derived for there to be a fixed point with $Y > \alpha\mu$. Of course, those sufficient conditions apply to when σ is exogenous and later we will endogenize σ.

Given $Y^* (\sigma, \eta)$, define

$$\phi^* (\sigma, \eta) \equiv \phi (Y^* (\sigma, \eta), \sigma, \eta),$$

as the maximum profit realization such that a type-η cartel is stable. $\phi^* (\sigma, \eta)$ is a measure of cartel stability since the cartel is stable iff $\pi \leq \phi^* (\sigma, \eta)$ and thus internally collapses with probability $1 - H (\phi^* (\sigma, \eta))$.

3.2 Stationary Distribution

Given $\phi^* (\sigma, \eta)$, the stochastic process by which cartels are born and die (either through internal collapse or being shut down by the AA) is characterized in this section. The random events driving this process are the opportunity to cartelize, market conditions, and conviction by the AA. We initially characterize the stationary distribution for type-η industries. The stationary distribution for the entire population of industries is then derived by integrating the type specific distributions over all types.

Consider an arbitrary type-η industry. If it is not cartelized at the end of the preceding period then, by the analysis in Section 3.1, it'll be cartelized at the end of the current period with probability $\kappa H (\phi^* (\sigma, \eta))$. With probability κ it has the opportunity to cartelize, and with probability $H (\phi^* (\sigma, \eta))$ the realization of π is such that collusion is incentive compatible. If instead the industry was cartelized at the end of the previous period, it'll still be cartelized at the end of this period with probability $(1 - \sigma) H (\phi^* (\sigma, \eta))$, which is the probability that it did not internally collapse and that it wasn't caught by the AA. Suppose there is a continuum of type-η industries with independent realizations of the stochastic events each period. The task is to characterize the stationary distribution with regards to the frequency of cartels.

Let $\beta (\eta)$ denote the proportion of type-η industries which are not cartelized. The stationary rate of non-cartels is defined by :

$$\begin{aligned} \beta (\eta) &= \beta (\eta) [(1 - \kappa) + \kappa (1 - H (\phi^*)) + \kappa\sigma H (\phi^*)] \\ &\quad + [1 - \beta (\eta)] [(1 - H (\phi^*)) + \sigma H (\phi^*)] \end{aligned} \tag{5}$$

Examining the rhs of (5), a fraction $\beta (\eta)$ of type-η industries were not cartelized in the previous period. Out of those industries, a fraction $1 - \kappa$ will not have the opportunity to cartelize in the current period. A fraction $\kappa (1 - H (\phi^*))$ will have the opportunity but, due to a high realization of π, find it is not incentive compatible to collude, while a fraction $\kappa\sigma H (\phi^*)$ will cartelize and collude but then are discovered by the AA. Of the industries that were colluding in the previous period, which have mass $1 - \beta (\eta)$, a fraction $1 - H (\phi^*)$ will collapse for internal reasons and a fraction $\sigma H (\phi^*)$ will instead be caught by the authorities and thus shutdown.

Solving (5) for $\beta (\eta)$:

$$\beta (\eta) = \frac{1 - (1 - \sigma) H (\phi^*)}{1 - (1 - \kappa)(1 - \sigma) H (\phi^*)}. \tag{6}$$

For the stationary distribution, the fraction of cartels among type-η industries is then:

$$1-\beta(\eta)=\frac{\kappa(1-\sigma)H(\phi^*)}{1-(1-\kappa)(1-\sigma)H(\phi^*)}. \tag{7}$$

Finally, the derivation of the entire population of industries is performed by integrating the type-η distribution over $\eta \in \left[\underline{\eta},\overline{\eta}\right]$. The mass of cartelized industries, which we refer to as the cartel rate C, is then defined by:

$$C(\sigma)\equiv\int_{\underline{\eta}}^{\overline{\eta}}[1-\beta(\eta)]g(\eta)\,d\eta=\int_{\underline{\eta}}^{\overline{\eta}}\left[\frac{\kappa(1-\sigma)H(\phi^*(\sigma,\eta))}{1-(1-\kappa)(1-\sigma)H(\phi^*(\sigma,\eta))}\right]g(\eta)\,d\eta. \tag{8}$$

3.3 Equilibrium Probability of Paying Penalties

Recall that $\sigma=qrs$ where q is the probability of a cartel being discovered, r is the probability that the AA chooses to prosecute a reported case, and s is the probability of a conviction. We now want to derive the equilibrium value of s, where $s=p(\lambda L+R)$, L is the mass of leniency cases, and R is the mass of non-leniency cases handled by the AA. As both L and R depend on the cartel rate C and the cartel rate depends on s (through σ), this is a fixed point problem. We need to find a value for s, call it s', such that, given $\sigma=qrs'$ (and recall that q is fixed and, for the time being, r is fixed), the induced cartel rate $C(qrs')$ is such that it generates L and R so that $p(\lambda L+R)=s'$.

With our expression for the cartel rate, we can provide expressions for L and R. The mass of cartel cases generated by the leniency program is:

$$L(\sigma)=\begin{cases}0 & \text{if } qrs\le\theta\\ \int_{\underline{\eta}}^{\overline{\eta}}(1-H(\phi^*(qrs,\eta)))\left[\frac{\kappa(1-\sigma)H(\phi^*(qrs,\eta))}{1-(1-\kappa)(1-qrs)H(\phi^*(qrs,\eta))}\right]g(\eta)\,d\eta & \text{if } \theta<qrs\end{cases}, \tag{9}$$

In (9), note that an industry does not apply for leniency when it is still effectively colluding. When collusion stops, leniency is used when the only equilibrium is that all firms apply for leniency, which is the case when $\theta<qrs$. Thus, when $\theta<qrs$, L equals the mass of cartels that collapse due to a high realization of π. This is consistent with a concern expressed by a European Commission official that many leniency applicants are from dying cartels.[11]

The mass of cartel cases generated without use of the leniency program is

$$R(\sigma)=\begin{cases}qr\int_{\underline{\eta}}^{\overline{\eta}}\left[\frac{\kappa(1-qrs)H(\phi^*(qrs,\eta))}{1-(1-\kappa)(1-qrs)H(\phi^*(qrs,\eta))}\right]g(\eta)\,d\eta & \text{if } qrs\le\theta\\ qr\int_{\underline{\eta}}^{\overline{\eta}}H(\phi^*(qrs,\eta))\left[\frac{\kappa(1-qrs)H(\phi^*(qrs,\eta))}{1-(1-\kappa)(1-qrs)H(\phi^*(qrs,\eta))}\right]g(\eta)\,d\eta & \text{if } \theta<qrs\end{cases}. \tag{10}$$

[11] This statement was made by Olivier Guersent at the 11th Annual EU Competition Law and Policy Workshop: Enforcement of Prohibition of Cartels in Florence, Italy in June 2006.

If the leniency program is not being used (that is, $qrs \leq \theta$), then, given the cartel rate is C or $\int_{\underline{\eta}}^{\overline{\eta}} \left[\frac{\kappa(1-qrs)H(\phi^*(qrs,\eta))}{1-(1-\kappa)(1-qrs)H(\phi^*(qrs,\eta))} \right] g(\eta)\, d\eta$, the mass of cases being handled by the AA is qrC. If instead $\theta < qrs$, so that dying cartels do use the leniency program, then the cartels left to be caught are those which have not collapsed in the current period which is $\int_{\underline{\eta}}^{\overline{\eta}} H(\phi^*(qrs,\eta)) \left[\frac{\kappa(1-qrs)H(\phi^*(qrs,\eta))}{1-(1-\kappa)(1-qrs)H(\phi^*(qrs,\eta))} \right] g(\eta)\, d\eta$. This expression comes from the fact that a fraction $\frac{\kappa(1-qrs)H(\phi^*(qrs,\eta))}{1-(1-\kappa)(1-qrs)H(\phi^*(qrs,\eta))}$ of type-η industries are cartelized and a fraction $H(\phi^*(qrs,\eta))$ of them get a realization of π that allows them to continue to collude. Multiplying by qr, we have the mass of non-leniency cases.

The fixed point for the probability of a conviction is defined by a value for s satisfying:

$$s = \begin{cases} p\left(qr \int_{\underline{\eta}}^{\overline{\eta}} \left[\frac{\kappa(1-qrs)H(\phi^*(qrs,\eta))}{1-(1-\kappa)(1-qrs)H(\phi^*(qrs,\eta))} \right] g(\eta)\, d\eta \right) & \text{if } qrs \leq \theta \\ p\left(\lambda \int_{\underline{\eta}}^{\overline{\eta}} (1 - H(\phi^*(qrs,\eta))) \left[\frac{\kappa(1-qrs)H(\phi^*(qrs,\eta))}{1-(1-\kappa)(1-qrs)H(\phi^*(qrs,\eta))} \right] g(\eta)\, d\eta \right. & \text{if } \theta < qrs \\ \left. + qr \int_{\underline{\eta}}^{\overline{\eta}} H(\phi^*(qrs,\eta)) \left[\frac{\kappa(1-qrs)H(\phi^*(qrs,\eta))}{1-(1-\kappa)(1-qrs)H(\phi^*(qrs,\eta))} \right] g(\eta)\, d\eta \right) & \end{cases} \tag{11}$$

where we have substituted for L using (9) and R using (10). A fixed point is denoted $s^*(r)$ where we explicitly recognize its dependence on r since it will be solved for next.

3.4 Optimal Antitrust Policy

Ideally, an AA would choose its caseload - which is controlled by r (the fraction of non-leniency cases pursued) - so as to minimize the cartel rate in the economy: $\min_{r\in[0,1]} C(qrs^*(r))$. However, such an objective is problematic. Unless those working for the AA are of a benevolent species, a career concerns perspective would tell us that the AA will not seek to minimize the cartel rate because it is not observable, much less verifiable. As AA employees cannot be rewarded based on something that is unobservable, presumably their behavior is not driven by the cartel rate.

We will assume that AA employees are rewarded according to observable performance measures. One such measure is the fraction of cases won. The problem with that measure is the AA will then prosecute a very small number of cases and pour lots of resources into them in order to produce a high conviction rate. (Or, in a richer model, the AA may only take on the really easy cases.)

