The New Keynesian Phillips Curve and the Cyclicality of Marginal Cost

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Abstract

Several authors have argued that if the labor share of income is used as the proxy for real marginal cost, then the sticky-price version of the New Keynesian Phillips Curve does a good job of approximating US inflation dynamics. However, this paper argues that the labor share is an inappropriate measure of real marginal cost for two reasons: it is countercyclical whereas theory predicts marginal cost should be procyclical, and it employs a counterfactual assumption about the behavior of labor over the business cycle. Relaxing this assumption to a more realistic one leads to a measure of marginal cost that is markedly procyclical. Testing this improved measure of marginal cost then produces results that are contradictory to the entire underlying model of the NKPC. Thus I conclude that the NKPC fails to give a sound explanation of inflation dynamics.

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1 Introduction

Fischer (1977), Taylor (1980), and Calvo (1983), amongst others, laid the foundations for modern econometric analysis of inflation by looking at nominal wage and price-setting by forward-looking individuals and firms. When we aggregate over such individual behavior, we are left with a relation between short-run inflation and real marginal cost. This is the 'New Keynesian Phillips Curve' (henceforth NKPC), which draws much interest since it is a macroeconomic relationship borne out of explicit micro-foundations. It also has the appeal of being simple enough to be useful for theoretical policy analysis. Although one can point out that there are overly restrictive assumptions that underly the NKPC, it continues to be commonly used for the lack of a better alternative. To this day, the partial equilibrium, sticky-price version of the NKPC is a widely popular model amongst many economists, leading numerous of them to conclude that it does a good job of explaining short-run inflation dynamics.

Gali and Gertler (1999) were the first to claim empirical success of the marginal cost-based NKPC, where they use labor's share of income as their proxy for real marginal cost. They find that *"real marginal costs are a significant and quantitatively important determinant of inflation."* Many authors have since tested the NKPC with the labor share, and there is plenty of evidence to suggest that the model is successful with this measure. However, some economists have taken the opposing view and questioned using a NKPC with the labor income share, such as Rudd and Whelan (2007), who correctly point out that the labor share represents average cost and not marginal cost. In this paper, I explicitly detail the reasons as to why the labor income share is a poor approximation to real marginal cost, and then improve upon existing work by developing a more reasonable method of computing marginal cost.

This paper argues that the labor income share is a poor proxy for real marginal cost for two main reasons: first, the labor share is countercyclical. However, standard microeconomic theory predicts that short-run marginal cost should be procyclical. This has been empirically confirmed by many authors, such as Bils (1987) and Rotemberg and Woodford (1999). Yet the labor income share is countercyclical, as it rises during times of recession, contrary to what intuition and theory tells us about marginal cost.

The second flaw of the labor share measure of marginal cost is that it is based on an overly restrictive assumption. Specifically, Gali and Gertler assume that labor input can be adjusted freely at a fixed real wage rate. However this assumption is too simplistic once we examine it closely. In practice, labor can be decomposed into the number of employees multiplied by the respective hours that they work, and as Oi (1962) shows, employment is *quasi*-fixed in the short run. This means that labor will not be perfectly flexible over the business cycle.

Instead, it is hours which can be adjusted flexibly, with little-to-no adjustment costs.

This is particularly true of some industries, such as the manufacturing sector, where hours are frequently varied. However hours are not adjusted at a fixed wage rate; instead, wages in these industries are dependent on the number of hours worked, since varying hours necessitates that firms give their workers overtime pay. Hence labor cannot be adjusted costlessly at a fixed wage rate.

This paper seeks to improve upon the measurement of marginal cost. To do this, I use Bils' idea that under cost minimization, the marginal cost of increasing output can be computed by looking at the cost of increasing any of the inputs of production, where that input is chosen freely, whilst holding the other inputs fixed at their optimal levels. Hence, we are free to choose any margin along which to measure marginal cost. Since labor can be decomposed into employment and hours, I examine hours adjusted at the margin, whilst holding the other inputs fixed. I then improve upon existing methods by combining this with a wage rate that is dependent on the number of hours worked, so that the payment to each extra hour worked is not fixed as it is in the Gali and Gertler framework. Indeed, the general expression for real marginal cost now includes overtime hours as one of its components.

Computing the general expression of marginal cost requires good data on overtime hours and overtime premia, but this data is not available for the aggregate economy. However, reliable data is published for the manufacturing sector, which is the industry that I therefore focus on. Applying the general expression for real marginal cost to manufacturing data produces a series that is highly procyclical, just as theory suggests it should be. The fact that we have a marginal cost variable that behaves as theory predicts it should, makes this a much more plausible proxy for real marginal cost than the countercyclical labor income share.

Since I look at marginal cost for the manufacturing sector, I need a sector-level version of the NKPC for testing purposes. I provide this by adapting the underlying equations of the NKPC to industry-level ones, and from this I derive the relationship that describes the behavior of inflation. Testing this 'disaggregated NKPC' with the improved measure of marginal cost yields a statistically significant negative coefficient on real marginal cost. In order for the NKPC to be valid, this coefficient must be positive and significant. Therefore the more reasonable measure of marginal cost produces results which strongly reject the model.

Finally, proponents of the NKPC respond to criticisms that the model does not fit the data too well by arguing that results are vastly improved by including a lag of inflation in the model: the so-called ‘hybrid NKPC’. However, I show how testing the hybrid NKPC with procyclical marginal cost also yields a negatively significant coefficient on marginal cost. Given these results, one must conclude that the NKPC with the labor income share does not do a good job of explaining inflation dynamics.

2 Labor Income Share in the NKPC

2.1 Background

The sticky-price NKPC[1], which is derived from Calvo’s model of random price adjustment, is commonly recognizable in the following form:

$$\pi_t = \lambda x_t + \beta E_t\{\pi_{t+1}\} \qquad \text{(NKPC)}$$

where π_t is the rate of inflation, and x_t is real marginal cost. Early attempts at examining the NKPC’s empirical performance, starting with Roberts (1995), tried to substitute marginal cost with the output gap, y_t, since under certain conditions one can assume that the output gap moves in the same way as marginal cost:

$$\pi_t = \lambda y_t + \beta E_t\{\pi_{t+1}\}$$

In their seminal 1999 paper, Gali and Gertler test the NKPC using the generalized methods of moments (GMM) estimator. They show that using detrended log GDP as the proxy for the output gap, applied to quarterly data, yields a negative and significant coefficient for λ. The finding of a negative coefficient on the output gap is robust across a wide range of possible instrument sets, measures of inflation, and GDP detrending procedures, as other authors have also demonstrated [Gali, Gertler, and Lopez-Salido (2005), Rudd and Whelan

[1]I note here that this is the partial equilibrium NKPC, which is what I use throughout this paper. DSGE versions of the NKPC exist also-for example Erceg, Henderson, and Levin (2000) derive a type of NKPC with sticky prices as well as sticky wages. This is similar to the NKPC I consider, except it is modified to include a cost-push shock.

(2007)]. The model predicts that λ must be positive, so the finding of a negative coefficient is evidence that the model is not working as theory predicts. In particular, Fuhrer (1997) argues that a negative coefficient on the output gap is at odds to the disinflation experiences of the US and other countries [Ball (1994)].

Gali and Gertler claim to find a solution to correct for the empirical failure of the NKPC by measuring real marginal cost directly, instead of using an ad hoc output gap as its proxy. A large literature exists which argues that conventional measures of the output gap may be poor, so using this variable may not be appropriate. In their paper, Gali and Gertler explain that real marginal cost, X, will be the ratio of the real wage rate to the marginal product of labor:

$$X = \frac{\omega}{\partial Y / \partial L} \tag{1}$$

where ω is the real wage rate, L is (total) labor, and Y is output. Note that all variables in this paper are in real terms where applicable, unless otherwise stated. To derive $\partial Y/\partial L$, Gali and Gertler use a Cobb-Douglas production function $Y = AK^{\alpha}L^{1-\alpha}$, where A is technology, K is capital, and $0 < \alpha < 1$. Solving for this expression of marginal cost in (1) now yields:

$$X = \frac{1}{1-\alpha}\frac{\omega L}{Y} \tag{2}$$

which is in effect $1/(1-\alpha)$ multiplied by the ratio of total labor compensation to real output. This ratio is also called the labor income share: $S = (\omega L/Y)$. Note that this can also be thought of as real average unit labor costs. From (2), we re-insert time subscripts to indicate that these variables are time-series leaving:

$$X_t = \frac{S_t}{1-\alpha} \tag{3}$$

Now letting lower case letters denote percent deviations from the steady state gives: $x_t = s_t$. This states that real marginal cost can be represented by the labor income share, so Gali and Gertler's NKPC becomes:

$$\pi_t = \lambda s_t + \beta E_t\{\pi_{t+1}\} \tag{4}$$

Using the non-farm business labor share for data on s_t, Gali and Gertler find that λ is positive and highly significant[2]. This result is robust to the hybrid version of the NKPC (lagged inflation included also), and to testing the model over different sub-samples. Hence they conclude that the labor income share is a good proxy for real marginal cost, and that this is an important determinant of inflation. Sbordone (2005) uses a slightly different estimation technique to show that the labor share drives inflation just as Gali and Gertler predict. Fuhrer and Olivei (2004) argue that conventional GMM estimates are unreliable, and they implement their 'optimal instrument' GMM approach to improve upon results. They find that expectations play a much smaller role in the determination of inflation than others suggest, but they still find the labor share to be positive and significant (in fact it is even more significant in their paper than in Gali and Gertler's). Gali, Gertler, and Lopez-Salido (2005) also show that their findings about the labor income share in the NKPC are robust to a variety of estimation procedures. It is apparent that Gali and Gertler's conclusion that the labor share of income drives inflation is a widely-used result.

