External Habit Formation and the Home Bias Puzzle

Stephen H. Shore[1] and Joshua S. White[234]

First draft: June, 2001.
This draft: January, 2006.

[1]University of Pennsylvania, Wharton School of Business. Email: sshore@wharton.upenn.edu. Please contact author at: 3641 Locust Walk, 303 Colonial Penn Center, Philadelphia, PA 19103; 215-898-7770.

[2]University of Illinois, Urbana-Champaign, College of Business. Email: jswhite@uiuc.edu.

[3]We thank John Campbell, Joshua Coval, George Chacko, David Laibson, Debbie Lucas, Tarun Ramadorai, Jeremy Stein, Luis Viceira, and two anonymous referees, as well as seminar participants at Harvard, NYU, the University of Illinois at Urbana Champaign, the Federal Reserve Board of Governors, and the University of Rochester for valuable comments. We thank Linda Tesar and Ingrid Werner for the use of their home bias data.

[4]JEL Classification: F30 (International Finance, General), G11 (Portfolio Choice), G15 (International Financial Markets)

Abstract

The equity home bias, the observed lack of international diversification in equity portfolios, is a persistent puzzle in international finance. We propose an explanation for this puzzle based on external habit formation preferences. Agents' utility depends on the difference between consumption and a slow-moving subsistence level. The subsistence level, or external habit, is a backward-looking moving average of national aggregate consumption. We assume that a small group of agents holds primarily domestic securities. We can think of these agents as small business owners who are forced to hold domestic assets for agency reasons. If the remaining agents have external habit formation utility, then they will mimic the domestic bias of this small group.

We perform a calibration using consumption and asset return moments from several countries and show that this effect can potentially explain a large aggregate holding of domestic assets. The model performs especially well in explaining high equity home bias in small countries, where the observed lack of international diversification is most puzzling.

1 Introduction

One of the puzzles of international finance is the equity home bias, the tendency of investors to hold more domestic equities than would be optimal under traditional mean-variance analysis. Home bias is pervasive and large. Cooper and Kaplanis (1994) report that domestic equity investment, as a fraction of the total equity portfolio, ranges from 65% in France to near 100% in Sweden. One particularly puzzling fact is the large home bias found in small countries. Small countries, whose equities comprise a small fraction of the global mean-variance efficient portfolio, presumably have the most to gain from international diversification.

We show that a form of preferences based on consumption externalities can generate a substantial equity home bias, even in small countries. We assume that an agent's utility depends on the difference between consumption and a standard of living. This standard of living, or "external habit," is a backward-looking average of the aggregate consumption of an agent's perceived peer group. Under these preferences, agents are particularly sensitive to peers' investment behavior. Because agents' future standard of living depends on investment performance of other agents in the peer group, it is optimal for them to partially mimic the group's aggregate portfolio choice.

Under external habit formation preferences, home bias arises when the reference group is national and a seed group of agents in the country is constrained to overinvest domestically. We assume that this seed group is composed of small business owners and company executives, agents who are unable to adequately diversify their domestic business risks for agency reasons. Heaton and Lucas (2000) show that a substantial fraction of wealth is held in proprietary businesses and that this wealth is very unevenly distributed in the population. In the presence of this seed group, unconstrained agents overinvest domestically, through a herding mechanism. Unconstrained agents mimic the portfolio holdings of the constrained group; they also mimic the portfolio holdings of other unconstrained agents. These effects combine to magnify the domestic equity bias of constrained agents in the aggregate portfolio.

Our model can explain a substantial home bias in small countries. Since agents have constant relative risk aversion preferences in the difference between consumption and habit, marginal utility approaches infinity as consumption approaches the habit level. Agents therefore hold a portfolio that guarantees their consumption need never fall below habit. Unconstrained agents must hold enough of the domestic asset to ensure that the habit obligations incurred by the consumption of the constrained agents are met. This requirement makes the composition of portfolio holdings relatively insensitive to the mean-variance properties of the domestic asset. Therefore, a large home bias arises even in small countries, where the domestic asset is likely to have large idiosyncratic variance.

We calibrate our model to unconditional consumption and asset return moments in the U.S., the U.K., and Sweden, countries for which high-quality, long-term data on consumption is available. We find that a large fraction of observed home bias can be explained by our model. Calibrated allocations range from 64% to 75% for the U.S., 51% to 84% for the U.K., and 30% to 67% for Sweden. In comparison, when domestic investors can invest in home equities and the MSCI EAFE index, mean-variance optimal domestic equity holdings are approximately 57% for the U.S., 35% for the U.K., and 8% for Sweden.