A more reasonable measure is the mass of successful cases. Since leniency cases are presumed to be won for sure and a fraction s of non-leniency cases are won then $L + sR$ measures the mass of successful cases. The AA will then be modelled as choosing $r \in [0,1]$ to maximize $L(qrs^*(r)) + s^*(r)\, R(qrs^*(r))$, which takes the form:

$$\begin{cases} qr\int_{\underline{\eta}}^{\bar{\eta}}\left[\frac{\kappa(1-qrs^*(r))H(\phi^*(qrs^*(r),\eta))}{1-(1-\kappa)(1-qrs^*(r))H(\phi^*(qrs^*(r),\eta))}\right]g(\eta)\,d\eta\times \\ p\left(qr\int_{\underline{\eta}}^{\bar{\eta}}\left[\frac{\kappa(1-qrs)H(\phi^*(qrs^*(r),\eta))}{1-(1-\kappa)(1-qrs^*(r))H(\phi^*(qrs^*(r),\eta))}\right]g(\eta)\,d\eta\right) & \text{if } qrs^*(r)\le\theta \\ \\ \left(\int_{\underline{\eta}}^{\bar{\eta}}(1-H(\phi^*(qrs^*(r),\eta)))\left[\frac{\kappa(1-qrs^*(r))H(\phi^*(qrs^*(r),\eta))}{1-(1-\kappa)(1-qrs^*(r))H(\phi^*(qrs^*(r),\eta))}\right]g(\eta)\,d\eta\right. & \text{if } \theta<qrs^*(r) \\ \left.+qr\int_{\underline{\eta}}^{\bar{\eta}}H(\phi^*(qrs^*(r),\eta))\left[\frac{\kappa(1-qrs^*(r))H(\phi^*(qrs^*(r),\eta))}{1-(1-\kappa)(1-qrs^*(r))H(\phi^*(qrs^*(r),\eta))}\right]g(\eta)\,d\eta\right)\times \\ p\left(\lambda\int_{\underline{\eta}}^{\bar{\eta}}(1-H(\phi^*(qrs^*(r),\eta)))\left[\frac{\kappa(1-qrs^*(r))H(\phi^*(qrs^*(r),\eta))}{1-(1-\kappa)(1-qrs^*(r))H(\phi^*(qrs^*(r),\eta))}\right]g(\eta)\,d\eta\right. \\ \left.+qr\int_{\underline{\eta}}^{\bar{\eta}}H(\phi^*(qrs^*(r),\eta))\left[\frac{\kappa(1-qrs^*(r))H(\phi^*(qrs^*(r),\eta))}{1-(1-\kappa)(1-qrs^*(r))H(\phi^*(qrs^*(r),\eta))}\right]g(\eta)\,d\eta\right) \end{cases} \quad (12)$$

When $qrs^*(r)\le\theta$, the leniency program is not used so the total mass of cases is made up of those cartels prosecuted without leniency,

$$R = qr\int_{\underline{\eta}}^{\bar{\eta}}\left[\frac{\kappa(1-qrs^*(r))H(\phi^*(qrs^*(r),\eta))}{1-(1-\kappa)(1-qrs^*(r))H(\phi^*(qrs^*(r),\eta))}\right]g(\eta)\,d\eta,$$

of which a fraction $p(R)$ are won. When leniency is used, which occurs when $\theta < qrs^*(r)$, the probability of success with non-leniency cases is $p(\lambda L + R)$.

4 Numerical Method

We start by specifying the values for the 13 parameters – n, α, ω, θ, κ, δ, γ, τ, ξ, υ, λ, q, and ρ – as well as the two probability distributions, $H(\pi)$ and $G(\eta)$, for market shocks and industry types, respectively. A log-normal distribution $LN(\mu,\sigma^2)$ with $\mu = 0$ and $\sigma = 1.5$ is used for both $H(\pi)$ and $G(\eta)$, though $H(\pi)$ is defined over $[1,\infty)$, while $G(\eta)$ is defined over $[1.1,\infty)$. Table 1 reports the range of feasible values for the parameters as well as the baseline set of values.

The numerical problem has a nested structure. Given a value of r, the underlying problem is to find a fixed point, $s^*(r)$, to $s = p(\lambda L(qrs) + R(qrs))$, where $L(qrs)$ is the mass of cartel cases generated by the leniency program defined in (9) and $R(qrs)$ is the mass of non-leniency cartel cases defined in (10). Note that the dependence of L and R on r and s is through the endogenous probability of paying penalties, $\sigma = qrs$, which affects the incentive compatibility of collusion.

The procedure for finding $s^*(r)$ begins by specifying an initial value for s. For each η, we need to solve for a fixed point to $\psi(Y,qrs,\eta)$,

$$Y^*(qrs,\eta) = \psi(Y^*(qrs,\eta),qrs,\eta).$$

As there may be multiple fixed points, we use the Pareto criterion to select among them and thus choose the largest fixed point. Since $\psi(Y,qrs,\eta)$ is increasing and $\psi(\mu,qrs,\eta) < \mu$ then, by setting $Y^0 = \mu$ and iterating on $Y^{t+1} = \psi(Y^t,qrs,\eta)$, this process converges to the largest fixed point, $Y^*(qrs,\eta)$.

In computing the stationary distributions from Section 3.2, we need to take the step of computationally searching for $\widehat{\eta}(qrs)$ which is the smallest industry type for which collusion is not incentive compatible for any market condition. $\widehat{\eta}(qrs)$ is defined by: $Y^*(qrs,\eta) > \alpha\mu$ for $\underline{\eta} < \eta \leq \widehat{\eta}(qrs)$ and $Y^*(qrs,\eta) = \alpha\mu$ for $\eta > \widehat{\eta}(qrs)$. To perform this step, we set $\underline{\eta} = 1.1$ and $\overline{\eta} = 10$ and use a 1000 element finite grid of values for η, denoted $\Gamma(\underline{\eta},\overline{\eta})$. $\widehat{\eta}(qrs)$ is located by applying the iterative bisection method on $\Gamma(\underline{\eta},\overline{\eta})$. As part of the bisection method, $\overline{\eta}$ needs to be set at a sufficiently high value so that $Y^*(qrs,\overline{\eta}) = \alpha\mu$. Once having identified $\widehat{\eta}(qrs)$ and using $Y^*(qrs,\eta)$ and (4), $\phi^*(qrs,\eta)$ is calculated for a finite grid over $[\underline{\eta},\widehat{\eta}(qrs)]$. These values are then used in computing $L(qrs)$ and $R(qrs)$. The integration uses the Newton-Cotes quadrature method with the trapezoid rule (see Miranda and Fackler, 2002).

Choosing an initial value for s and using our derived expressions for $L(qrs)$ and $R(qrs)$, we then compute:

$$\widehat{p}(qrs) \equiv p(L(qrs),R(qrs)) = \frac{\tau}{\xi + \upsilon\,[\lambda L(qrs) + R(qrs)]^{\rho}}. \tag{13}$$

After specifying a tolerance level ϵ, if $|s - \widehat{p}(qrs)| > \epsilon$ then a new value for s is selected using the iterative bisection method. Note that once a new value for s is specified, the entire preceding procedure must be repeated. This procedure is repeated until the process converges to the fixed point value of $s^*(r)$ such that

$$|s^*(r) - \widehat{p}(qrs^*(r))| \leq \epsilon. \tag{14}$$

ϵ is set at .001.[12]

With the equilibrium probability of a conviction, $s^*(r)$, we have the equilibrium probability of paying penalties, $qrs^*(r)$, from which we can calculate the stationary distribution on cartels using the formulas in Section 3.2, especially (8) which provides the frequency of cartels. Note that the equilibrium cartel rate, mass of leniency cases, and mass of non-leniency cases are, respectively, $C\left(qrs^*(r)\right)$, $L\left(qrs^*(r)\right)$, and $R\left(qrs^*\left(r\right)\right)$.

To derive the AA's optimal policy, we allow $r \in \{0,.1,\ldots,1\}$ and perform the procedure described above for each of those values. The AA's optimal policy is the value of r which maximizes $L(qrs^*(r)) + s^*(r)R(qrs^*(r))$. To test whether the coarseness of the grid was driving some results, we re-ran the model for a limited number of parameter configurations when $r \in \{0,.01,\ldots,1\}$. While quantitative results did change, none of the properties we describe below were altered.

5 Results

Equilibrium was solved for a baseline parameter configuration and variations off of that baseline; the range of parameter values are provided in Table 1. Table 2 reports

[12] To explore whether the fixed point is unique, we plotted $\widehat{p}(qrs)$ for all $s \in \{0,.1,\ldots,1\}$ and $r \in \{0,.1,\ldots,1\}$; for $\rho \in \{1.3,1.5,1.7\}$ and $\theta \in \{0,1\}$. In all cases, $\widehat{p}(qrs)$ is a concave function of s and there is no reason to think that there is more than one fixed point.

the results for the baseline configuration, both when there is no leniency program (Table 2a) and when full leniency is available (Table 2b). For enforcement policies $r \in \{0, .1, \ldots, 1\}$, the reported variables are the equilibrium values for the probability of conviction, $s^*(r)$; the probability of paying penalties, $q \times r \times s^*(r)$; the mass of leniency cases, $L(qrs^*(r))$; the mass of non-leniency cases, $R(qrs^*(r))$; the performance measure of the AA, $AA \equiv L(qrs^*(r)) + s^*(r) \cdot R(qrs^*(r))$; the cartel rate or fraction of industries cartelized, $C(qrs^*(r))$; the probability that a cartel is penalized, $\frac{L(qrs^*(r))+s^*(r)\cdot R(qrs^*(r))}{C(qrs^*(r))}$; and the average duration of a cartel.

The case of no antitrust enforcement is $r = 0$ so that the AA does not prosecute any of the non-leniency cases. It also means that no firm applies for leniency because there is no concern of being convicted otherwise.[13] Thus, there is no enforcement when $r = 0$, with or without a leniency program. In this pre-Sherman Act pre-Article 81 world, almost 33% of industries are cartelized at anytime; the remainder do not collude because it is not incentive compatible or they have not had the opportunity to form a cartel since their most recent one collapsed.[14] The average duration of a cartel is 156 periods with a cartel's demise solely being due to internal collapse as a result of strong market conditions.

Now suppose the AA has a mild enforcement policy in that it prosecutes 10% of reported cartels, $r = .1$. When there is no leniency program, 23% of industries are cartelized; hence, enforcement has reduced the cartel rate from its laissez faire level of 33%. 20% of cartels are discovered in any period (as $q = .2$), 10% are prosecuted (as $r = .1$), and the AA gains a conviction in 69% of those cases (see s^* in Table 2a). The compounding of these three probabilities means that there is a 1.4% chance each period that a cartel will be caught and convicted or, in short, penalized. The average cartel duration has dropped from 156 periods (when there is no enforcement) to 43 periods.

To identify the AA's optimal enforcement policy, we need to find the value for r that maximizes AA. The maximal fraction of industries that are successfully prosecuted is .00556 and is achieved when the AA prosecutes 60% of reported cartels. With that prosecution policy, about 9% of industries are cartelized at anytime and each of them faces a 6% chance of being penalized in any period. Average cartel duration is only 12 periods. Note, however, that the policy which minimizes the cartel rate is $r = 1$ so that the AA prosecutes all cases. It is not optimal for the AA to take such an aggressive stance because the reduction in cartels means fewer convicted cartels and the AA is presumed to maximize the number of successful cases. Note that the probability of gaining a conviction is higher at .62 when the AA sets $r = 1$, compared to .51 when $r = .6$. However, if the AA was to be rewarded based on its success rate in court then it would set r very low. If restricted to selecting r from $\{.1, \ldots, 1\}$, it

[13] Of importance here is the assumption that, whenever it exists, firms achieve the equilibrium in which no firm applies for leniency. The rationale for that selection is that it Pareto dominates the equilibrium when all firms do apply for leniency. If the latter equilibrium is selected under some circumstances, then the results would change.