2.2 Cyclicality of Marginal Cost

However, there are two fundamental problems that we can attribute to using the labor income share as our proxy for real marginal cost: firstly, the labor share of income is countercyclical, whereas theory suggests marginal cost should be procyclical; and second, the assumption used in order to obtain the labor income share as the proxy for real marginal cost is overly restrictive.

2.2.1 Countercyclical Labor Income Share

Standard economic theory predicts that real marginal cost should be procyclical. During an expansion firms will raise production, and since some factors of production are fixed, the short-run marginal cost curve will be strictly upward-sloping. We can also consider situations where the economy is experiencing a downturn: in this scenario, they will want to decrease production, which will lead to a reduction in costs for that marginal decrease in output, as certain factors become idle. Empirically, many authors have shown that marginal cost is upward-sloping in the short-run, although the degree of the slope of this curve has been

[2]They also find the coefficient, β, on expected inflation to be positive and highly significant.

debated, for example between Bils (1987) and Hall (1988).

Now consider Figure 1, which shows the non-farm business labor income share that Gali and Gertler use: we can see that this variable moves countercyclically. In periods of recession, marginal cost should be falling. Yet the labor income share does the opposite and rises. For instance, the labor share reaches its highest point since 1974 in 1980Q2, during a recession when GDP growth fell by 2.0% and unemployment rose an entire percentage point to 7.3%. Furthermore, after the end of each recession we would expect production to slowly rise and for marginal cost to hence increase as well; however, the labor share does the opposite and falls. Having this countercyclical proxy for real marginal cost is in direct violation of how theory predicts marginal cost should behave: marginal cost should be procyclical.

Why the Labor Income Share is Countercyclical

Recall the expression for the labor share is from (2), re-written as: $S = \frac{1}{1-\alpha}\frac{\omega}{Y/L}$. To understand why the labor share is countercyclical, consider the time series for the real wage ω and the average product of labor Y/L in Figure 2. We can see that the real wage, ω, has a general upward trend from 1960 to 2007, but it does not respond to the business cycle. In particular, if we examine the recessions periods, the real wage seems not to react at all to the economy's downturn, since it stays roughly constant during each recession. This empirical fact has been demonstrated by many authors (see Abraham and Haltiwanger's (1995) survey of the literature).

Now consider Y/L, the average product of labor. This has an upward trend over the time period in question as well, however this variable does react to periods of recession. In fact during each recession, Y/L falls, and then starts to rise again immediately after the recession has ended. In other words, Y/L is procyclical, perhaps due to phenomena such as labor hoarding.

If we now re-consider $\frac{\omega}{Y/L}$, we have the acyclical real wage divided by the procyclical average product of labor. This is what makes the labor income share countercyclical. Since marginal cost should be procyclical, the countercyclical labor income share is thus a poor proxy for real marginal cost.

2.2.2 Assumption Needed to Obtain Labor Share as Proxy for Marginal Cost

The expression for marginal cost from which the labor income share is derived is: $X = \frac{\omega}{\partial Y/\partial L}$. By using this expression for marginal cost, Gali and Gertler make the basic assumption that labor can be flexibly adjusted at a fixed real wage. However, this assumption is too restrictive and is not a reasonable description of reality.

To see why, note that we can think of labor, L, as total hours. Therefore it can be broken down into the following components: employment, N, and hours per worker, H. The simplest and most obvious decomposition of labor is then to say that it is number of employees multiplied by the number of hours each of them works:

$$L_t = N_t H_t \tag{5}$$

Given (5), Gali and Gertler's key assumption of being able to freely adjust labor at a fixed real wage rate, requires that if L changes, then both or one of N and H will change in the same direction. Now the question becomes: can N and/or H be flexibly adjusted at a fixed real wage rate?

First consider employment, N: can this be flexibly adjusted at a fixed real wage rate? As Oi (1962) shows, employment is a *quasi*-fixed factor of production, meaning it is partially fixed and partially variable. In other words, employment has costs of adjustment, such as recruitment and training costs. Therefore employment cannot be flexibly adjusted a fixed real wage rate.

Secondly we can ask whether hours, H, can be flexibly adjusted at a fixed real wage rate? The consensus amongst economists is that hours can be adjusted with little-to-no adjustment costs. Intuitively this makes sense, since a firm which asks an existing worker to work an extra hour does not have to pay any recruitment or training costs, only the wage rate for that hour. However, in order to induce the worker to work that extra hour requires that the firm pays an overtime wage. For example, in times of expansion, firms will have to increase wages at the margin to compensate for the increased disutility of working a higher number of hours. Particularly in industries where hourly wages are paid, such as manufacturing, hours are frequently adjusted and overtime pay is given to workers. So we can conclude that hours are flexible, but not at a fixed real wage rate. Instead, if we vary hours, the real wage rate must be dependent on the number of hours worked. Thus the real wage rate is not fixed as Gali and Gertler assume. Theoretically, it is also simple to show this must be true; for example, Lewis (1969) demonstrated that having the wage rate independent of hours, in the presence of quasi-fixed employment, produces implausible and unintuitive results. He concludes that we can only make a firm's cost minimization problem meaningful by making wages a function of hours.

To sum, Gali and Gertler make an overly restrictive assumption when they flexibly adjust labor at a fixed real wage rate, since employment cannot be adjusted costlessly, and varying hours requires overtime pay.

3 General Expression for Marginal Cost

Having discussed the weaknesses of the labor income share as a proxy for real marginal cost, I will now show how we can improve on the limitations of the labor income share to produce an improved and more general expression for real marginal cost.

The starting point is to use Bils' insight that a necessary condition for cost minimization is that the relative marginal products of inputs are equal to their relative costs. In other words, the marginal cost of increasing output can be computed as the cost of increasing input i, where we freely choose i to produce the marginal increase in output, whilst holding the other inputs fixed at their optimal levels. Hence we can chose which input i we wish to vary, whilst holding the other factors fixed.

Given that we have now divided labor into employment and hours, we have two margins along which we can examine marginal cost. We have established that employment is a quasi-fixed factor, which means that it will be hard to measure marginal cost along this margin due to the presence of adjustment costs.

Instead I will measure marginal cost along the hours margin. In particular, I will look at hours in the manufacturing sector where hours are frequently varied with no obvious costs of adjustment. So marginal cost will be derived by varying average hours of work for production workers, H, whilst holding employment and all other inputs constant at their optimal levels.

Without time subscripts for easier notation, marginal cost will then be:

$$X = \frac{dCosts}{dY} = \left(\frac{\partial Costs}{\partial H}\right)\left(\frac{\partial H}{\partial Y}\right)|Y^*, H^*, N^* \tag{6}$$

where '*' terms denote optimal levels. In order to compute what (6) actually looks like, we need to examine what the two derivatives $(\frac{\partial Costs}{\partial H})$ and $(\frac{\partial H}{\partial Y})$ will be. For $(\frac{\partial H}{\partial Y})$, I use the same Cobb-Douglas production function as Gali and Gertler do, but I combine it with (5) to get:

$$Y = AK^{\alpha}(NH)^{1-\alpha} \tag{7}$$

which yields:

$$\frac{\partial Y}{\partial H} = (1-\alpha)\frac{Y}{H} \tag{8}$$

Now we need to derive $(\frac{\partial Costs}{\partial H})$, which first requires a definition for the '$Costs$' function. The derivation of the labor share as the proxy for marginal cost assumes that costs are equal to the real wage rate multiplied by labor: $Costs = \omega L$. However firms must be able to compensate workers for varying the number of hours they work, as discussed in the previous section. Thus we need to change ω to $\omega(H)$, which is the average hourly real wage. One can also think of this expression stemming from having a fixed labor supply curve, while labor demand shifts to determine where labor market equilibrium occurs. In addition to writing the wage rate as $\omega(H)$, we also use (5) to decompose labor into employment and hours. These changes give us the new cost function:

$$Costs = \omega(H)NH \tag{9}$$

Given this form of our cost function, we can compute its derivative with respect to hours:

$$\frac{\partial Costs}{\partial H} = \omega(H)N + \omega'(H)HN = N[\omega(H) + \omega'(H)H] \tag{10}$$

Finally substitute (10) and (8) into (6) to get our general expression for marginal cost:

$$X = \frac{1}{1-\alpha}\left(\frac{NH}{Y}\right)[\omega(H) + \omega'(H)H] \tag{11}$$

Now notice an interesting observation that we get if we set $\omega'(H) = 0$ in (11): $\omega(H)$ just becomes ω in Gali and Gertler's notation, and $NH = L$. So setting $\omega'(H) = 0$ leaves:

$$X_{(\omega'=0)} = \frac{1}{1-\alpha}\left(\frac{\omega(H)NH}{Y}\right) = \frac{1}{1-\alpha}\left(\frac{\omega L}{Y}\right) = \frac{S}{1-\alpha} \tag{12}$$

In other words $\omega'(H) = 0$ reduces the general expression for marginal cost to the labor income share. The labor share is thus a special case of the general expression for marginal cost, where $\omega'(H) = 0$. Therefore if we find that $\omega'(H) \neq 0$, then the labor income share cannot be an accurate measure of marginal cost.