Although our paper focuses on the international home bias, our model has implications for related local non-diversification puzzles. Heaton and Lucas (2000) find that, among stockholders, an average of 10% of total stockholdings are in stocks of companies where they are employeed. Benartzi (2001), in a study of 401(k) plan portfolio allocation, finds that a high level of investment in own-company stock is commonplace. Our model can rationalize these findings if we interpret the reference group as local, rather than national, so that agents mimic investment behavior of their coworkers. A plausible seed group of constrained agents in this case are company executives, who are required to hold options or shares as part of an incentive plan.[1] Also consistent with these types of local peer effects, Hong, Kubik and Stein (2004) have highlighted the importance of social interactions as determinants of equity market participation among Health and Retirement Study respondents.

External habit formation has implications for the dynamics of portfolio holdings. Although the accuracy of existing time series holdings data has been questioned (Warnock, 2001), we present the time series implications of our model here, with two purposes. First, higher quality time series data on the home bias will become available in the future. Time series data on holdings will be an important testing ground for existing home bias theories. Second, the pattern of trading implied by our model corresponds with the findings of Benartzi (2001). Our model offers an explanation not only for high own-company stock ownership, but also for the observation that employees buy more stock when it appreciates.

1.1 Equity Home Bias

Early literature on the equity home bias demonstrates the benefits of international diversification (Levy and Sarnat, 1970) and the tendency to underinvest in foreign securities (Solnik, 1974). More recent work shows that the equity home bias has not gone away (French and Poterba, 1991), although it has diminished somewhat in the 1990's (Tesar and Werner, 1998). Lewis (1999) surveys and evaluates proposed explanations of the equity home bias. The consensus of the literature is that there is no "smoking gun," a single explanation that entirely explains home bias. Many existing explanations either explain insufficient home bias or generate counterfactual auxiliary predictions.

A large class of home bias explanations focuses on the role of domestic equities in hedging a country-specific risk factor. Leading candidate risk factors are the output of domestic non-traded goods and the value of a non-traded asset, human capital. Neither of these risk factors seems to conclusively explain portfolio holdings of domestic equities. A related set of explanations does not identify an exogenous risk factor, but rather makes assumptions that cause domestic wealth to effectively become a risk factor. In DeMarzo, Kaniel, and Kremer (2004), local aggregate wealth determines the price of a scarce "local resource," and agents' desire to hedge local price risk leads them to invest in domestic equities.

In non-tradeable goods models, agents hold equity in the domestic producers of non-traded goods to hedge non-traded goods output. Stockman and Dellas (1989) propose a model of this type in which preferences are separable in non-traded and traded goods. This assumption implies complete home bias in the equity of non-traded goods producers and perfect diversification in the equity of traded goods producers. Tesar (1993), Pesenti and

[1] Although each employee may not have a company executive in her personal reference group, another agent in her reference group may. Overlapping reference groups can distribute the seed for own-company investment throughout the company.

Van Wincoop (1994), and Serrat (2001) show that bias in holdings of non-traded goods producers may be echoed in holdings of traded-goods producers if investors' utility over traded and non-traded goods is non-separable. Their conclusions are highly sensitive to the specification of the utility function. Equilibrium non-tradeables models universally imply that foreigners will not hold shares of domestic non-tradeables producers, which is clearly counterfactual. Further, Lewis (2000) points out that first-order conditions for risk sharing when utility is defined over tradeable and non-tradeable goods consumption are strongly rejected.

A second set of models examines the role of non-traded assets in explaining the home bias. Bottazzi, Pesenti and Van Wincoop (1996) use a continuous-time VAR to model the dynamic interaction between wage rates and returns on traded assets and show empirically that domestic assets can provide a hedge against poor current or future wage realizations. In contrast, Baxter and Jermann (1997) argue that human capital returns make the equity home bias even "worse than you think." Imputing returns to human capital, they argue that domestic equity returns are positively correlated with human capital returns, making foreign equities more attractive.