[14] In a richer model, we may want to allow the stochastic process on the opportunity to form a cartel to depend on the enforcement regime. Managers may be more brazen and less inhibited about communicating if enforcement is weaker. This would entail endogenizing κ.

would prosecute only 10% of cases and have a 69% win record.

Table 2b reports results for when there is a leniency program in which the first firm to come forward has all penalties waived. With a leniency program, the optimal policy of the AA is to prosecute 30% of the non-leniency cases and this results in an 8% cartel rate. In any period, a cartel faces a 5.6% chance of being penalized. This penalization rate comes from the leniency program and also conviction through other means. Average cartel duration is 19 periods. If the AA was to prosecute all reported cartels, only 1/100th of 1% of industries would be cartelized; effectively, cartelization would be eliminated.

Before deriving general conclusions regarding the impact of a leniency program, it is useful to discuss the various ways in which a leniency program affects the calculus to form and maintain a cartel. Let us treat σ as fixed and suppose $\theta < \sigma$ so that firms would potentially want to use the leniency program (which is the case with our numerical results because $\theta = 0$). Previous work has shown that the introduction of a leniency program has three effects; these effects are all present in Harrington (2008). First, it makes cartels less stable by reducing the penalties that a firm receives when it cheats; expected penalties to a deviating firm decline from σF to θF, where F is the penalty if convicted in court. This tightens the incentive compatibility constraint in (3) and thereby reduces the maximum market state for which collusion is incentive compatible, ϕ^*. This effect is referred to as the Deviator Amnesty effect. Second, the probability of paying penalties is higher because firms in a collapsing cartel will apply for leniency. Thus, the probability of paying penalties rises from σ to $\sigma H(\phi^*) + (1 - H(\phi^*))$ where $\sigma H(\phi^*)$ is the probability that a cartel does not collapse but is caught and convicted by the AA and $1 - H(\phi^*)$ is the probability that a cartel internally collapses and firms subsequently apply for leniency. This effect is called the Race to the Courthouse effect. Third, a leniency program affects the penalties that cartel members pay in equilibrium. When firms apply for leniency (which occurs when the cartel collapses), penalties are reduced from F to ωF. This is the Cartel Amnesty effect and it serves to promote cartel formation. Ex ante, it is unclear whether a leniency program will make collusion more or less difficult since there are counter-acting forces. Of course, these three effects have been described for when σ is fixed and we are endogenizing σ in the current model by assuming it is lower when the AA's caseload is bigger. This introduces a potentially significant feedback effect in that more effective enforcement (either through a leniency program or the value of r) can reduce the cartel rate, which can then make enforcement more effective (by raising the probability of a conviction) which can lower the cartel rate more, and so forth.

The ensuing analysis will work through three steps. First, the effect of a leniency program on the cartel rate is examined holding fixed the enforcement policy, r. Recall that the enforcement policy refers to the fraction of cartels reported outside of the leniency program which the AA chooses to prosecute. Second, how the AA adjusts its optimal enforcement policy in response to having a leniency program is then characterized. Third, the effect of a leniency program on the cartel rate is investigated while allowing the AA to appropriately adjust its enforcement policy.

The key parameters that we vary are ρ (a higher value for which extends the range of caseloads for which the probability of conviction is concave in the caseload), υ (which impacts the sensitivity of the probability of a conviction to the caseload), λ (which measures the burden of prosecuting a leniency case relative to a non-leniency case), and α (which measures the profitability of the non-collusive solution relative to the collusive solution). Preliminary numerical runs revealed that ρ is a particularly influential parameter so we vary υ, λ, and α for all $\rho \in \{1.3, 1.5, 1.7\}$.

Figure 1 provides information relevant to assessing the impact of a leniency program when the enforcement policy is unchanged. For policies ranging from no enforcement to prosecuting all reported cartels, the cartel rate is reported for the baseline parameterization ($\rho = 1.5$) and also $\rho = 1.3, 1.7$. The cases of no leniency ($\theta = 1$), partial leniency ($\theta = .05, .1$), and full leniency ($\theta = 0$) are considered.[15] For any enforcement policy, the introduction of a full leniency program always reduces the cartel rate. A partial leniency program either has no effect - as it is not utilized by cartels - or it reduces the cartel rate. That a leniency program reduces the cartel rate is confirmed for other parameter values in Figures 2-4 (where we only compare full and no leniency). Unambiguously, we find that a leniency program is effective against collusion. While, as summarized above, there are counter-acting forces with a leniency program, our numerical analysis thus far finds that they net out so that collusion is more difficult.[16]

Property 1 Given the antitrust authority's prosecution policy (that is, r is fixed), the introduction of a leniency program reduces the frequency of cartels.

To examine Property 1 in greater depth, let us draw upon Table 2 which reports results for the baseline configuration. Going hand in hand with the cartel rate declining is that the probability a cartel assigns to being penalized (either through the leniency program or other means) goes up when a leniency program is put in place. For example, when $r = .5$, the introduction of a leniency program increases the probability of a cartel paying penalties from .051 to .095. This is not surprising. When there is no leniency program, the probability of paying penalties is the compounding of the probability of being discovered (which is $q = .2$), of being prosecuted (which is r), and of being convicted, which is endogenous and depends on the caseload. When there is a leniency program, the probability of paying penalties is the sum of the previous probability (though it'll take a different value because the caseload is different) and the probability of applying for leniency, which is the probability that the cartel internally collapses (as all dying cartels apply for leniency). Thus, unless the probability of gaining a conviction in a non-leniency case is sufficiently lower when there is a leniency program, we would expect that the probability of a cartel being penalized is higher when there is a leniency program as a cartel can be penalized by being caught and convicted *or* by applying for leniency after the cartel has collapsed. In fact, with a leniency program in place, the AA not only shuts

[15] Note that leniency is ineffective when $\theta > qrs^* (r)$ as then the only equilibrium is for firms not to apply for leniency. Thus, we only considered low values for θ since $qrs^* (r)$ is generally low.

[16] We have also run the model when σ is fixed - so the AA has no resource constraint - and again a leniency program always reduces the cartel rate.

down cartels by virtue of the leniency program, but the AA is also more successful in winning non-leniency cases. For example, when $r = .5$, the probability of winning a non-leniency case, s^*, rises from .512 to .711. This higher success rate is most likely due to a combination of fewer cartels - and thus fewer cases to prosecute - and that some cases are handled through the leniency program which uses up fewer resources, which leaves more resources to try non-leniency cases.

Holding the enforcement policy fixed, a leniency program always increases the chances of a cartel being penalized, but it may or may not reduce the average cartel duration. In Table 2, if $r = .5$ then the introduction of a leniency program causes the average cartel duration to fall from 14.2 to 10.8 periods. When instead enforcement policy is weak ($r = .1$), the average cartel duration rises from 42.5 to 46.3 periods. With the latter, note that the cartel rate still declines, from 23% to 16.3%. We will later return to the issue of cartel duration.

To explore the impact of a leniency program on the aggressiveness of enforcement by the AA, Figure 5 reports the optimal enforcement policy for various parameter configurations. Let us begin with Figure 5a. With the exception of $\rho = 1.2$, the AA takes on a smaller fraction of non-leniency cases after a leniency program is instituted.[17] For example, for $\rho = 1.5$, it takes on 60% of such cases when there is no leniency program, but only 30% when there is a leniency program. This is not particularly surprising but its implications will prove to be significant. Given that the AA is required to take on all leniency cases, those cases use up resources which, through the probability of conviction function $p(\lambda L + R)$, lowers the probability of winning a conviction in a non-leniency case. However, at the same time, there are fewer cartels when there is a leniency program, holding r fixed (Property 1), which could allow the AA to take on a larger fraction of non-leniency cases. Indeed, this is what occurs for $\rho = 1.2$, but that case is more of an exception. Allowing for different values of υ, λ, and α from the baseline configuration, we again find that a leniency program results in the AA prosecuting a smaller fraction of non-leniency cases.

Property 2 Generally, the introduction of a leniency program results in the antitrust authority pursuing a less aggressive enforcement policy in that it prosecutes a smaller fraction of cartels discovered outside of the leniency program.

Next we turn to the issue of greatest interest: Once the AA adjusts its enforcement policy, what is the impact of a corporate leniency program on the frequency of cartels? Let us begin by examining Table 3. First consider $\rho = 1.3$. In the pre-leniency environment, the AA optimally prosecutes 60% of its cases. The resulting cartel rate is 20.3% and the per period chances that a cartel is penalized is 1.9%. Introducing a leniency program causes the AA to lower its prosecution rate so that it pursues 40% of non-leniency cases. Still, the penalization rate is higher at 4.5% and cartel duration is shorter at 24 periods as compared to 33 periods. Most importantly, the cartel rate is halved from 20.3% to 10.1% with the leniency program. Now suppose

[17] There is also the exception of $\rho = 1.1$ in that there are multiple optima when there is no leniency program.

$\rho = 1.5$. Introducing a leniency program substantially reduces the fraction of non-leniency cases prosecuted from 60% to 30%. The penalization rate falls from 6.1% to 5.6%, and cartel duration rises from 12 to 19 periods. Nevertheless, a leniency program lowers the cartel rate from 9.1% to 8.1%. Finally, suppose $\rho = 1.7$. In the absence of a leniency program, the AA prosecutes 40% of reported cartels, while enforcement declines to 20% with a leniency program. The weaker enforcement lowers the penalization rate from 5.7% to 4.4%, and cartel duration rises significantly from 13 to 24 periods. The most striking finding is that there are now more cartels in the economy. Introducing a leniency program *raises* the cartel rate from 9.9% to 10.5%.

That a leniency program can result in more cartels is not a pathological finding for our model. Figure 6 compares the cartel rate with and without leniency for a wide range of parameter values. Though it is difficult to identify any systematic relationship between these parameters and the incremental impact of a leniency program on the cartel rate, it is observed that a leniency program often ends up raising the cartel rate. Additional results are reported in Tables 4-9 where we find that whenever the cartel rate rises in response to a leniency program, there is also an increase in average cartel duration.

Property 3 When the antitrust authority chooses its optimal prosecution policy, the introduction of a leniency program can either lower or raise the cartel rate, depending on the parameter configuration. It can raise the cartel rate because the antitrust authority less aggressively pursues those cases not generated through the leniency program. Whenever the introduction of a leniency program raises the cartel rate, average cartel duration also rises.

To understand how a leniency program can raise the cartel rate, it is crucial to take account of industry heterogeneity. Recall that industries differ in the parameter η which controls the short-run profit gain from a cartel member deviating from the collusive outcome. Thus, when η is higher, a cartel is less stable in the sense that it'll internally collapse for a wider set of market conditions, which implies shorter average duration. In fact, when η is sufficiently high, a cartel never forms in that it is not incentive compatible for any market conditions. ɧ denotes the highest value for η such that a cartel forms with positive probability.