3.1 Estimating $\omega'(H)$

Looking at the general expression of marginal cost in (11), we can get everything from the data except for $\omega'(H)$. This term now remains to be estimated to derive a series for marginal cost, in its generalized form.

To estimate $\omega'(H)$ we require some sort of functional form for $\omega(H)$. I specify the real wage rate in the manufacturing sector to be dependent on two parts - a straight-time wage rate which is invariant to the number of hours worked, and an overtime premium which is dependent on the number of hours worked. We can think of the total pay per worker as being: $\overline{\omega}H + p\overline{\omega}V$, where $\overline{\omega}$ is the straight-time component of the real wage rate, and p is the overtime premium paid on top of the straight-time wage for V overtime hours per worker. Therefore the average hourly real wage rate will be the total pay divided by the number of hours worked:

$$\omega(H) = \overline{\omega}[1 + p\nu(H)] \tag{13}$$

where $\nu(H) = \frac{V}{H}$ is ratio of overtime hours to average hours per worker, which is dependent on the average number of hours worked. Given our functional form for the real wage schedule in (13), we now derive $\omega'(H)$:

$$\omega'(H) = \overline{\omega}p\nu'(H) \tag{14}$$

Finally we can substitute (13) and (14) into (11), which makes the general expression for marginal cost become:

$$X = \frac{1}{1-\alpha}\left(\frac{NH}{Y}\right)\overline{\omega}[1 + p(\nu(H) + H\nu'(H))] \tag{15}$$

From (15) the problem of estimating marginal cost has been reduced even further: α is a constant; and $\left(\frac{NH}{Y}\right)$, $\overline{\omega}$, p, and $\nu(H)$ are all variables that can be obtained from the data, including $\nu(H)$. Therefore, the only thing that remains to be estimated is $\nu'(H)$, and once we have this we can derive our series for real marginal cost in the manufacturing sector.

4 Manufacturing Data

Data for overtime hours and hourly wages, including overtime premia, is available for the manufacturing sector. Many authors have attempted to approximate overtime hours for non-manufacturing industries by saying that $V = (H - 40)$. However it is not clear as to what constitutes overtime for workers who are paid by salary instead of an hourly wage. This makes it unclear as to how marginal cost should be measured for these industries. However, the manufacturing sector involves frequent changes in hours, with workers paid a straight-time hourly wage and an overtime premium for overtime hours. Hence this sector certainly lends itself well to the measurement of (15).

4.1 The $\nu(H)$ function

In order to estimate (15), we need to know what $\nu'(H)$ is, which first requires estimating the $\nu(H)$ function. We can reasonably expect that the higher the number of straight-time hours that employees work, the more likely it is that they will work overtime hours too. Alternatively, we can also predict that when we add hours of work to the manufacturing industry, some are overtime hours and some just regular hours. Hence when we aggregate across the manufacturing sector, we can expect a plot of ν (or V/H) against H to display an upward-sloping trend. In other words, $\nu(H)$ is increasing in H.

Taking quarterly data for the period 1960:1 to 2007:3 on V and H for the manufacturing sector from the Bureau of Labor Statistics, I then plot ν against H. To determine the line of best fit for these datapoints, I conduct various regressions of ν on H and various powers of

H, using simple OLS with robust standard errors[3]. Out of all the specifications that I test, the best fit is given by the quadratic one:

$$\nu(H) = a + bH + cH^2 \tag{16}$$

This regression gives the highest adjusted-R^2 out of all the specifications tested, with coefficient values $b = -0.4362$, and $c = 0.0057$, both of which are highly significant. Results of the linear (which has the second-best fit) and quadratic $\nu(H)$ regressions are given in Table 1. Looking at the fitted line against the scatter-plot of actual data-points, in Figure 3, shows how well the quadratic specification matches against the data.

Knowing that $\nu(H) = a + bH + cH^2$ is a good description of overtime hours, we can now easily compute the derivative with respect to H:

$$\nu'(H) = b + 2cH \tag{17}$$

I use my estimates of b and c in (16) to compute the time series for ν' using (17), which I in turn use in my general expression for marginal cost.

4.2 Procyclical Marginal Cost

Now we have all the terms needed to estimate manufacturing real marginal cost in (15). Data for manufacturing variables is taken from the BLS, except for output data which is from the Bureau of Economic Analysis. The BLS data reports overtime hours and reports wages with and without overtime earnings. This allows me to derive the overtime premium, p, which is defined by the Code of Federal Regulations as something that is paid when: (i) hours worked in a single day or workweek are more than required by the employee's contract; or, (ii) overtime premia can be paid on 'special' workdays, such as weekends or holidays.

The time-series for real marginal cost in the manufacturing sector can be seen in Figure 4. From this figure, we can see that the new measure of marginal cost moves noticeably with the business cycle: it falls sharply during each recession, and also reacts very quickly after the recession has ended by rising immediately. In other words marginal cost is markedly pro-

[3]Plotting the time series of ν for 1960:1-2007:3 shows no discernible trend over time (Also supported by the small sum of squared deviations from the mean: $\sum_0^T(\nu_t - \overline{\nu})^2$=0.022).

cyclical, since workers do not work overtime hours during recessions. Or put more generally, we can think of the hours that employees work being lower during times of recession, leading to decreased marginal cost during those periods. Now compare my measure of manufacturing marginal cost to the manufacturing labor income share, seen in Figure 5. Manufacturing's labor share is countercyclical, just as the aggregate labor share is[4], since it rises during recessions and falls during periods of expansion. Economic theory predicts that marginal cost should move procyclically, and as Figure 4 shows, the new measure of marginal cost behaves in precisely this way, whereas the labor income share moves counter to what theory predicts.

5 A Disaggregated NKPC

We now have a sector level measure of marginal cost to test in the NKPC, whereas previous studies have tested marginal cost using aggregate data. Thus we need a NKPC model that describes inflation at the sector level as opposed to the aggregate level. To do this, I go back to the original equations of the model and see whether they will be valid at the sector level. It turns out that the disaggregated NKPC that we derive is a little different from the aggregate version of the model.

Recall the three equations that we use to derive the NKPC in its modern form [see Mankiw and Reis (2002) for further detail]:

$$p_t^* = p_t + x_t \tag{a}$$

$$q_t = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t p_{t+k}^* \tag{b}$$

$$p_t = (1 - \theta) q_t + \theta p_{t-1} \tag{c}$$

p_t^* is the optimal profit-maximizing price, p_t is the actual price level, x_t is real marginal cost, and q_t is the firms' adjustment price (all variables in logs). (a) states that a firm's desired (or profit-maximizing) price is the sum of the actual aggregate price level and some measure of aggregate marginal cost [see Blanchard and Kiyotaki (1987) for further elaboration]. The second equation, (b), states that the adjustment price for firms will be the weighted average of current and all future optimal prices. Finally the third equation, (c), is Calvo's random

[4] $Corr(s_t, s_t^m) = 0.8681$.

price adjustment, which states that every period a certain fraction of firms, $(1-\theta)$, will adjust prices, and the remaining θ of the firms don't adjust prices and continue to use the previous period's price. Solving (a), (b), and (c) gives the usual aggregate NKPC.

Now we must ask whether (a), (b), and (c) hold at the sector level. First let us consider equations (b) and (c). The equation describing firms' adjustment price holds true for each firm, and is aggregated across all firms in the economy to get (b). Hence we can also aggregate across all firms in one sector, meaning we can change the variables in (b) to sector-level ones, where there is no influence of aggregate variables. The same can be said for (c) which holds for any group of firms: under Calvo's model of random price adjustment, we can say that a fraction of firms in an industry adjusts prices each period, while the remaining firms in the industry leave prices at the previous period's level. Using Calvo-type pricing for multiple sectors is something which is often done, such as in Bils and Klenow (2004).