Related to models of non-traded goods, but qualitatively distinct, is a model of local investment bias based on "community effects" (DeMarzo, Kaniel and Kremer, 2004). Demarzo et al. show that a tilt towards local assets results when agents compete for scarce "local resources" in fixed supply, such as real estate. Demand for local resources depends on the performance of the local aggregate portfolio. Therefore, the locally held portfolio serves as a hedge for the price of local resources. If a subset of agents in a community are inherently locally biased in their investment portfolios, other agents will "herd," mimicking this local bias. The Demarzo et al. model is theoretically appealing because it derives herd behavior under standard assumptions about investors' utility. The model has auxiliary testable implications. The local assets must effectively hedge the price of local resources. Local price risk must be large, with the price of local goods fluctuating more than the payoffs of the non-diversified portfolio. While this assumption has not been tested formally, it is difficult to square with our intuition that local goods prices are substantially less volatile than equity prices, and that equity prices may not be a good hedge for such local price risk. Second, the community in which herding occurs is defined by commonality of the local resource. Agents will mimic the portfolios of all other agents who influence prices of the local resource, but not agents who do not demand this local resource for reasons of preference or geography.

A strength of our external habit model, relative to other explanations of the equity home bias, is the ability to explain a large home bias is small countries, where the benefits of diversification under a canonical risk-sharing model are quite large. Other explanations for the home bias do not have similar implications. Non-traded goods and assets models do not explain this fact, unless we assume that small countries have, for example, more volatile streams of non-traded goods or human capital. Whether this is true is an unresolved empirical question. Nonetheless, it seems unlikely that the characteristics of these factors are sufficiently different across countries to generate the extreme observed home bias in small countries. Transaction costs explanations do not imply a larger home bias in small countries unless costs of international diversification are larger in small countries. To the extent that the structure of transaction costs is endogeneous, this seems like an unlikely explanation – capital markets should be endogeneously structured to enable diversification by those who benefit the most from it. Furthermore, transaction costs have been found to be generally too small in magnitude to explain the lack of international diversification. A noteable exception is Hentschel and Long (2004), who argue that small transaction costs could substantially reduce international diversification.

In contrast to other explanations of the home bias, our model and to some degree the Demarzo et al. model, imply that the size of the equity home bias will be positively related to the size of the seed group in the population. The size of the non-traded sector in the economy is positively related to the size of the inherently domestically biased seed group. Although we should be careful in drawing inferences from cross-country analysis using the limited number of countries for which home bias data is available, it is interesting to note that there is a negative relationship in the data between home bias and the ratio of market capitalization to GDP (Figure 1). Here, home bias is measured as the difference between the percentage allocation to domestic equities and the market capitalization of the equity market relative to the world equity market. This relationship in the figure suggests that, if a country has a large seed group of domestically constrained agents, it will have large home bias. This is suggestive evidence in favor of herding models of the home bias.

In work related to this paper, several authors have used consumption or investment externalities to explain the home bias. Wheatley (2000) investigates a static model in which agents have preferences over mean and variance of return relative to peers. He finds that if agents have labor income and short-sale constraints and the human capital to wealth ratio differs across agents, agents' portfolios will exhibit home bias. Lauterbach and Reisman (2004) investigates a similar model. Chue (2002) uses a continuous time model in which utility depends on the ratio of individual consumption to aggregate consumption.[2] In contrast to this work, this paper uses a dynamic model that, by virtue of the slow-moving nature of habit, has the potential to better fit the observed smoothness of consumption relative to asset returns. Furthermore, the specific form of habit formation preferences we use enables us to intuitively explain the high level of equity home bias in small countries.

1.2 Habit and Comparative Utility

The literature on external habit formation and other forms of comparative utility is relatively old in economics (*e.g.*, Duesenberry, 1949). In finance, more recent work has attempted to use habit formation to explain puzzles in financial markets, such as the equity premium puzzle (Constantinides, 1990; Abel, 1990) and the stock market volatility puzzle (Campbell and Cochrane, 1999), and to explain time-varying expected returns (Campbell and Cochrane, 1999; Chan and Kogan, 2002).

There are several important divisions in the literature relating to how habit formation is specified. Most important for this paper is whether habit is formed internally or externally. In internal habit models, habit is based on an agent's own consumption; in external habit models, it is based on the consumption of peers. The idea that happiness is based on

[2] In contrast to Wheatley (2000) and Reismann (2004), our model is dynamic and uses external habit formation preferences. We obtain a result that illustrates in closed form the impact of external habit formation for highly generic price processes. There are two differences from Chue (2002) in our utility specification: we use a difference model instead of a ratio model, and our model allows a slow-moving reference point instead of relying on instantaneous comparison of consumption values. The ratio model allows closed-form analytic solutions only in this pure comparison form.

relative consumption or wealth is intuitively appealing. Easterlin (1974, 1995) examines international survey evidence on happiness. He finds that while the rich seem happier than the poor within societies, there seem to be no substantial differences in happiness across societies. While any direct survey on utility values should not be taken too literally, this work lends credence to the notion that utility may have a comparative component, supporting external habit formation, and that this comparison is local.