Figure 7 reports the average cartel duration for each industry type $\eta \leq$ ɧ. An examination of Figure 7a ($\rho = 1.3$) reveals two notable effects from introducing a leniency program. First, it reduces ɧ and thereby shrinks the range of industry types for which a cartel forms with positive probability. This finding holds for all of our parameterizations, as evidenced by the output in Tables 3-9; ɧ is always lower with a leniency program. Second, for those industries that do cartelize with positive probability (that is, $\eta \leq$ ɧ), the average duration of a cartel is shorter. This latter result, however, is specific to that parameterization. In Figure 7c ($\rho = 1.7$), average duration for those industries that still cartelize actually increases with the introduction of a leniency program. When $\rho = 1.5$, this holds true for all values except those close to ɧ (see Figure 7b). This finding is confirmed for other parameterizations in Figures 8-10. In sum, the institution of a leniency program causes fewer cartels to form but those that do form can last for a longer time.

To understand the determination of duration, recall that a cartel shuts down when it internally collapses (due to strong market conditions) or it is discovered and successfully prosecuted by the AA; the latter occurs with probability $\sigma^{*}(\mathbf{b}) \equiv q\mathbf{b}s^{*}(\mathbf{b})$ where $\mathbf{b}$ is the AA's optimal enforcement policy. Holding r fixed, a leniency program can make collusion less profitable because, in response to a cartel collapsing, all firms apply for leniency and this raises expected penalties.[18] With higher penalties, the collusive value is reduced which means market conditions don't have to be as strong for collusion to no longer be incentive compatible. With internal collapse being more likely, average cartel duration is shorter. This effect, however, is weaker for industries with a lower value for η. Since cartels in those industries are more stable, they are less likely to collapse and thus the leniency program is less likely to be used. All this adds up to expected penalties not rising as much for industries with a lower value for η. In unreported results, we show that, holding the enforcement policy fixed, the availability of leniency decreases average cartel duration for all industry types, but the decrease isn't as large when η is lower.

Thus far, we've argued that a leniency program reduces cartel duration, holding the enforcement policy fixed. To explain how a leniency program can actually increase cartel duration - as shown, for example, in Figure 7b - we need to take account of how the AA adjusts its enforcement policy in response to having a leniency program. By Property 2, we know that the AA will lower r which can mean weaker enforcement in terms of a lower probability a cartel is caught outside of the leniency program and successfully prosecuted by the AA. That effect is clearly beneficial to cartels from all industries but is especially advantageous for industries with low values of η because a more stable cartel is more affected by detection through non-leniency means than through the leniency program (which is used only when the cartel collapses). For consider the extreme case when η is so low that a cartel in such an industry never collapses on its own, so shut down occurs only when it is caught and convicted by the AA. The leniency program's only effect is then through a weaker enforcement policy, in which case collusion is easier for those cartels and this translates into longer duration. Consistent with this explanation, the increase in cartel duration from a leniency program is greater when η is lower and thus cartels are more stable (see, for example, Figure 7c).

Property 4 When the antitrust authority chooses its optimal prosecution policy, the introduction of a leniency program: 1) reduces the range of industries that cartelize; and 2) can either raise or lower the average cartel duration, depending on the parameter configuration and the industry type. A leniency program reduces average cartel duration more for cartels arising in an industry with a higher value of η so that cartels are less stable.

In sum, a leniency program is highly detrimental to marginally stable cartels as

[18] In resonse to the discontinuation of collusion, firms want to minimize expected penalties. It is not an equilibrium for all firms not to apply since, with full leniency, any single firm can avoid all penalties by applying. But once one firm is expected to apply then it is optimal for all to do so.

they no longer form (as reflected in þ declining). This effect serves to reduce the cartel rate. At the same time, a leniency program can result in highly stable cartels having even longer lifetimes because enforcement outside of the leniency program is weaker. It is the latter effect that can cause a leniency program to result in a higher cartel rate. Fewer cartels form but those that do form last for a longer time, which translates into a higher frequency of cartels at any moment in time.

Finally, we conclude by describing what could be achieved with a leniency program if instead the AA implemented the policy that minimizes the cartel rate. Whether all cartels should be prosecuted depends on how a more aggressive enforcement policy affects the size of the caseload. The marginal effect of an increase in enforcement on the caseload is

$$\frac{\partial qrC\left(\sigma^{*}\left(r\right)\right)}{\partial r} = qC\left(\sigma^{*}\left(r\right)\right) + qrC'\left(\sigma^{*}\left(r\right)\right)\frac{\partial \sigma^{*}\left(r\right)}{\partial r}. \tag{15}$$

Raising the enforcement rate has two effects on the caseload which are the two terms in (15). It has a direct effect in that the caseload goes up by $qC\left(\sigma^{*}\left(r\right)\right)$. It also has an indirect effect, the sign of which is the opposite of the sign of $\partial \sigma^{*}\left(r\right)/\partial r$ since $C' < 0$. If $\partial \sigma^{*}\left(r\right)/\partial r > 0$ then more aggressive enforcement raises the penalty rate which serves to reduce the cartel rate and thereby lower the caseload. As more aggressive enforcement lowers the cartel rate then it actually reduces the size of the first effect, $qC\left(\sigma^{*}\left(r\right)\right)$, which makes it attractive to be even more aggressive. In other words, prosecuting a bigger fraction of cartels reduces the cartel rate which makes it easier to prosecute an even higher fraction of cartels which lowers the cartel rate more, and so on. In that case, the socially optimal policy would be to prosecute all cartels. Alternatively, if $\partial \sigma^{*}\left(r\right)/\partial r < 0$ then more aggressive enforcement lowers the penalty rate for cartels - because the conviction rate $s^{*}\left(r\right)$ is falling - which then increases the cartel rate and thereby expands the caseload. As both the direct and indirect effects have more aggressive enforcement increasing the caseload, this limits how much enforcement is socially optimal.

Table 10 reports the enforcement policy that minimizes the cartel rate along with the resulting cartel rate.[19] Without a leniency program, it is often socially optimal to prosecute all cartels though that need not always be the case. In particular, it can be best to prosecute only some cartels when the probability of conviction is very sensitive to the caseload. Recall that the probability of conviction is concave and then convex in the caseload with the inflection point increasing in ρ. Thus, when ρ is lower, the probability of conviction is more convex in the caseload so that it declines more significantly as the caseload increases. When υ is high, the probability of conviction is higher and it falls faster with the caseload (at least when the caseload is low). In those cases, the cartel rate is minimized by not prosecuting all reported cartels.

Since, by Property 1, a leniency program reduces the cartel rate (holding the enforcement policy fixed), the direct effect of enforcement on caseload, $qC\left(\sigma^{*}\left(r\right)\right)$, will be smaller with a leniency program and that makes it more likely that the

[19] Where the first-best policy is reported as a range of values, all of them are optima.

socially optimal policy is to prosecute all reported cartels. Indeed, that is what we find in Table 10. At least for the parameterizations considered, all cartels should be prosecuted when there is a leniency program. Note that if the AA pursued the socially optimal policy then a leniency program always seems to reduce the cartel rate.

Property 5 When the antitrust authority chooses a socially optimal prosecution policy, the introduction of a leniency program reduces the cartel rate.

6 Concluding Remarks

Theare two primary contributions of this paper. First, it provides a more complete assessment of the effects of a corporate leniency program by taking account of its impact on enforcement policy with respect to cartels discovered outside of the program. We find that the antitrust authority weakens its enforcement policy with respect to the prosecution of cases not involving leniency, and this can be so severe as to actually cause the frequency of cartels to rise after the adoption of a leniency program. A surfeit of leniency applications need not mean a lower cartel rate. A second contribution is to offer a more comprehensive framework for investigating the implications of antitrust policy. By blending the population model of cartels in Harrington and Chang (2008) with a model of an optimizing antitrust authority, the effect of different policies on the cartel rate can be examined.

Using this framework, there are many other issues that can be addressed. In this paper, we considered a leniency program that only applies prior to an investigation. However, many programs allow at least partial leniency if a firm comes forward after an investigation has begun. Within our model, this could be handled by providing an option for leniency after the authority decides to prosecute. Insight can be gained into whether or not leniency should only be awarded prior to an investigation. Another policy issue concerns the budget of the antitrust authority. A bigger budget makes the probability of conviction less sensitive to the caseload, and that lower sensitivity ought to mean a lower cartel rate. The question is whether the marginal effect of a bigger budget on the cartel rate is higher or lower after a leniency program is adopted. It is quite possible that an effective leniency program could significantly lower the cartel rate that a bigger budget could compound its efficacy and effectively eliminate collusion. Finally, more thought needs to be given to specifying the preferences of an antitrust authority. This is an unexplored area that warrants research. Whatever those preferences are, the framework developed here can be modified to encompass them and evaluate what are their implications for the efficacy of antitrust policy.

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Table 2: Baseline Results
(ρ = 1.5)

(A) no leniency (θ = 1)

r	$s^*(r)$	$q \times r \times s^*(r)$	$L(qrs^*(r))$	$R(qrs^*(r))$	AA	cartel rate	% cartels penalized	avg. duration of a cartel
0.	.800781	0.	0.	0.	0.	.326347	0.	155.568
.1	.691406	.0138281	0.	.0045959	.00317763	.229795	1.38281	42.5436
.2	.613281	.0245313	0.	.00721283	.0044235	.180321	2.45312	26.9193
.3	.5625	.03375	0.	.00896896	.00504504	.149483	3.375	20.4948
.4	.529297	.0423438	0.	.0101603	.00537783	.127004	4.23437	16.7763
.5	.511719	.0511719	0.	.0108177	.00553563	.108177	5.11719	14.1663
.6	.507813	.0609375	0.	.0109584	.00556482	.0913201	6.09375	12.097
.7	.519531	.0727344	0.	.0105471	.00547952	.0753361	7.27344	10.2748
.8	.546875	.0875	0.	.00951043	.00520102	.0594402	8.75	8.68384
.9	.578125	.104063	0.	.008388	.00484931	.0466	10.4062	7.36874
1.	.615234	.123047	0.	.00712106	.00438112	.0356053	12.3047	6.28207