This leaves equation (a), which isn't quite a matter of changing aggregate variables to disaggregated ones. Equation (a) comes from assuming Dixit-Stiglitz CES utility, combined with monopolistic competition. This gives us the familiar looking result that the optimal, profit-maximizing price for a given firm j, is a mark-up over nominal marginal cost:

$$P_{j,t}^{*} = \mu \widehat{X}_{j,t} \tag{18}$$

where μ is the mark-up, and $\widehat{X}_{j,t}$ is *nominal* marginal cost. When we convert this equation into real terms, and set the log of the mark-up term to zero (which we can do without loss of generality), we get equation (a). Now suppose, we aggregate across all manufacturing firms such that $\sum_j (P_{j,t}^m)^* = (P_t^m)^*$, where the '$m$' superscript denotes the manufacturing sector. These manufacturing firms also have optimal price as a mark-up over nominal marginal cost:

$$(P_t^m)^* = \mu \widehat{X}_t^m \tag{19}$$

To convert into real variables we divide both sides of the equation by the aggregate price level:

$$\frac{(P_t^m)^*}{P_t} = \mu \frac{\widehat{X}_t^m}{P_t} \tag{20}$$

where P_t (with no superscript) is the aggregate price level. The term on the right-hand side is nominal marginal cost divided by the aggregate price level, which is just the same as real marginal cost for manufacturing:

$$\frac{(P_t^m)^*}{P_t} = \mu X_t^m \tag{21}$$

where X_t^m is manufacturing's real marginal cost. Now take logs, and let lower case letters represent log variables to get:

$$(p_t^m)^* = p_t + x_t^m \tag{22}$$

where the log of the mark-up is set to zero, without loss of generality. Notice that this is almost the same as equation (a), except we have an aggregate price term remaining, rather than simply switching all variables to disaggregated ones as we could do with (b) and (c). Finally we have the three equations we need to derive the NKPC at a disaggregated level, which I apply to the manufacturing sector:

$$(p_t^m)^* = p_t + x_t^m \tag{I}$$

$$q_t^m = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t(p_{t+k}^m)^* \tag{II}$$

$$p_t^m = (1-\theta)q_t^m + \theta p_{t-1}^m \tag{III}$$

$(p_t^m)^*$ represents the manufacturing firms' desired price, p_t is the aggregate price level, x_t^m is real marginal cost in manufacturing, q_t^m is the adjustment price in the manufacturing sector, and p_t^m is the actual manufacturing price. In other words, the disaggregated model is virtually identical to the aggregate one, except in (I) where the aggregate price level, p_t, remains.

Solving equations (I), (II), and (III) gives the 'disaggregated' NKPC (see Appendix A for details of the derivation):

$$\pi_t^m = \lambda[x_t^m + (p_t - p_t^m)] + \beta E_t\{\pi_{t+1}^m\} \tag{Dis. NKPC}$$

where π_t^m represents manufacturing inflation. Also note that $\lambda = (1-\theta)(1-\beta\theta)/\theta$, just as it is in the aggregate NKPC. We can see that the disaggregated NKPC is not quite the same as the aggregate NKPC, due to the addition of the $(p_t - p_t^m)$ term. In other words, manufacturing inflation is also dependent on the relative price between the manufacturing sector and the whole economy, which will affect how prices change in this sector. This relative price term is the result of the aggregate price level, p_t, remaining in equation (I). Intuitively it is not surprising that aggregate price affects sectoral inflation. In particular, we know that the aggregate price level affects nominal marginal cost in each sector, and it therefore affects the price that manufacturers want to charge. So the inflation rate in one sector is partially dependent on the overall price level prevailing in the economy.

In summary: to test the NKPC at a disaggregated level requires going back to the model and examining each equation carefully at the sector level. The end result is a NKPC which has an additional relative price term in it, making the disaggregated NKPC different from the usual aggregate-version of the model.

6 Testing the NKPC

6.1 Tests Using the Labor Share

Gali and Gertler test the aggregate NKPC, $\pi_t = \lambda s_t + \beta E_t\{\pi_{t+1}\}$, using quarterly data over the period 1960:1 to 1997:4, with conventional GMM[5]. Their results show that λ is positive and significant (β is also positive and highly significant), indicating that the NKPC fits the data. This leads them to conclude that the NKPC is a good first approximation to the dynamics of inflation.

The econometric problem with Gali and Gertler's test is that the use of conventional GMM with this large number of seemingly arbitrary instruments is subject to the problem of weak instruments[6]. Weak instruments are those which are only weakly correlated with the included endogenous variables [see Stock, Wright, and Yogo (2002)], and these in turn lead to GMM point estimates, hypothesis tests and confidence intervals which are unreliable [see Fuhrer and Olivei (2004)].

Fuhrer and Olivei develop a method of 'optimal instruments GMM' in their paper to adjust for the weak instruments bias. This method uses projections that impose the dynamic

[5] Inflation is the percentage change in GDP deflator. Instruments are four lags of inflation, the non-farm business labor share, output gap, long-short interest rate spread, wage inflation, and commodity price inflation.

[6] This is confirmed when I use the Anderson and Rubin (1949) approach to test for weak instruments. Under the $H_0 : \lambda = \lambda_0, \beta = \beta_0$, I compute the A-R statistic , getting a p-value of 0.00, i.e. the null is rejected.

constraints implied by the forward-looking relation, instead of simply instrumenting with a linear projection on the instrument set. In other words, they enter conditional projections of each instrument given the reduced form implied by the model, instead of entering the instruments as raw lags. For the rest of this paper I will use Fuhrer and Olivei's optimal instruments GMM[7] instead of conventional GMM that Gali and Gertler use[8]. To verify Gali and Gertler's claims, I conduct optimal instruments GMM for the aggregate NKPC using the labor share. Their conclusions are confirmed with this method too, the only notable exception being that expectations of inflation are much less significant (see Table 2 for results).

Next I check the manufacturing labor share in the disaggregated NKPC, which is countercyclical just as the aggregate labor share is. I use a similar choice of instruments to those used by Fuhrer and Olivei, namely: three lags of manufacturing labor share, inflation, output gap, and the long-short run interest rate spread[9], to which I apply the optimal instruments methodology. Applying this approach yields the following result:

$$\pi_t^m = \underset{(0.002)}{0.005}[s_t^m + (p_t - p_t^m)] + \underset{(0.060)}{0.691} E_t \pi_{t+1}^m$$

where s_t^m has been de-meaned since this variable is expressed in terms of deviation from its steady-state value. Table 3 contains full results, including results for treating s_t^m and $(\overline{p}_t - p_t^m)$ separately, and results for the omission of the relative price term. The result we get from all specifications is that the coefficient on manufacturing's labor share is positive and significant, just as Gali and Gertler's paper predicts. In addition, expectations of future inflation are also a major determinant of inflation. Indeed if we relax the constraint that the coefficient is the same for s_t^m and $(p_t - p_t^m)$, the same conclusions apply. The omission of the relative price term altogether also produces a positive and significant coefficient for the labor share as well. Hence we can say that the relative price term is not driving these results.

In this exercise, I performed the same test as Gali and Gertler do, but with manufacturing data instead of aggregate data. In both cases, we reach the same conclusions about the

[7]I am extremely grateful to Fuhrer and Olivei for sharing their MATLAB code.

[8]Note that all results in this paper give the same conclusions if conventional GMM is used, with the exception of larger coefficients for expected inflation.

[9]All instruments are manufacturing variables except the interest rate spread.

labor income share: that its coefficient is positive and significant. Hence changing from the aggregated NKPC to the disaggregated NKPC does not make any difference to Gali and Gertler's claim that the labor share is a good proxy for real marginal cost.

6.2 Disaggregated NKPC with the Improved Measure of Marginal Cost

In principle it would be good to be able to use the procyclical measure of marginal cost in the aggregate NKPC, however it is hard to measure aggregate marginal cost. For this reason we proceed by testing a sector's marginal cost in the disaggregated NKPC. Specifically, I use the improved measure of marginal cost that I derived for the manufacturing sector[10]. My data on manufacturing variables are taken from the BLS, and is quarterly over the period 1960:1 to 2007:3.

Optimal instruments GMM is used again, with initial instruments of three lags of manufacturing marginal cost, inflation, output gap, and the long-short run interest rate spread. Using my procyclical measure of marginal cost gives the following result:

$$\pi_t^m = \underset{(0.0015)}{-0.0062}[x_t^m + (p_t - p_t^m)] + \underset{(0.072)}{0.627} E_t \pi_{t+1}^m$$

This result shows that the coefficient on $[x_t^m + (p_t - p_t^m)]$ is negative and significant (at the 1% level), with a t-statistic of -4.17. This means that the procyclical measure of marginal cost is driving inflation, but in a negative way; a result that rejects the validity of the model. This is the same conclusion that Gali and Gertler make when they see the negative coefficient for the output gap in the NKPC. Moreover when we recall that $\lambda = (1-\theta)(1-\beta\theta)/\theta$, then for reasonable values of β and θ it must the case that $\lambda > 0$. So the finding of a negative and significant λ means that the NKPC is simply not working with the improved measure of marginal cost.

I also run two other regressions with the new measure of manufacturing marginal cost. Firstly, I allow the coefficients on marginal cost and the relative price term to differ. This allows me to directly compare the coefficient on manufacturing marginal cost with the coefficient on the labor income share from the aggregate NKPC. In this specification, the

[10]Where marginal cost is de-meaned to show deviation from its steady-state value.

coefficients on both manufacturing marginal cost and the relative price term are negative and significant. Since NKPC theory implies that λ should be positive, the coefficient on the relative price term should be positive as well. Hence the negative coefficient on $(p_t - p_t^m)$ is also contrary to theory. Secondly, I run the same regression again but omitting the relative price term this time. This gives the same conclusions, which means that the relative price term is not driving our results. All of these results are detailed in Table 4.

Whichever way we test this improved measure of marginal cost in the disaggregated NKPC, we get a negative and significant coefficient for λ. This represents a strong rejection of the model, as Gali and Gertler describe in their own words: *"The model clearly doesn't work in this case: the coefficient associated with the output gap (as a proxy for marginal cost) is negative and significant, which is at odds with the prediction of the theory."* So once we improve upon the derivation of marginal cost to a more reasonable one, the series for marginal cost turns out to be procyclical. However when this is tested in the NKPC it yields a negative and significant coefficient, rendering the model as invalid.