Recent research by Schroder and Skiadas (2002) and Gollier (2004), respectively, has shown that both internal and external habit models show isomorphisms to traditional models without comparative utility. For example, Gollier demonstrates that given certain assumptions, the equilibrium consumption and portfolio choice rules with comparative utility are the same as those without comparative utility, with an adjusted degree of risk aversion.

Another important dichotomy is the distinction between difference and ratio utility. Abel (1990) and Chan and Kogan (2002) use ratio utility, in which the argument of the instantaneous utility function the ratio of consumption to habit, C/X:

$$u(C,X) = \frac{\left(\frac{C}{X}\right)^{1-\gamma}}{1-\gamma}. \tag{1}$$

A practical advantage of this approach is that utility is defined whenever consumption and habit are positive. Constantinides (1990) and Campbell and Cochrane (1999) argue for a specification based on the difference between consumption and habit, $C - X$:

$$u(C,X) = \frac{(C-X)^{1-\gamma}}{1-\gamma}. \tag{2}$$

This approach requires that $C_t > X_t \; \forall t$. In general equilibrium in an exchange economy, this specification requires a careful choice for the evolution of habit, as in Campbell and Cochrane (1999). An advantage of difference-based utility, however, is that relative risk aversion is time-varying and countercyclical. In equilibrium, this implies a countercyclical risk premium.[3]

Difference-based habit formation has an intuitive appeal, if we think of habit as a subsitence level, in the spirit of Rubinstein (1976a, 1976b). In a growing economy, any static subsistence level eventually becomes trivially small. We can think of habit as simply a subsistence level that evolves over time, growing with the economy. By using difference-style external habit, we convey the intuition that agents' consumption creates externalities and that these externalities enter the utility function in the form of a slow-moving subsistence level.

This paper is divided into five sections. Section 2 develops a novel closed-form solution to the general portfolio choice problem for an external habit formation agent, where constrained individuals are allowed to have arbitrary portfolio and consumption rules, and prices follow an arbitrary diffusion process. Section 3 shows how this framework can help to explain the level and time series behavior of the equity home bias. Section 4 presents a calibration of the model. Section 5 concludes.

[3] Constantinides (1990) and Lax (2002) also use difference-style utility. Constantinides employs difference-style utility with internal habit in a continuous-time, portfolio choice framework. Lax uses a discrete-time difference-style internal habit model to explain increasing conservatism in portfolios as agents age.

2 An External Habit Formation Model

This section considers the decision problem of an agent with external habit formation utility who takes as given both prices and the portfolio and consumption choices of other agents in the economy. We derive the consumption and portfolio rules of the agent in relation to the optimal policy of a hypothetical agent with power utility facing identical investment opportunities. We show that agents optimally mimic the portfolio rule of their peers. Intuitively, if agents are risk averse in their performance relative to a peer-determined benchmark, mimicking peers' investments provides a hedge, so that agents will never be forced to consume below their habit level.

We assume that the agent values consumption relative to habit, X, which is a geometrically weighted, backward-looking moving average of aggregate consumption,

$$X_t = b \int_{-\infty}^{t} e^{-a(t-s)} C_s^A ds.$$

Here, C_t^A represents aggregate consumption in the agent's reference group at time t, and a and b are parameters governing the evolution of habit. This habit formation process is also used in Chan and Kogan (2002) and is an external habit version of the process used in Constantinides (1990). An important implication of this habit process is that the evolution of X is instantaneously deterministic:

$$dX = \left(bC^A - aX\right) dt \tag{3}$$

The agent's preferences are of the difference type employed in Constantinides (1990) and Campbell and Cochrane (1999):

$$U_0 = \mathrm{E}_0 \int_0^\infty e^{-\delta t} \frac{\left(C_t - X_t\right)^{1-\gamma}}{1-\gamma} dt,\ \gamma \neq 1; \tag{4}$$

$$U_0 = \mathrm{E}_0 \int_0^\infty e^{-\delta t} \ln\left(C_t - X_t\right) dt,\ \gamma = 1. \tag{5}$$

We assume that other agents in the reference group follow a known investment and consumption rule, in aggregate. This rule is an arbitrarily time- and state-dependent policy $\{\alpha_A, C_A\}$ subject to a budget constraint over aggregate wealth, W_A. We refer only to the average policy – we do not require that all other agents are identical or need to know the distribution of policies in the economy.