(B) full leniency (θ = 0)

r	$s^*(r)$	$q \times r \times s^*(r)$	$L(qrs^*(r))$	$R(qrs^*(r))$	AA	cartel rate	% cartels penalized	avg. duration of a cartel
0.	.800781	0.	0.	0.	0.	.326347	0.	155.568
.1	.707031	.0141406	.00172931	.00322326	.00400825	.162892	2.46068	46.2674
.2	.673828	.0269531	.00146561	.00447425	.00448049	.113322	3.95377	26.9705
.3	.666016	.0399609	.0013304	.00478344	.00451625	.0810545	5.57187	18.8118
.4	.681641	.0545313	.00114561	.0043661	.00412173	.0557219	7.39696	14.0095
.5	.710938	.0710938	.000915516	.00348462	.00339286	.0357617	9.48742	10.819
.6	.748047	.0897656	.000641522	.00236427	.0024101	.0203437	11.8469	8.58553
.7	.775391	.108555	.000423745	.00141355	.0015198	.0105205	14.446	7.00619
.8	.789063	.12625	.000263309	.000736697	.000844609	.00486767	17.3514	5.82197
.9	.798828	.143789	.0000908122	.000214449	.00026212	.0012822	20.4431	4.91776
1.	.800781	.160156	.0000150377	.0000186852	.0000300005	.000108464	27.6594	3.6154

Table 3: Impact of the Leniency Policy on Cartel Behavior
for $\rho \in \{1.0, 1.1, ..., 1.9, 2.0\}$

	Optimal Enforcement Policy		Cartel Rate		Penalization Rate		Cartel Duration		bη	
ρ	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency
1.0	.9	.8	.280756	.198025	.005625	.0167513	80.2996	74.9596	1.7764	1.4916
1.1	1. (*)	.9	.265363	.173012	.0078125	.0218537	64.6766	52.9967	1.7586	1.4649
1.2	.5	.9	.240116	.133414	.0119141	.03268	47.6928	33.3012	1.7319	1.4204
1.3	.6	.4	.202972	.100516	.0192187	.0448551	32.8335	23.5487	1.6874	1.3759
1.4	.8	.3	.138895	.0938382	.0375	.0482288	18.701	21.8319	1.5895	1.367
1.5	.6	.3	.0913201	.0810545	.0609375	.0557187	12.097	18.8118	1.5005	1.3492
1.6	.5	.2	.0870105	.106969	.0638672	.0420231	11.5838	25.2437	1.4916	1.3848
1.7	.4	.2	.0986173	.104659	.0565625	.0438959	12.9266	24.2417	1.5183	1.3848
1.8	.4	.2	.092889	.101227	.0596875	.0442401	12.358	23.8562	1.5005	1.3759
1.9	.4	.2	.0907345	.100516	.0614062	.0448551	12.0019	23.5487	1.5005	1.3759
2.0	.4	.2	.0886194	.10009	.0625	.0452272	11.8467	23.3676	1.4916	1.3759

* At $\rho = 1.1$, there are other optima at .4, .5, and .8 for the case of no leniency. However, they all yield the identical values for the variables reported above.

Table 4: Impact of the Leniency Policy on Cartel Behavior
for $\upsilon \in \{100, 200, ..., 1000\}$

	Optimal Enforcement Policy		Cartel Rate		Penalization Rate		Cartel Duration		bη	
υ	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency
100	.4	.2	.0922987	.101227	.0601562	.0442401	12.2589	23.8562	1.5005	1.3759
200	.4	.2	.0992805	.105277	.0560937	.0433882	13.0359	24.5019	1.5183	1.3848
300	.5	.2	.0863227	.107153	.0644531	.041878	11.4747	25.3169	1.4916	1.3848
400	.5	.2	.0956224	.111326	.0582031	.0410243	12.6165	26.0574	1.5094	1.3937
500	.6	.3	.0913201	.0810545	.0609375	.0557187	12.097	18.8118	1.5005	1.3492
600	.7	.2	.0959345	.117878	.0579687	.0385085	12.6687	27.9204	1.5094	1.4026
700	.8	.3	.126689	.0879494	.0425	.051208	16.7166	20.4895	1.5717	1.3581
800	.6	.3	.158486	.0932563	.0309375	.0487899	22.0555	21.5996	1.6251	1.367
900	.5	.3	.176231	.0983806	.0257812	.0467452	25.7532	22.6667	1.6518	1.3759
1000	.4	.3	.190102	.101156	.0221875	.0443013	29.2032	23.8251	1.6696	1.3759

Table 5: Impact of the Leniency Policy on Cartel Behavior for $\lambda \in \{0.1, 0.2, ..., 1.0\}$

	Optimal Enforcement Policy		Cartel Rate		Penalization Rate		Cartel Duration		ɧ	
λ	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency
.1	.6	.2	.0913201	.111659	.0609375	.0407727	12.097	26.2057	1.5005	1.3937
.2	.6	.2	.0913201	.112159	.0609375	.0403987	12.097	26.431	1.5005	1.3937
.3	.6	.2	.0913201	.112491	.0609375	.0401512	12.097	26.5831	1.5005	1.3937
.4	.6	.3	.0913201	.0805455	.0609375	.0563209	12.097	18.6315	1.5005	1.3492
.5	.6	.3	.0913201	.0810545	.0609375	.0557187	12.097	18.8118	1.5005	1.3492
.6	.6	.3	.0913201	.0813973	.0609375	.055316	12.097	18.9348	1.5005	1.3492
.7	.6	.3	.0913201	.0818917	.0609375	.0547391	12.097	19.1189	1.5005	1.3492
.8	.6	.3	.0913201	.0822252	.0609375	.0543531	12.097	19.244	1.5005	1.3492
.9	.6	.2	.0913201	.117514	.0609375	.0387619	12.097	27.7535	1.5005	1.4026
1.	.6	.2	.0913201	.118059	.0609375	.0383829	12.097	28.0047	1.5005	1.4026

Table 6: Impact of the Leniency Policy on Cartel Behavior for $\alpha \in \{0, 0.1, 0.2, ..., 0.9\}$

	Optimal Enforcement Policy		Cartel Rate		Penalization Rate		Cartel Duration		ɧ	
α	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency
.0	.6	.3	.0913201	.0810545	.0609375	.0557187	12.097	18.8118	1.5005	1.3492
.1	.5	.2	.0863133	.0947043	.0570312	.0415374	12.5562	25.5612	1.4649	1.3492
.2	.4	.2	.0837056	.0783083	.0503125	.0445457	13.7115	24.0163	1.4293	1.3136
.3	.4	.2	.0647443	.0592745	.0539062	.0470116	12.4517	22.5497	1.367	1.2691
.4	.3	.1	.0641951	.0722964	.0428906	.0304881	14.5004	36.562	1.3403	1.2691
.5	.3	.1	.0439759	.0484535	.045	.0337216	13.0493	32.9225	1.278	1.2246
.6	.2	.1	.039026	.0224783	.0310156	.0354393	16.0839	29.758	1.2424	1.1712
.7	.2	.1	.0166839	.00416149	.0317187	.0427619	13.2716	23.903	1.1801	1.1267
.8	.1	.0	.00500245	.0165807	.0159766	.0	14.0517	26.5775	1.1356	1.1623
.9	.0	.0	.0	.0	.0	.0	.0	.0	1.1	1.1

Table 7: Impact of the Leniency Policy on Cartel Behavior
for $\delta \in \{.75, .85, .9\}$

	Optimal Enforcement Policy		Cartel Rate		Penalization Rate		Cartel Duration		$\mathfrak{h}$	
δ	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency
.75	.5	.2	.0656841	.0770546	.0632812	.0496593	10.8533	22.0817	1.4293	1.3225
.85	.6	.3	.0913201	.0810545	.0609375	.0557187	12.097	18.8118	1.5005	1.3492
.90	.6	.2	.119522	.131551	.0515625	.0357735	14.535	29.4743	1.6607	1.4293

Table 8: Impact of the Leniency Policy on Cartel Behavior
for $\kappa \in \{.025, .05, .1\}$

	Optimal Enforcement Policy		Cartel Rate		Penalization Rate		Cartel Duration		$\mathfrak{h}$	
κ	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency
.025	.4	.2	.073827	.0902084	.0521875	.0397653	14.4871	26.7658	1.5272	1.4471
.05	.6	.3	.0913201	.0810545	.0609375	.0557187	12.097	18.8118	1.5005	1.3492
.1	1.	.2	.10126	.103686	.065625	.0440132	10.5324	24.5037	1.4827	1.3136

Table 9: Impact of the Leniency Policy on Cartel Behavior
for $\tau \in \{.6, .8, 1\}$

	Optimal Enforcement Policy		Cartel Rate		Penalization Rate		Cartel Duration		$\mathfrak{h}$	
τ	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency
.6	.9	.3	.149483	.112574	.03375	.0400895	20.4948	26.6214	1.6162	1.3937
.8	.6	.3	.0913201	.0810545	.0609375	.0557187	12.097	18.8118	1.5005	1.3492
1	.4	.2	.0913201	.0923258	.0609375	.0496988	12.097	21.238	1.367	1.367

Table 10: First-Best Enforcement Policy and the Cartel Rate

	w/o leniency		w/leniency	
ρ	first-best policy	cartel rate	first-best policy	cartel rate
1.0	.9	.280756	1.	.194444
1.1	1.	.265363	.9	.173012
1.2	.5	.240116	1.	.00011248
1.3	.6	.202972	1.	.00011248
1.4	.8	.138895	1.	.000108464
1.5	1.	.0356053	1.	.000108464
1.6	1.	.0266876	1.	.000108464
1.7	1.	.0241605	1.	.000108464
1.8	1.	.0225701	1.	.000108464
1.9	1.	.0221739	1.	.000108464
2.0	1.	.0219361	1.	.000108464

	w/o leniency		w/leniency	
ν	first-best policy	cartel rate	first-best policy	cartel rate
100	1.	.0225701	1.	.000108464
200	1.	.0245118	1.	.000108464
300	1.	.0267849	1.	.000108464
400	1.	.0305144	1.	.000108464
500	1.	.0356053	1.	.000108464
600	1.	.0466786	1.	.000108464
700	.8	.126689	1.	.000108464
800	.6	.158486	1.	.000108464
900	.5	.176231	1.	.000108464
1000	.4	.190102	1.	.000108464

	w/o leniency		w/leniency	
λ	first-best policy	cartel rate	first-best policy	cartel rate
.1	1.	.0356053	1.	.000108464
.2	1.	.0356053	1.	.000108464
.3	1.	.0356053	1.	.000108464
.4	1.	.0356053	1.	.000108464
.5	1.	.0356053	1.	.000108464
.6	1.	.0356053	1.	.000108464
.7	1.	.0356053	1.	.000108464
.8	1.	.0356053	1.	.000108464
.9	1.	.0356053	1.	.000108464
1.	1.	.0356053	1.	.000108464

	w/o leniency		w/leniency	
α	first-best policy	cartel rate	first-best policy	cartel rate
.0	1.	.0356053	1.	.000108464
.1	1.	.0249287	1.	.0
.2	1.	.0171529	.9-1.	.0
.3	1.	.0113936	.8-1.	.0
.4	1.	.00647866	.7-1.	.0
.5	1.	.00249799	.6-1.	.0
.6	1.	.000394471	.5-1.	.0
.7	.8-1.	.0	.3-1.	.0
.8	.4-1.	.0	.1-1.	.0
.9	.0-1.	.0	.0-1.	.0