6.3 Robustness: Marginal Cost in the Aggregate NKPC

As mentioned earlier, it would be good if we could test the more reasonable measure of marginal cost at the aggregate level, but unfortunately it is difficult to get a reliable series for the aggregate economy. That being said, let us make a couple of assumptions that will allow us to test manufacturing marginal cost at the aggregate level, to serve as a robustness check for the previous results.

The two assumptions I will test in the aggregate NKPC are: (i) using manufacturing marginal cost directly; and (ii) a weighted average marginal cost, x_t^w. For (i), I simply use the procyclical manufacturing marginal cost in the standard aggregate NKPC using the same instruments as in section 6.1. For (ii), I derive a series which is the weighted average between the improved measure of marginal cost for the manufacturing sector and non-manufacturing's labor income share, since overtime data is not available for non-manufacturing sectors. The weights used are manufacturing's and non-manufacturing's shares in national income, with manufacturing's share declining from just under 30% in 1960 to just over 10% in 2007. x_t^w is an improvement over the labor share, where we've used manufacturing marginal cost instead of the labor share for that sector's marginal cost. The series for x_t^w can be seen in Figure 6, and is sometimes countercyclical and sometimes procyclical. We get the following results from running these regressions with optimal instruments GMM as before (see Table 5 for full results):

$$\pi_t = \underset{(0.039)}{-0.069} x_t^m + \underset{(0.092)}{0.780} E_t \pi_{t+1} \tag{i}$$

$$\pi_t = \underset{(0.007)}{-0.012} x_t^w + \underset{(0.100)}{0.798} E_t \pi_{t+1} \tag{ii}$$

In both cases, the coefficient on marginal cost is negative and significant at the 10% level, while expectations of inflation continue to be positive and significant. Hence the procyclical marginal cost contradicts what theory suggests should happen at the aggregate level as well, whether we try manufacturing marginal cost directly or take a weighted-average between manufacturing marginal cost and non-manufacturing labor share.

As one final robustness check, I re-run the disaggregated NKPC with the Consumer Price Index as the measure of the price level, instead of the GDP deflator which we've used until now, and the inflation rate is then the percentage change in the CPI. The coefficient on marginal cost remains negative and significant (see Table 6). So the result that the better measure of marginal cost causes the NKPC not to work is also robust to what measure of inflation we use.

6.4 Why the Model Does Not Work

We now know that the countercyclical labor income share works in the disaggregated and aggregated versions of the NKPC, but procyclical marginal cost does not. What causes one variable to work and the other one not to work? To answer this question in a way which is easy to understand, let us consider the NKPC where $\beta = 1$. In the aggregate NKPC, we have $\pi_t = \lambda x_t + E_t \pi_{t+1}$, where x_t is de-meaned. Now let us also assume that expected inflation is just actual inflation plus an error term: $E_t \pi_{t+1} = \pi_{t+1} + \epsilon_{t+1}$, and re-write the NKPC as

$\pi_t - \pi_{t+1} = \lambda x_t + \epsilon_{t+1}$, where the left-hand side is -1 times the change in inflation. If we apply OLS estimation to this equation, basic econometric theory tells us that:

$$\lambda = \frac{Cov(\pi_t - \pi_{t+1}, x_t)}{Var(x_t)}$$

where the sign of λ depends on the covariance between the change in inflation and marginal cost, since the variance of marginal cost will certainly be positive. At the aggregate level, setting marginal cost as manufacturing marginal cost yields: $Cov(\pi_t - \pi_{t+1}, x_t^m) = -0.000863$. In other words, this negative covariance ensures that $\lambda < 0$. Setting marginal cost to be the labor share yields: $Cov(\pi_t - \pi_{t+1}, s_t)$=0.000072, which ensures that $\lambda > 0$. Hence the countercyclical labor share works by achieving a positive covariance with $(\pi_t - \pi_{t+1})$, but if a more realistic procyclical measure of marginal cost is used, then $\lambda < 0$ due to the negative covariance that is produced.

In the disaggregated NKPC, λ is similarly dependent on $Cov(\pi_t^m - \pi_{t+1}^m, [x_t^m + (p_t - p_t^m)])$ divided by the variance of $[x_t^m + (p_t - p_t^m)]$. Using the procyclical measure of marginal cost in manufacturing yields this covariance as -0.001335, whereas manufacturing's labor share gives a covariance of 0.000017. Thus the intuition here is the same as the aggregate level: procyclical marginal cost cannot work due to the negative covariance between $(\pi_t^m - \pi_{t+1}^m)$ and $[x_t^m + (p_t - p_t^m)]$.

Appendix B illustrates the similar exercise where we now let $\beta \neq 1$, in which case it can be shown that the sign of λ depends on the covariance between marginal cost and inflation. And once again, procyclical marginal cost makes this covariance negative to ensure that $\lambda < 0$, while the countercyclical labor share makes the covariance positive enough to make $\lambda > 0$.

7 Hybrid Disaggregated NKPC

Many economists, such as Rudd and Whelan (2007), criticize the NKPC since the model does not seem to fit the data too well. The consensus seems to be that the best way to improve the model is to include a lag of inflation in the NKPC, called the 'hybrid' NKPC. Gali and Gertler also find that this generalization improves the fit of the model to the data. Therefore we must also consider the hybrid NKPC, which takes the following form:

$$\pi_t = \lambda s_t + \gamma_f E_t\{\pi_{t+1}\} + \gamma_b \pi_{t-1} \tag{H.NKPC}$$

where we have added a lag of inflation to the model. The inclusion of π_{t-1} can be justified by having firms who are rule-of-thumb price setters (see Gali and Gertler's paper for more detail).

Before I test manufacturing's marginal cost in the hybrid setting, I confirm that the hybrid NKPC performs well when the labor share is used as the proxy for marginal cost. The results for the aggregate version of the hybrid NKPC with optimal instruments GMM are (displayed also in Table 7):

$$\pi_t = \underset{(0.013)}{0.059} s_t + \underset{(0.051)}{0.729} E_t\{\pi_{t+1}\} + \underset{(0.053)}{0.251} \pi_{t-1}$$

where the coefficient on the labor income share is highly positive and significant. Expectations continue to play an important role, and lagged inflation also is significant. So the labor income share's performance in the NKPC is robust to the inclusion of lagged inflation.

Now I will do the parallel hybrid exercise for my model based at the disaggregated level. As before, in order to test the hybrid NKPC at a disaggregated level, we must go back to the original model and re-derive the NKPC using sector-level variables. These equations and the derivation of the model are in Appendix C, where we get the following hybrid disaggregated NKPC:

$$\pi_t^m = \lambda[x_t^m + (p_t - p_t^m)] + \gamma_f E_t \pi_{t+1}^m + \gamma_b \pi_{t-1}^m \tag{H-D.NKPC}$$

So we can see that the disaggregated hybrid NKPC is the same as the aggregate version, except we have to add a relative price term, just as we do in the standard disaggregated model. Now I test procyclical manufacturing marginal cost in the hybrid disaggregated NKPC, using optimal instruments GMM with the same instruments as before, yielding the following result:

$$\pi_t^m = -0.00084[x_t^m + (p_t - p_t^m)] + 0.532 E_t\{\pi_{t+1}^m\} + 0.408\pi_{t-1}^m$$
$$(0.00038) \qquad\qquad (0.081) \qquad (0.082)$$

Expectations of future inflation and lagged inflation are both positive and significant, as they are in the aggregate version of the hybrid NKPC. The coefficient on real marginal cost plus relative prices also remains negative in the hybrid disaggregated NKPC, and it is significant with a p-value of 0.03. Table 8 contains the hybrid disaggregated NKPC results, including the regression where we allow x_t^m and $(p_t - p_t^m)$ to take different coefficients, as well as the regression where the relative price term is omitted. The exact same conclusions apply to these regressions, indicating that it is not the relative price term that is driving results.

Hence the most popular generalization of the NKPC, the hybrid NKPC, still produces a negative coefficient on marginal cost when a more reasonable measure of marginal cost is used. Therefore the model still fails when tested with an improved measure of marginal cost.

8 Conclusion

The New Keynesian Phillips Curve is a widely popular model of inflation dynamics, which predicts that inflation will depend on real marginal cost, and expectations of future inflation. This model holds much appeal as it is a macro relationship which is derived from microfoundations, and is simple enough to be applicable to policy-making decisions. Indeed a consensus about an alternative model that performs better seems to be lacking in the literature, providing greater reason to use the NKPC. Empirically, the model seems to work well when labor's share of income is used as the proxy for real marginal cost, shown in Gali and Gertler's seminal paper about the empirical performance of the NKPC.

However the use of the labor income share as a proxy for real marginal cost is flawed for two reasons: firstly, it is countercyclical, whereas theory dictates that marginal cost should be procyclical; and second, the assumption used to obtain the labor share as a proxy for marginal cost is too restrictive. Specifically, the labor income share assumes that each additional unit of labor can be hired in a perfectly flexible manner at a fixed wage rate, irrespective of the number of hours worked. To improve this assumption in a reasonable way, we must recognize that employment is not costless but is quasi-fixed, and that the real wage rate is a function of hours.

Implementation of this more realistic assumption about the behavior of labor over the business cycle results in an improved measure of real marginal cost. In particular, this marginal cost depends on the number of overtime hours worked, and the overtime premium that is paid for those extra hours. The manufacturing sector lends itself best to the measurement of this specification of marginal cost because of its frequent variation in hours, and due to the existence of overtime that can be observed in this industry. Applying the general expression for marginal cost to manufacturing data then produces a series that is markedly procyclical.