The agent's investment opportunities are characterized by N assets, with price vector P, and K state variables, denoted by the vector H. Changes to both are driven by an $S \times 1$ vector of orthogonal shocks, dZ, where $S \geq N$. We allow drift and diffusion terms to depend on H:

$$\frac{dP_i}{P_i} = \mu_i\left(H\right) dt + \Gamma_i\left(H\right) dZ, \forall i = 1, ..., N; \tag{6}$$

$$dZ_i dZ_j = 0, i \neq j;\quad dZ_i dZ_j = dt, i = j; \tag{7}$$

$$dH = \upsilon\left(H\right) dt + \Phi\left(H\right) dZ. \tag{8}$$

Here, μ_i is the i-th element of an $N \times 1$ vector μ and Γ_i is the i-th row of an $N \times S$ matrix Γ. We assume also that the risk-free interest rate is constant,

$$\frac{dB}{B} = rdt, \tag{9}$$

where B is the price of the risk-free asset. We denote by the $N \times 1$ vector α the fraction of the agent's portfolio held in each risky asset; consequently, $1 - \alpha'\iota$ is the fraction invested in the risk-free asset. In the sense that we take prices as given, we can interpret this model as a partial equilibrium model. Alternatively, we can interpret it as a general equilibrium model, in the spirit of Constantinides (1990), where the price processes above reflect the characteristics of risky production technologies.

The choice problem of the external habit formation agent is therefore

$$\begin{aligned} &\max_{C,\alpha} \mathrm{E}_t \int_t^\infty e^{-\delta(\tau-t)} \frac{(C_\tau - X_\tau)^{1-\gamma}}{1-\gamma} d\tau \\ \text{s.t. } &dW = W\left(rdt + \alpha^T \left(\mu(H) - r\iota\right) dt + \alpha^T \Gamma(H)\, dZ\right) - Cdt, \\ &W_t > 0 \,\forall t. \end{aligned} \tag{10}$$

Proposition 1 solves this problem in terms of the solution of an analogous problem, the maximization problem of a power utility agent under the identical budget constraint.

Proposition 1 *Suppose that the power utility problem*

$$\begin{aligned} &\max_{C,\alpha} \mathrm{E}_t \int_t^\infty e^{-\delta(\tau-t)} \frac{C_\tau^{1-\gamma}}{1-\gamma} d\tau \\ s.t. \ &dW = W\left(rdt + \alpha^T \left(\mu(H) - r\iota\right) dt + \alpha^T \Gamma(H)\, dZ\right) - Cdt, \\ &W_t > 0 \,\forall t. \end{aligned}$$

has a value function of the form

$$J = f(H) \frac{W^{1-\gamma}}{1-\gamma}. \tag{11}$$

Then the choice problem (10) has the value function

$$I = f(H) \frac{\left(W - \frac{X + bW_A}{r+a}\right)^{1-\gamma}}{1-\gamma}. \tag{12}$$

The corresponding optimal consumption and investment rules are

$$C = X + f^{-\frac{1}{\gamma}} \left(W - \frac{X + bW_A}{r+a}\right); \tag{13}$$

$$\begin{aligned} \alpha = &\left(1 - \frac{\frac{X}{W} + b\frac{W_A}{W}}{r+a}\right) \frac{1}{\gamma} \left(\Gamma\Gamma^T\right)^{-1} \left[(\mu - r\iota) + \Gamma\Phi^T \frac{1}{f}\frac{df^T}{dH}\right] \\ &+ \alpha_A \frac{b\frac{W_A}{W}}{r+a} \end{aligned} \tag{14}$$

Proof. *See Appendix A.* ■

Proposition 1 shows that whenever the power utility solution to the choice problem takes the standard form, we can solve the analogous external habit formation problem. The optimal policies depend only on instantaneous knowledge of habit, wealth, aggregate wealth, and aggregate portfolio choice. Our results here are similar to those obtained in Schroder and Skiadas (2002), who prove an isomorphism between internal habit problems and their dual, non-habit problem.[4] It is important to note that we require the agent to be able to hold the aggregate portfolio; otherwise, without imposing significant restrictions on asset returns, consumption cannot be assured to stay above habit in all states of the world.