Figure 1 : Effects of Prosecution Policy on the Cartel Rate
(Baseline Case)

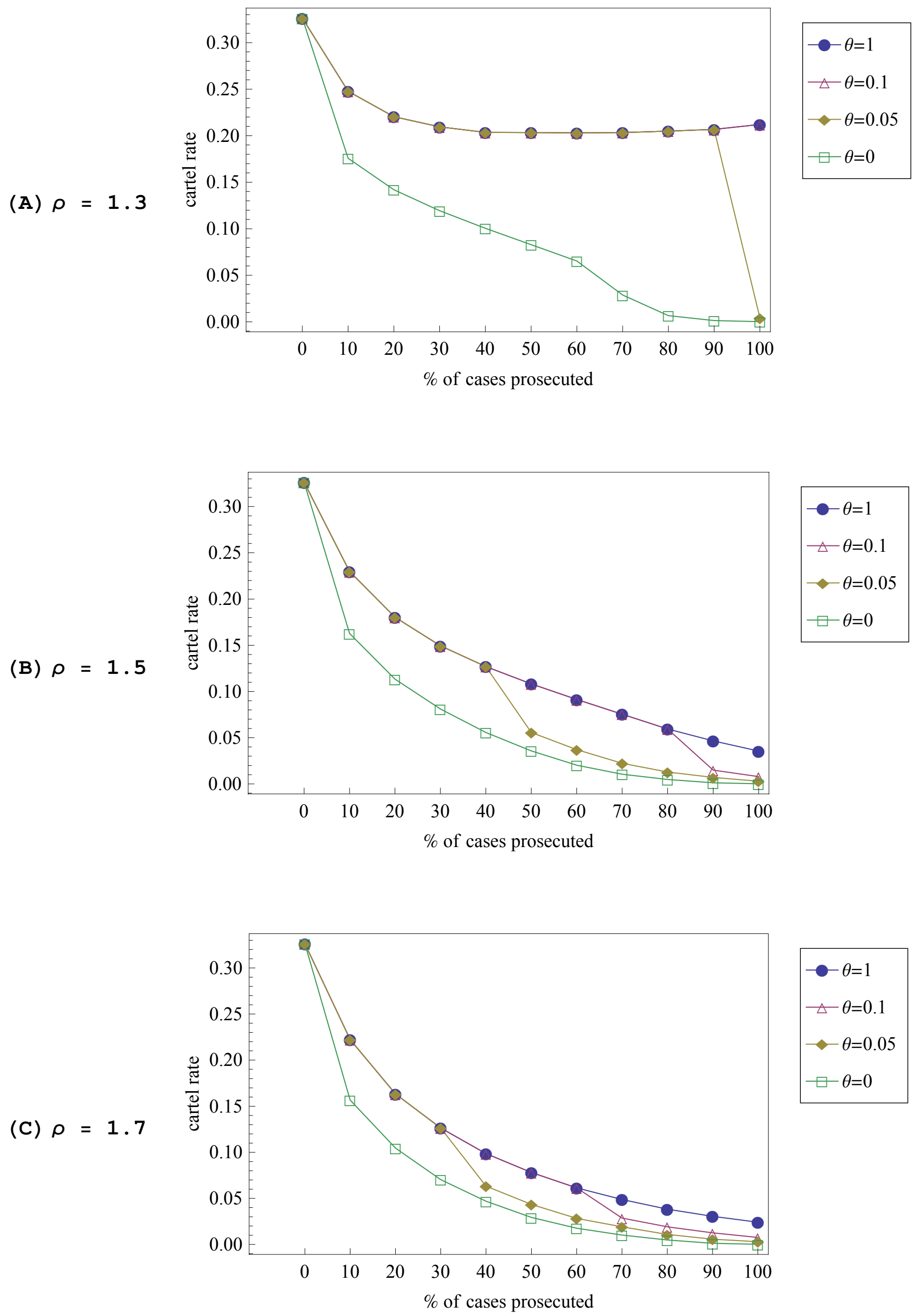


Figure 2 : Effects of Prosecution Policy on the Cartel Rate
for $\nu \in \{100, 1000\}$

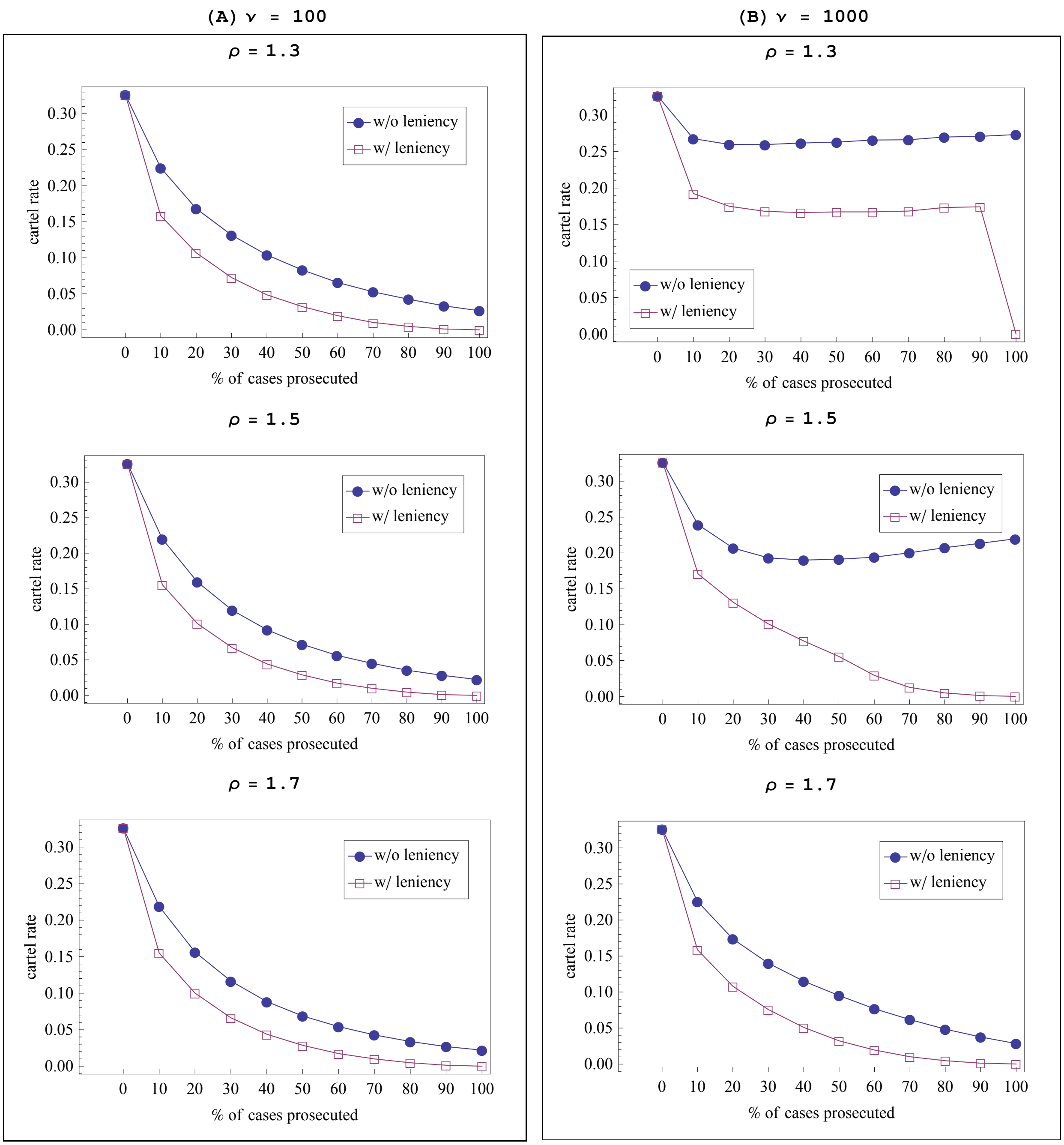


Figure 3 : Effects of Prosecution Policy on the Cartel Rate
for $\lambda \in \{0.2, 0.8\}$

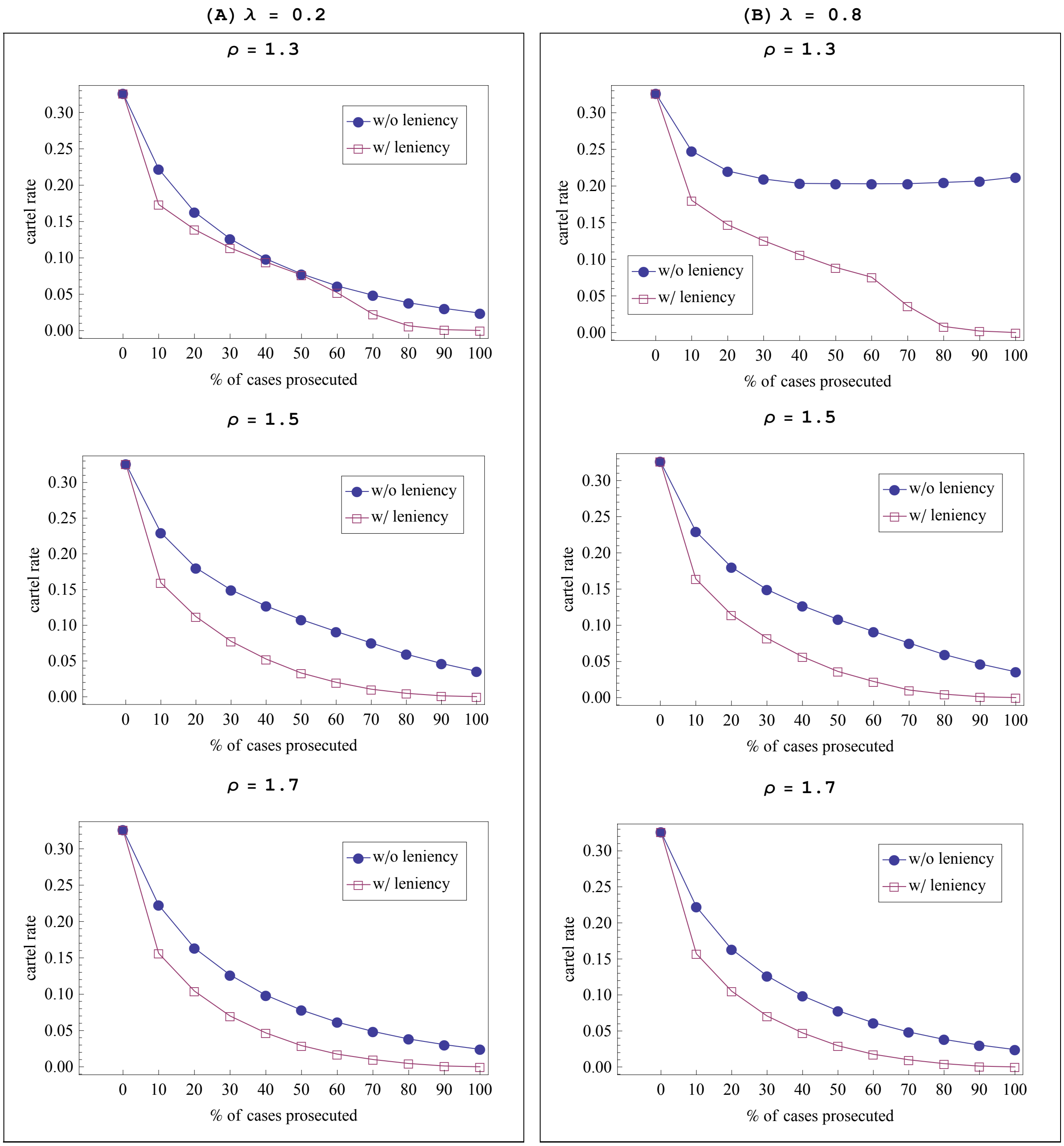


Figure 4 : Effects of Prosecution Policy on the Cartel Rate
for $\alpha \in \{0.2, 0.5\}$