Testing the disaggregated NKPC with this improved measure of real marginal cost produces a negative and significant coefficient, which contradicts the underlying model which the NKPC is based upon. Even if we consider one important generalization of the model, the hybrid NKPC, the model still does not work. All of these results suggest that there is a fundamental problem with the model, which is something that might only be fixed by a radical change.

In summary, the NKPC is the leading macroeconomic model which describes inflation dynamics, but when the explanatory variables are measured realistically the NKPC does not work as it is supposed to. Hence the NKPC does *not* provide a reasonable description of inflation dynamics.

Appendix A: Disaggregated NKPC Derivation

The model is given by (A-1), (A-2), and (A-3):

$$(p_t^m)^* = p_t + x_t^m \tag{A-1}$$

$$q_t^m = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t(p_{t+k}^m)^* \tag{A-2}$$

$$p_t^m = (1-\theta)q_t^m + \theta p_{t-1}^m \tag{A-3}$$

Now break the sum in (A-2), take expectations at time t, and apply the Law of Iterated Expectations to get:

$$q_t^m = (1-\beta\theta)(p_t^m)^* + \beta\theta E_t q_{t+1}^m \tag{A-4}$$

Rearrange (A-3) in terms of q_t^m:

$$q_t^m = \frac{1}{1-\theta}(p_t^m - \theta p_{t-1}^m) \tag{A-5}$$

Then move (A-5) one period forward and take expectations at time t:

$$E_t q_{t+1}^m = \frac{1}{1-\theta}(E_t p_{t+1}^m - \theta E_t p_t^m) \tag{A-6}$$

Now equate (A-5) with (A-4), whilst plugging in (A-1) and (A-6). All that is required now is some algebraic manipulation. Specifically, add θp_t^m to both sides, subtract p_t^m from both sides, add and subtract $\beta\theta E_t p_t^m$ to the RHS, and define $\pi_t^m = p_t^m - p_{t-1}^m$. Collecting terms, and simplifying the algebra results in the disaggregated NKPC:

$$\pi_t^m = \frac{(1-\theta)(1-\beta\theta)}{\theta}[x_t^m + (p_t - p_t^m)] + \beta E_t \pi_{t+1}^m \tag{A-7}$$

which is similar to the aggregate NKPC, except with an additional term which is the sectoral price level relative to the aggregate price level.

Appendix B: Why the Model Does Not Work ($\beta \neq 1$)

Re-write the NKPC purely in terms of time t variables: $\pi = \lambda x + \beta E + \epsilon$, where x is the marginal cost variable, and E is expectations of inflation (which is just next period's actual inflation plus an error, ϵ). Now we do some algebra to figure out what expression will determine the sign of λ. First de-mean the variables without loss of generality, then apply the OLS estimator, $\lambda = (D'D)^{-1}D'\pi$, where D is a matrix containing x and E. Doing the matrix algebra will show that λ depends on the sign of:

$$\sum_{i=1}^{T}(E_i - \overline{E})(E_i - \overline{E})\sum_{i=1}^{T}(x_i - \overline{x})(\pi_i - \overline{\pi}) - \sum_{i=1}^{T}(x_i - \overline{x})(E_i - \overline{E})\sum_{i=1}^{T}(E_i - \overline{E})(\pi_i - \overline{\pi}) \quad \text{(A-8)}$$

which we can then restate in terms of covariances:

$$T^2[Cov(E,E)Cov(x,\pi) - Cov(x,E)Cov(E,\pi)] \quad \text{(A-9)}$$

We know that $Cov(E,E) > 0$ for certain, and can make inferences about $Cov(x,E)$ and $Cov(E,\pi)$. Standard theory implies that higher marginal cost today will imply a higher discounted present value of inflation. This is intuitive since higher marginal costs will get passed onto higher prices, which has the tendency to raise future expected inflation. This means that $Cov(x,E) > 0$. By similar logic, if we expect a higher future rate of inflation, that will have an upward pressure on the current inflation rate. Certainly a higher current rate of inflation will imply a higher present discounted value of future inflation. This means that $Cov(E,\pi) > 0$. The term that remains to be examined is then $Cov(x,\pi)$, i.e. the covariance between marginal cost and inflation. To investigate this, we need to compute the variance-covariance matrix from the manufacturing dataset.

Setting marginal cost, x, as manufacturing marginal cost plus the relative price yields $Cov([x_t^m + (p_t - p_t^m)], \pi_t^m) = -0.000208$. Using just procyclical marginal cost, without the relative price term, yields $Cov(x_t^m, \pi_t^m) = -0.000011$. In either case, since $Cov(x,\pi) < 0$, this means that (A-9) must be negative. In other words the negative covariance between procyclical marginal cost and inflation means that the coefficient on marginal cost, λ, has to be negative. Now changing marginal cost, to be the manufacturing labor income share, we get $Cov(s_t^m, \pi_t^m) = 0.007898$. With $Cov(x, \pi) > 0$, if $Cov(E, E)Cov(x, \pi) > Cov(x, E)Cov(E, \pi)$, then (A-9) will be positive, which is exactly what happens. These signs of covariances can be easily confirmed with aggregate data as well, where $Cov(x_t^m, \pi_t) < 0$, and $Cov(s_t, \pi_t) > 0$.

Appendix C: Disaggregated Hybrid NKPC Derivation

The four equations we start with are (variables are in logs):

$$p_t^m = \theta p_{t-1}^m + (1-\theta)(\overline{p}_t^m)^* \tag{A-10}$$

$$(\overline{p}_t^m)^* = (1-\tau)(p_t^m)^f + \tau(p_t^m)^b \tag{A-11}$$

$$(p_t^m)^f = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\widehat{x}_{t+k}^m\} \tag{A-12}$$

$$(p_t^m)^b = (\overline{p}_{t-1}^m)^* + \pi_{t-1}^m \tag{A-13}$$

(A-10) shows how the manufacturing price level evolves: θ of the manufacturing firms do not adjust price, and $(1-\theta)$ do adjust price to $(\overline{p}_t^m)^*$. (A-11) states that out of the manufacturing firms who do adjust price, τ of them use backward-looking prices as a rule of thumb, $(p_t^m)^b$, and $(1-\tau)$ set prices optimally, in a forward-looking manner, $(p_t^m)^f$. (A-12) shows how forward-looking manufacturing price-setters set price equal to discounted expected future marginal costs in the manufacturing sector, where $\widehat{x}_t^m$ is *nominal* marginal cost in manufacturing. And (A-13) states that backward-looking price-setters set prices equal to the average price set in the most recent round of price adjustments, $(\overline{p}_{t-1}^m)^*$, with a correction for manufacturing inflation. To solve for the NKPC, first substitute (A-11) into (A-10):

$$p_t^m = \theta p_{t-1}^m + (1-\theta)[(1-\tau)(p_t^m)^f + \tau(p_t^m)^b] \tag{A-14}$$

Use the definition $\pi_{t-1}^m = p_{t-1}^m - p_{t-2}^m$ in (A-13), and then substitute into (A-14):

$$p_t^m = (1-\theta)[(1-\tau)(p_t^m)^f + \tau((\overline{p}_{t-1}^m)^* - p_{t-2}^m)] + (\theta + \tau - \tau\theta)p_{t-1}^m \tag{A-15}$$

Now subtract p_{t-1}^m from both sides and factor the coefficient on p_{t-1}^m. Next, subtract p_t^m from both sides, and add θp_t^m, τp_t^m, $-\tau\theta p_t^m$ to both sides to leave:

$$[\theta + \tau(1-\theta)]\pi_t^m = (1-\theta)[(1-\tau)(p_t^m)^f + \tau((\overline{p}_{t-1}^m)^* - p_{t-2}^m)] - (1-\tau)(1-\theta)p_t^m \quad \text{(A-16)}$$

Lag (A-10) by one period and rearrange to leave $(\overline{p}_{t-1}^m)^*$ on its own, and plug this into (A-16). Simplifying gives:

$$[\theta + \tau(1-\theta)]\pi_t^m = \tau\pi_{t-1}^m + (1-\tau)(1-\theta)[(p_t^m)^f - p_t^m] \quad \text{(A-17)}$$

Now plug (A-12) into (A-17). Move this equation forward by one period, take expectations at time t, and multiply both sides by $\beta\theta$. Let $\phi = [\theta + \tau(1-\theta(1-\beta))]$, then add and subtract $\beta\theta(1-\tau)(1-\theta)p_t^m$ to the RHS. Doing all of this algebraic manipulation carefully, leaves:

$$\phi\pi_t^m - \beta\theta E_t\pi_{t+1}^m = \tau\pi_{t-1}^m + (1-\tau)(1-\theta)(1-\beta\theta)[\widehat{x}_t^m - p_t^m] \quad \text{(A-18)}$$

Finally, define real marginal cost as $x_t^m = \widehat{x}_t^m - p_t$, where p_t is the aggregate price level. Hence, $\widehat{x}_t^m = x_t^m + p_t$. Using this, and rearranging (A-18) yields the disaggregated hybrid NKPC:

$$\pi_t^m = \lambda[x_t^m + (p_t - p_t^m)] + \gamma_f E_t\pi_{t+1}^m + \gamma_b\pi_{t-1}^m \quad \text{(A-19)}$$

where $\lambda = (1-\tau)(1-\theta)(1-\beta\theta)/\phi$, $\gamma_f = \beta\theta/\phi$, and $\gamma_b = \tau/\phi$.