The portfolio rule can be decomposed into three parts: an investment in bonds required to finance existing habit, an investment in the aggregate portfolio required to finance the habit obligations incurred by future aggregate consumption, and a surplus component invested as an agent with power utility.

The first two components together can be thought of as an asset that pays habit at its dividend. The first component is an investment in bonds costing

$$\frac{X}{r+a}.$$

This asset is the price of a bond paying a coupon of an initial amount X and declining at the constant rate a. In the absence of future aggregate consumption, habit will decay certainly at the rate a. In this sense, consumption obligations caused by existing habit can be thought of as riskless, and can be financed entirely by holdings of riskless assets.

The second component is an investment of

$$\frac{bW_A}{r+a}$$

in the aggregate portfolio. This component completes the synthesis of habit. When a unit of aggregate consumption occurs, it generates riskless obligations going forward instantaneously in the amount b, decaying at the rate a. Since per-unit financing of this obligation will cost $\frac{b}{r+a}$ (since the riskless rate is assumed to be constant), the agent must be assured that, whenever consumption occurs in the future, she will have enough wealth to buy bonds worth $\frac{b}{r+a}C_A$. The only way to achieve this is to hold $\frac{bW_A}{r+a}$ in the aggregate portfolio, α_A. These two components together form an asset that pays habit as its dividend. Thus, the first two components of the portfolio rule guarantee that the agent will always be able to consume at least habit, even in the worst state of the world, as long as wealth is large enough to allow investment in these assets. That is, surplus wealth, W_S must be positive:

$$W_S \equiv W - \frac{X + bW_A}{r+a} > 0.$$

If W_S is positive initially, it will positive at all subsequent points in time.

[4] In contrast to Schroder and Skiadas, our model uses external habit formation. Unlike Schroder and Skiadas, we cannot obtain tractable solutions when interest rates are allowed to vary. Schroder and Skiadas make a slightly broader point in drawing an analogy between the internal habit and dual problems: they derive results for all utility functions $u(C - X)$ where the solution to the dual problem, with preferences $u(C)$, is well-defined. Our results can be extended in the same manner, although we choose to focus on power utility for expositional simplicity.

When the agent holds this asset, she is guaranteed a payment of exactly X in each period. Since the utility function takes the form $\frac{(C-X)^{1-\gamma}}{1-\gamma}$, once the consumption of X is guaranteed, policy choices are made as a power agent consuming out of surplus wealth. Surplus wealth is invested according to the power utility rule

$$\alpha_{power} = \frac{1}{\gamma}\left(\Gamma\Gamma^T\right)^{-1}(\mu - r\iota) + \frac{1}{\gamma}\left(\Gamma\Gamma^T\right)^{-1}\Gamma\Phi^T\frac{1}{f}\frac{df^T}{dH},$$

with myopic and hedging terms, respectively. Thus, the portfolio rule can be restated in a more intuitive form:

$$\alpha = \frac{W_S}{W}\alpha_{power} + \frac{W_A}{W}\frac{b}{r+a}\alpha_A$$

Also, surplus consumption, $C - X$, is linear in surplus wealth in the proportion that a power utility agent would choose:

$$\begin{aligned} C - X &= f^{-\frac{1}{\gamma}}\left(W - \frac{X + bW_A}{r+a}\right) \\ &= f^{-\frac{1}{\gamma}}W_S \end{aligned}$$

Since the power utility consumption-wealth ratio is $f^{-\frac{1}{\gamma}}$, excess consumption of the habit agent can be seen as analogous to power utility consumption out of surplus wealth. This disaggregation of wealth is similar to the discrete-time, internal habit formation model of Lax (2001), who decomposes wealth into the portion required to sustain habit and the portion invested as a power utility agent.