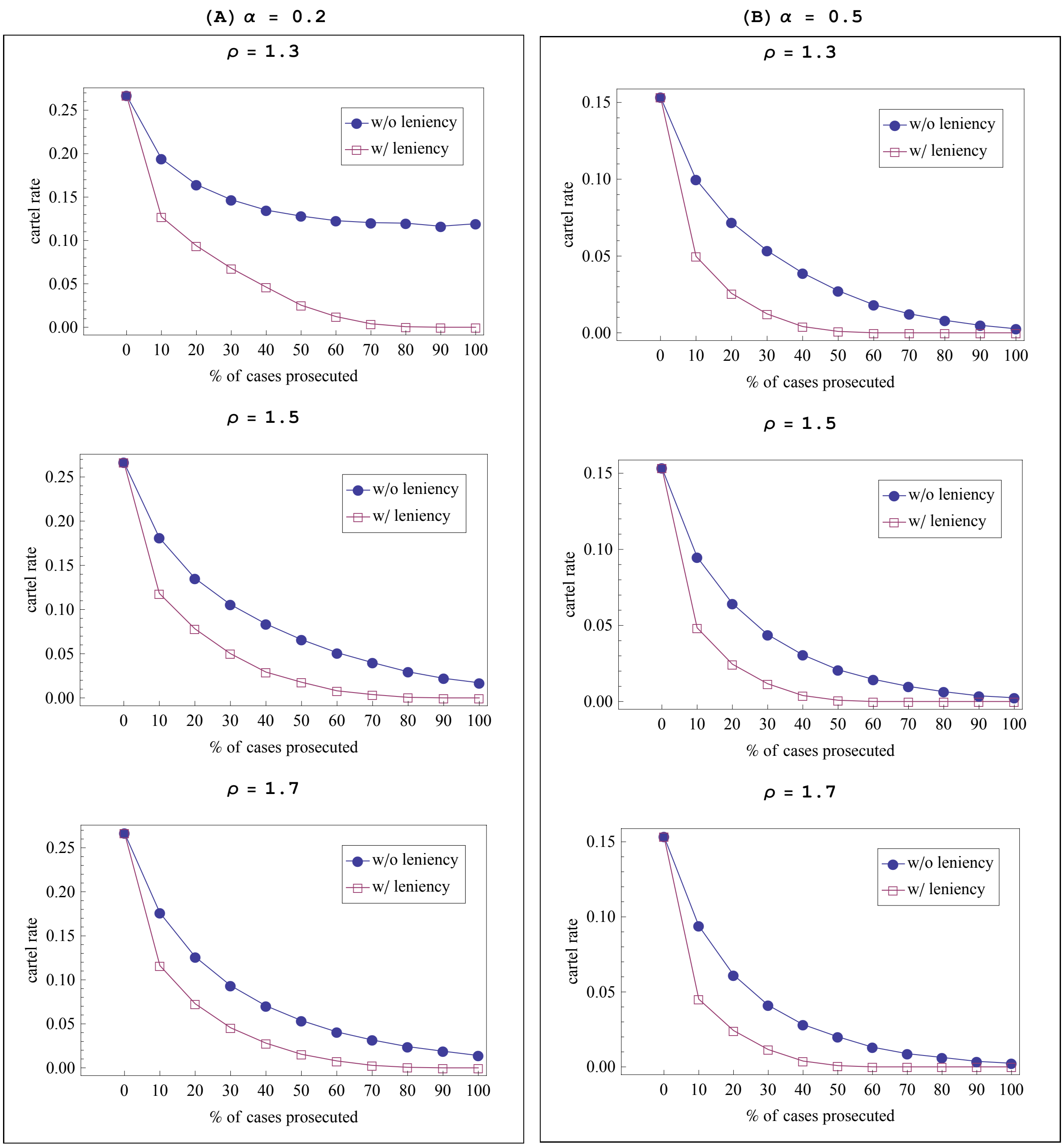
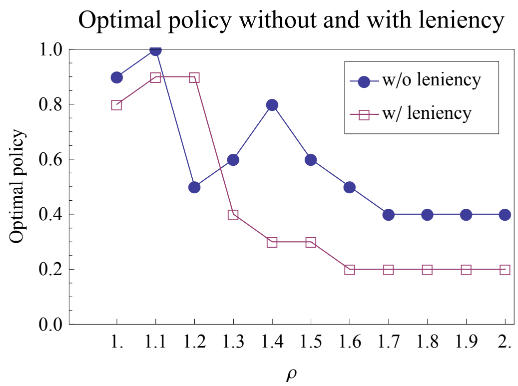
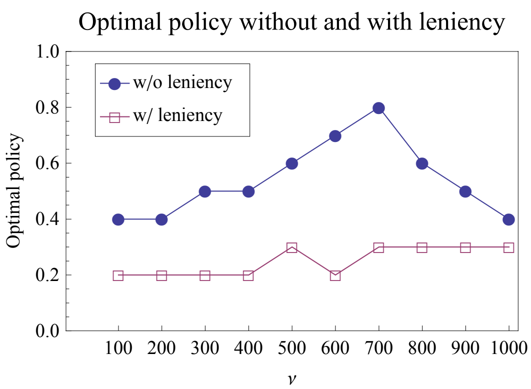


Figure 5 : Optimal Enforcement Policy with and without Leniency
for ρ, ν, λ, and α

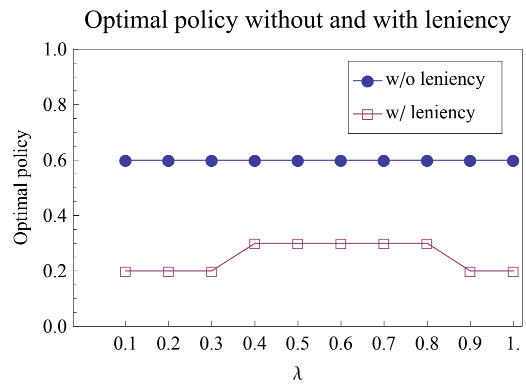
(A) ρ



(B) ν



(C) λ



(D) α

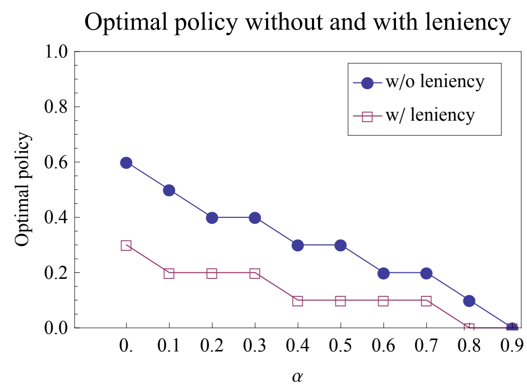


Figure 6 : Endogenous Cartel Rates with and without Leniency
for ρ, ν, λ, and α

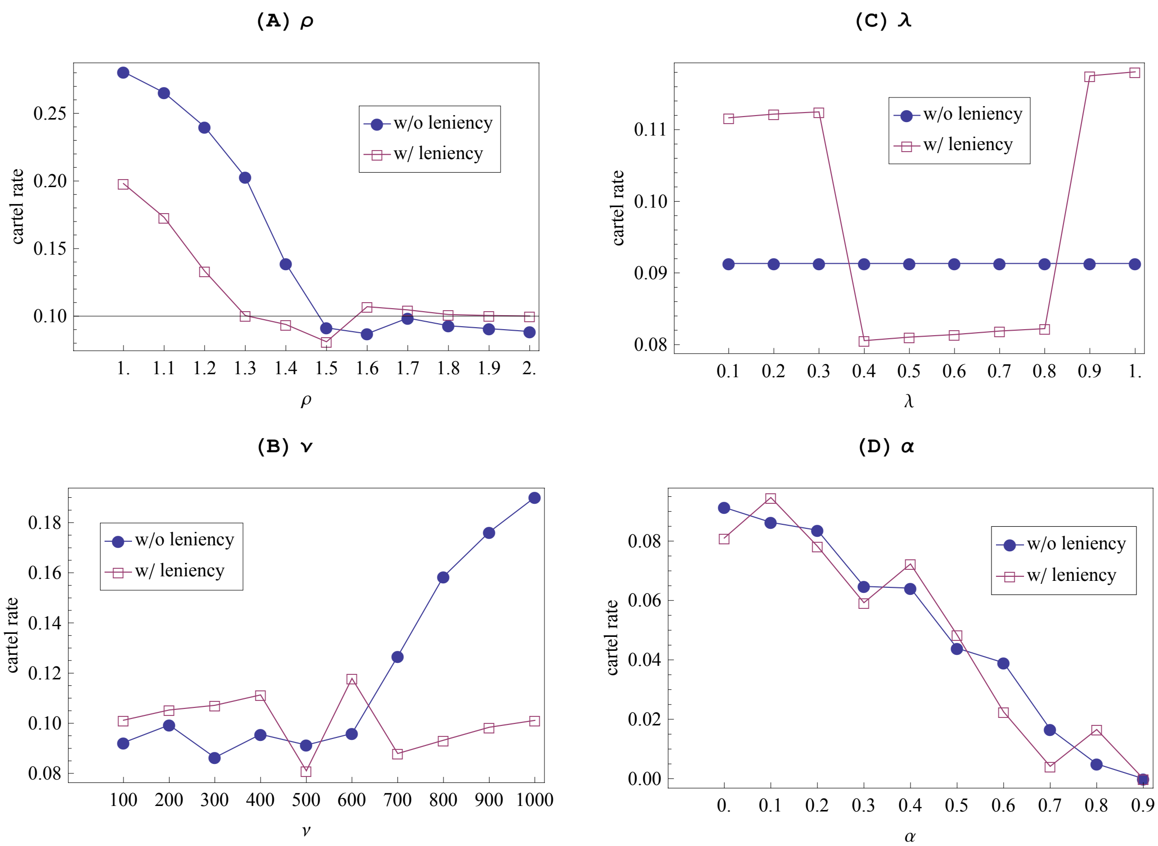
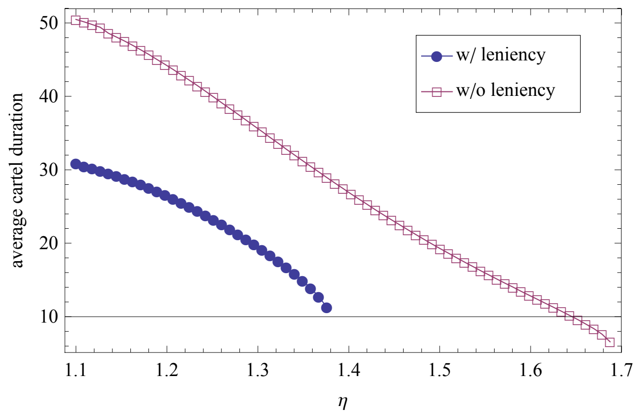
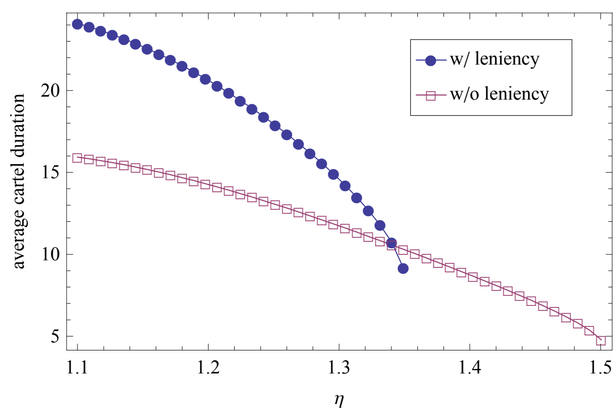