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Table 1: $\nu(H)$ Regressions

	$\nu(H) = a + bH$		$\nu(H) = a + bH + cH^2$		
	a	b	a	b	c
Coefficient	-.9760154	.0264254	8.396427	-.4361621	.0057049
Standard Error	.0272385	.0006697	1.071294	.0528657	.0006519
p-value	0.0000	0.0000	0.0000	0.0000	0.0000
Adj-R^2	0.8917		0.9228		

Table 2: Gali and Gertler's Aggregate NKPC (w/ Optimal Instruments)

$\pi_t = \lambda s_t + \beta E_t\{\pi_{t+1}\}$				
Time period	1960:1-1997:4		1960:1-2007:3	
	λ	β	λ	β
Coefficient	0.0362	0.6978	0.0318	0.7148
Standard Error	0.0093	0.0964	0.0086	0.0937
p-value	0.0000	0.0000	0.0003	0.0000
Adj-R^2	0.759245		0.777227	

Table 3: Disaggregated NKPC with Manufacturing Labor Share

$\pi_t^m = \lambda[s_t^m + (p_t - p_t^m)] + \beta E_t\{\pi_{t+1}^m\}$		
	λ	β
Coefficient	0.005037	0.691172
Standard Error	0.002265	0.060446
p-value	0.0273	0.0000
Adj-R^2	0.547907	

$\pi_t^m = \lambda_1 s_t^m + \lambda_2(p_t - p_t^m) + \beta E_t\{\pi_{t+1}^m\}$			
	λ_1	λ_2	β
Coefficient	0.033669	0.003960	0.744825
Standard Error	0.014640	0.002136	0.064459
p-value	0.0226	0.0653	0.0000
Adj-R^2	0.559852		

$\pi_t^m = \lambda s_t^m + \beta E_t\{\pi_{t+1}^m\}$		
	λ	β
Coefficient	0.051235	0.806253
Standard Error	0.014214	0.068721
p-value	0.0004	0.0000
Adj-R^2	0.556174	

Table 4: Disaggregated NKPC with Procyclical Manufacturing Marginal Cost

$\pi_t^m = \lambda[x_t^m + (p_t - p_t^m)] + \beta E_t\{\pi_{t+1}^m\}$		
	λ	β
Coefficient	-0.006240	0.626886
Standard Error	0.001498	0.071947
p-value	0.0000	0.0000
Adj-R^2	0.531813	

$\pi_t^m = \lambda_1 x_t^m + \lambda_2(p_t - p_t^m) + \beta E_t\{\pi_{t+1}^m\}$			
	λ_1	λ_2	β
Coefficient	-0.029393	-0.005471	0.644971
Standard Error	0.011617	0.001639	0.071424
p-value	0.0122	0.0010	0.0000
Adj-R^2	0.533016		

$\pi_t^m = \lambda x_t^m + \beta E_t\{\pi_{t+1}^m\}$		
	λ	β
Coefficient	-0.042072	0.689201
Standard Error	0.010454	0.075903
p-value	0.0001	0.0000
Adj-R^2	0.531342	

Table 5: Procyclical Marginal Cost in the Aggregate NKPC

(i) $\pi_t = \lambda x_t^m + \beta E_t\{\pi_{t+1}\}$		
	λ	β
Coefficient	-0.068880	0.779947
Standard Error	0.039477	0.092077
p-value	0.0826	0.0000
Adj-R^2	0.779458	

(ii) $\pi_t = \lambda x_t^w + \beta E_t\{\pi_{t+1}\}$		
	λ	β
Coefficient	-0.012036	0.797501
Standard Error	0.006521	0.099903
p-value	0.0665	0.0000
Adj-R^2	0.772169	

Table 6: Disaggregated NKPC w/ CPI as Inflation

$\pi_t^m = \lambda[x_t^m + (p_t - p_t^m)] + \beta E_t\{\pi_{t+1}^m\}$		
	λ	β
Coefficient	-0.073604	0.776224
Standard Error	0.031197	0.057074
p-value	0.0193	0.0000
Adj-R^2	0.510712	

Table 7: Hybrid (Aggregate) NKPC

$\pi_t = \lambda s_t + \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1}$			
	λ	γ_f	γ_b
Coefficient	0.058541	0.728678	0.250688
Standard Error	0.013340	0.050638	0.052731
p-value	0.0000	0.0000	0.0000
Adj-R^2	0.832647		

Table 8: Hybrid Disaggregated NKPC

$\pi_t^m = \lambda[x_t^m + (p_t - p_t^m)] + \gamma_f E_t \pi_{t+1}^m + \gamma_b \pi_{t-1}^m$			
	λ	γ_f	γ_b
Coefficient	-0.000837	0.531787	0.407753
Standard Error	0.000383	0.081203	0.081947
p-value	0.0299	0.0000	0.0000
Adj-R^2	0.707743		

$\pi_t^m = \lambda_1 x_t^m + \lambda_2(p_t - p_t^m) + \gamma_f E_t \pi_{t+1}^m + \gamma_b \pi_{t-1}^m$				
	λ_1	λ_2	γ_f	γ_b
Coefficient	-0.007759	-0.000395	0.529964	0.447399
Standard Error	0.003872	0.000353	0.081109	0.082058
p-value	0.0465	0.2641	0.0000	0.0000
Adj-R^2	0.706970			

$\pi_t^m = \lambda x_t^m + \gamma_f E_t \pi_{t+1}^m + \gamma_b \pi_{t-1}^m$			
	λ	γ_f	γ_b
Coefficient	-0.009082	0.531752	0.460608
Standard Error	0.003671	0.081252	0.082542
p-value	0.0143	0.0000	0.0000
Adj-R^2	0.707820		

Fig. 1: Labor Income Share

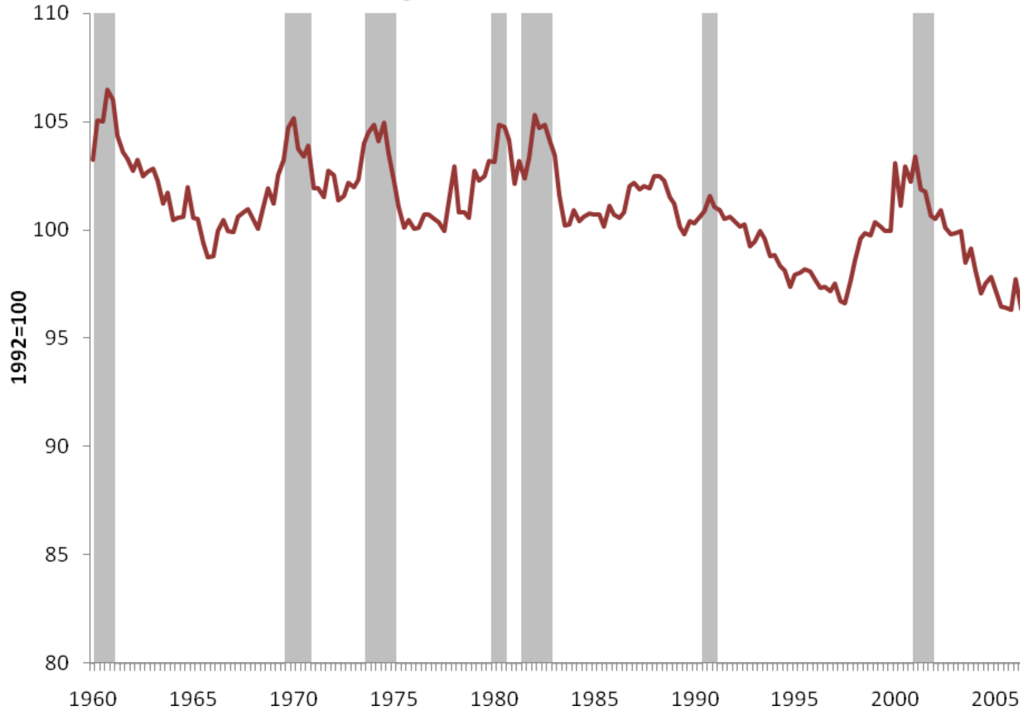
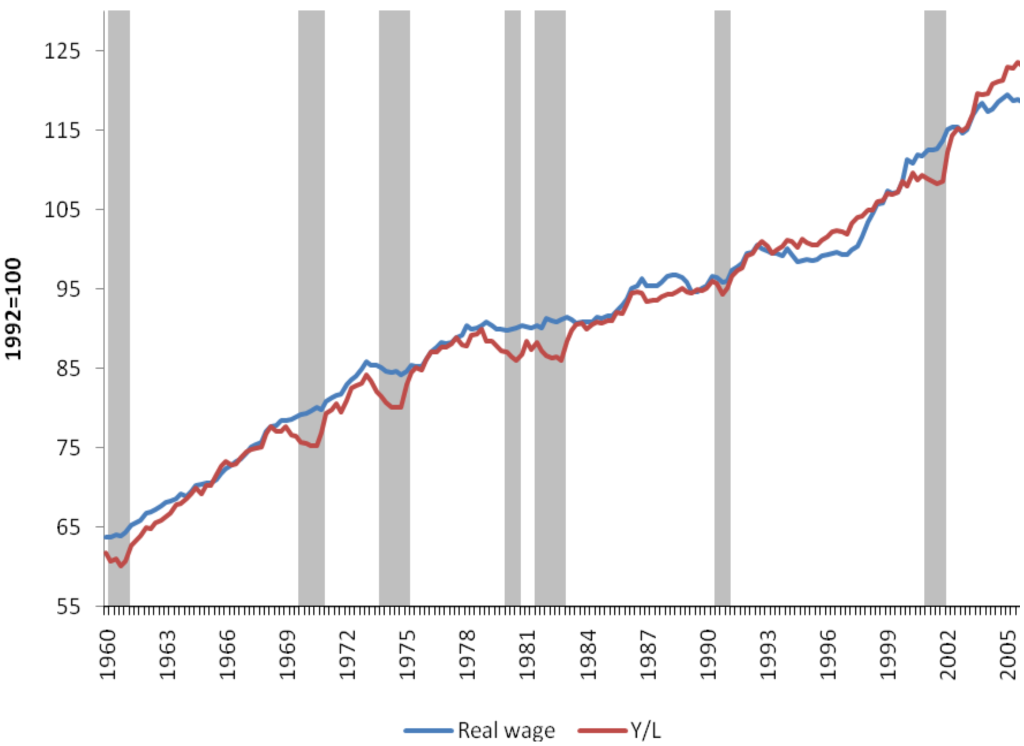