Figure 7 : Average Cartel Duration Conditional on η
(Baseline Case)

(A) ρ = 1.3



(B) ρ = 1.5



(C) ρ = 1.7

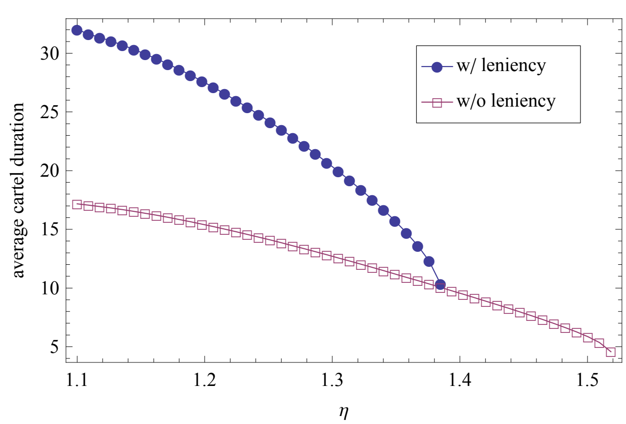


Figure 8 : Average Cartel Duration Conditional on η
for $\nu \in \{100, 1000\}$

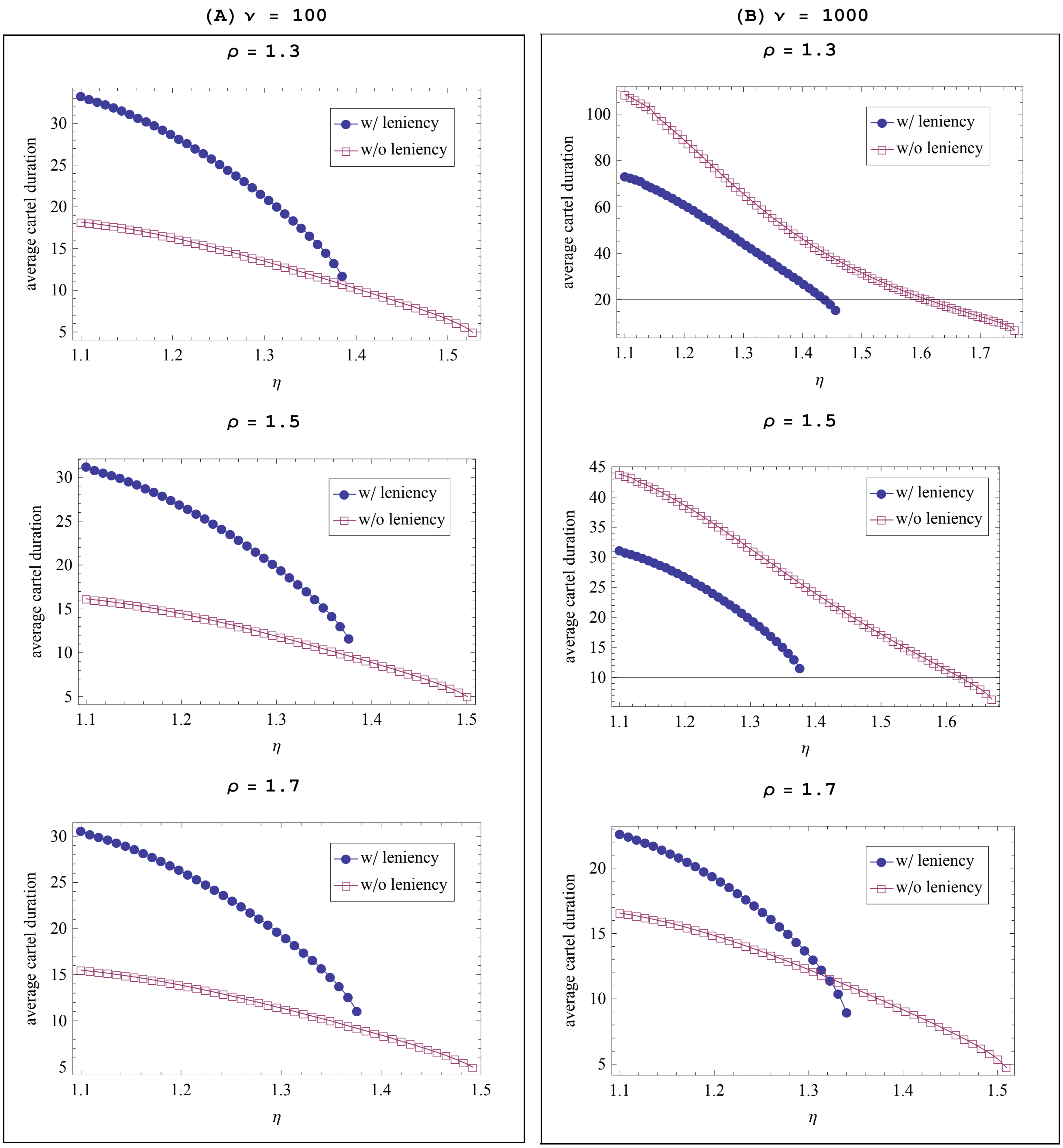


Figure 9 : Average Cartel Duration Conditional on η
for $\lambda \in \{0.2, 0.8\}$

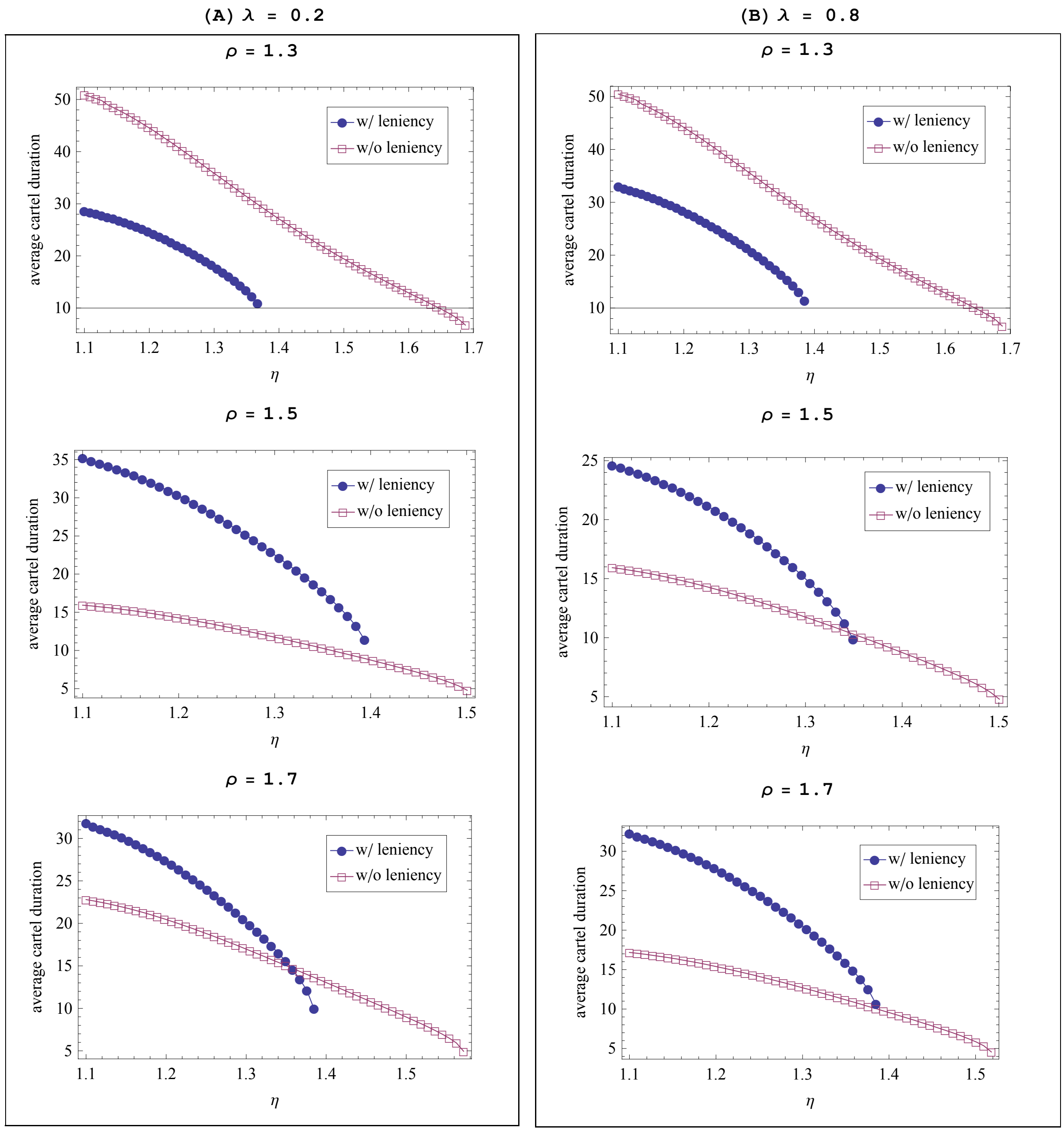


Figure 10 : Average Cartel Duration Conditional on η
for $\alpha \in \{0.2, 0.5\}$