Fig 2: ω vs. Y/L



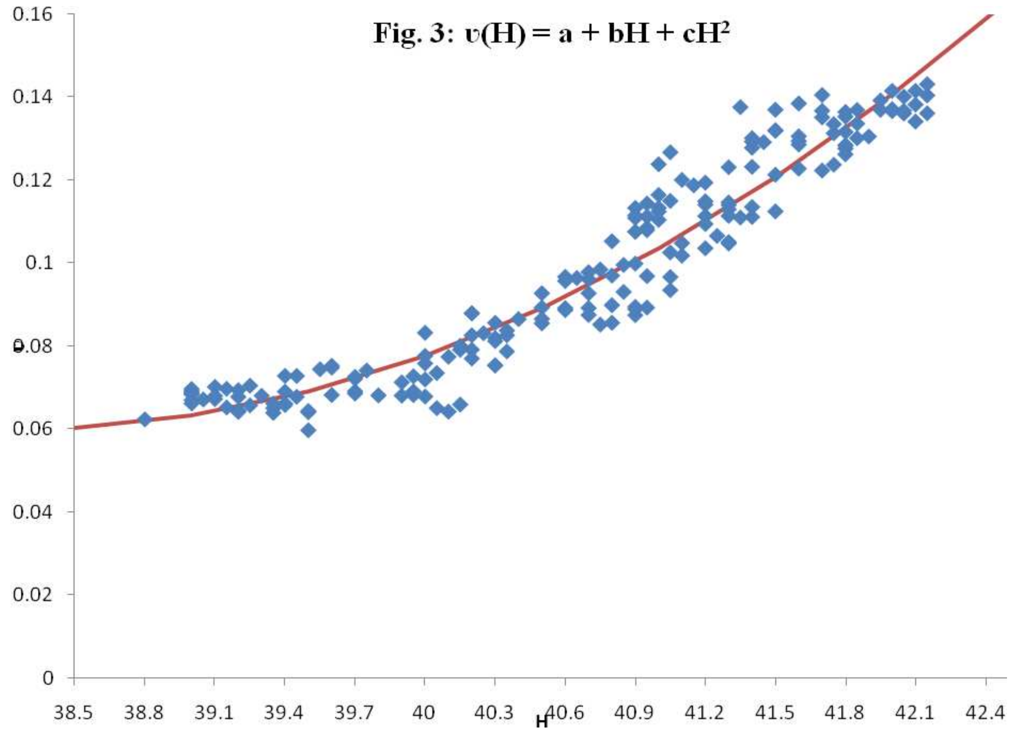


Fig. 4: Manufacturing Marginal Cost

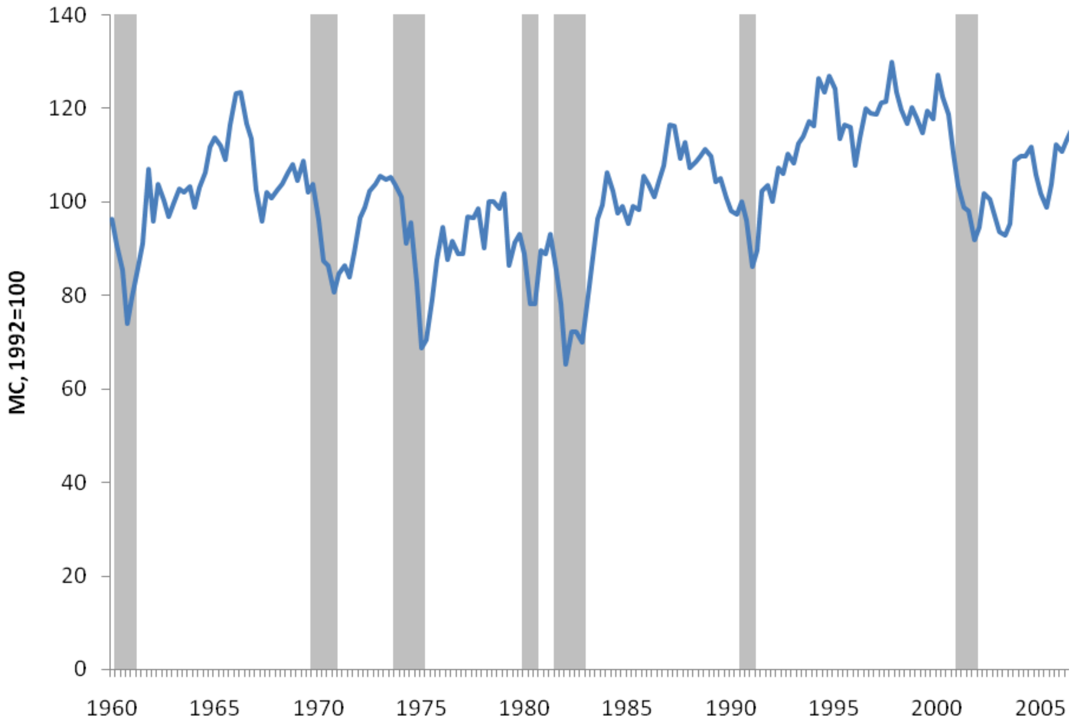


Fig. 5: Manufacturing Labor Share

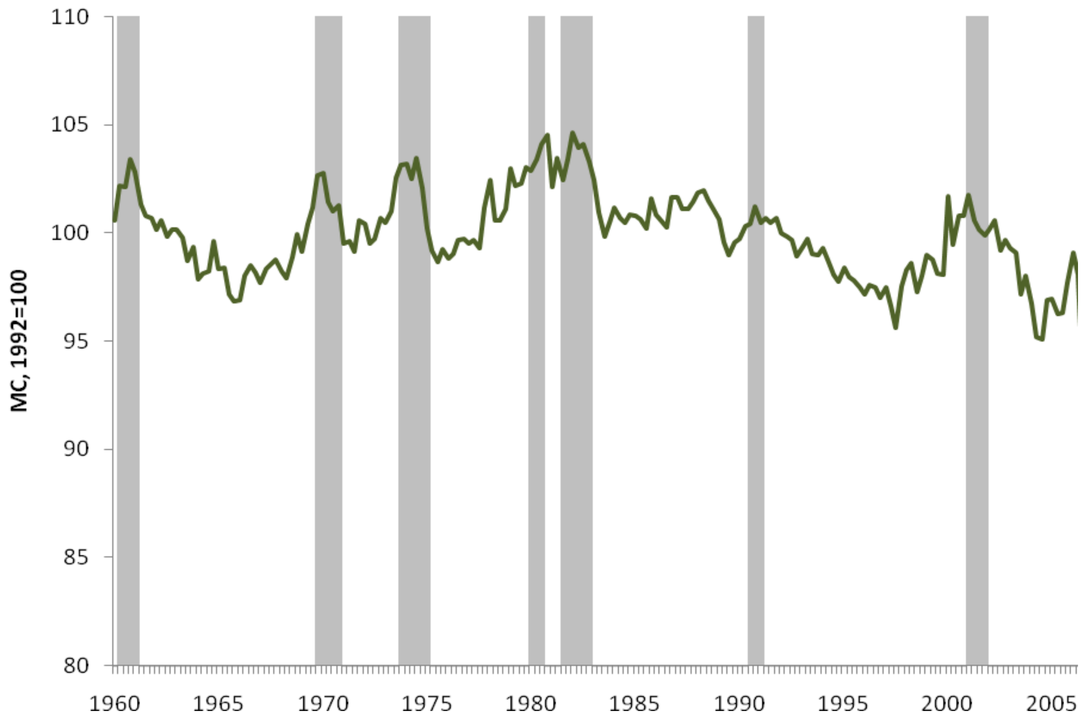


Fig. 6: Weighted-average Marginal Cost