An interesting special case of this problem is pure comparison utility, in which utility of consumption is measured relative to a aggregate consumption, instead of a slow-moving habit:

$$U\left(C, C^A\right) = \frac{\left(C - \psi C^A\right)^{1-\gamma}}{1-\gamma}. \tag{15}$$

This model arises as a limiting case where b and a approach infinity in a fixed proportion. Specifically, if $\psi = \frac{b}{a}$, habit is

$$X_t = \psi\int_{-\infty}^{t} ae^{-a(t-s)}C_s^A ds, \tag{16}$$

and

$$\begin{aligned} \lim_{a\to\infty} X_t &= \lim_{a\to\infty}\psi\int_{-\infty}^{t} ae^{-a(t-s)}C_s^A ds \\ &= \psi C_t^A. \end{aligned} \tag{17}$$

We can reinterpret the consumption and portfolio rules in the pure comparison case as

$$C = \psi C_A + f^{-\frac{1}{\gamma}}\left(W - \psi W_A\right); \tag{18}$$

$$\alpha = \left(1 - \psi\frac{W_A}{W}\right)\frac{1}{\gamma}\left(\Gamma\Gamma^T\right)^{-1}\left[(\mu - r\iota) + \Gamma\Phi^T\frac{1}{f}\frac{df^T}{dH}\right] + \alpha_A\psi\frac{W_A}{W}. \tag{19}$$

Note that, when agents are identical, pure comparison preferences lead to consumption and investment rules that are identical to the power case.

3 Habit Formation and the Home Bias

In the previous section, we show that external habit formation agents will optimally mimic aggregate portfolio weights. This section focuses on the implications of this result when a seed group in the population is constrained to hold domestic assets. For generality, the results in this section leave the identity of the seed group unspecified, while in the Section 4, for purposes of calibration, we assume that the seed group consists of small business owners.

Unconstrained agents with external habit formation utility mimic aggregate investment behavior. A certain portion of this behavior is specified by the seed group, and the remainder by other unconstrained agents. Therefore, there is a feedback effect: the effect of constrained agents is multiplied, since unconstrained agents mimic the mimicking behavior of other unconstrained agents. Section 3.1 derives the implications of this effect for the equity home bias. In Section 3.2, we illustrate the effect using a small country example, in which a power utility agent would optimally choose to hold none of the domestic asset. We show that, in this example, home bias is entirely independent of the mean-variance characteristics of the domestic asset. Section 3.3 considers a simplified version of the model in an exchange economy and demonstrates that the portfolio allocation results in this case survive when we take dividends as exogenous and solve for prices. Section 3.4 examines the time series implications of the portfolio choice model.

3.1 Portfolio Choice

Proposition 1 implies that external habit formation agents mimic other agents in the economy. We consider an economy with two types of agents, external habit formation agents (Type E) and other agents (Type O) with arbitrary portfolio weights. Type E agents mimic the investment behavior of each other and of the Type O agents and therefore tilt their portfolio in the direction of the Type O portfolio. If we assume that Type O agents are inherently home biased (*e.g.*, are constrained to hold domestic assets) then Type E agents will have a potentially large home bias as well. Any intrinsic home bias in the Type O portfolio is amplified in two ways: first, Type E agents must copy Type O agents; second, Type E agents must copy other Type E agents, who are, in turn, copying Type O agents. This leads to a feedback effect in portfolio allocations.

Proposition 2 *Suppose that the assumptions of Proposition* 1 *hold. Let a fraction* $(1-\beta)$ *of the population be identical external habit formation agents (Type E) as described in Proposition* 1*, with habit* X *and per-capita wealth* W_E*. Suppose that the remaining agents (Type O) have average per-capita wealth* W_O *and arbitrary time- and state-dependent aggregate portfolio holdings* α_O*. Optimal portfolio choice of Type E agents is*

$$\alpha_E = \left(1 - \frac{\frac{X}{W_E} + b\beta\frac{W_O}{W_E}}{r+a-b\,(1-\beta)}\right)\frac{1}{\gamma}\left(\Gamma\Gamma^T\right)^{-1}\left[(\mu - r\iota) + \Gamma\Phi^T\frac{1}{f}\frac{df^T}{dH}\right]$$
$$+\alpha_O\frac{b\beta\frac{W_O}{W_E}}{r+a-b\,(1-\beta)} \tag{20}$$

and optimal consumption of Type E agents is

$$C_E = X + f^{-\frac{1}{\gamma}}\left(\frac{r+a-b\,(1-\beta)}{r+a}\right)\left(W_E - \frac{X + b\beta W_O}{r+a-b\,(1-\beta)}\right). \tag{21}$$

Proof. *See Appendix A.* ■

Proposition 2 generates intuition for examining the impact of a fraction of agents, Type O, who are intrinsically biased in their portfolio choice. In the context of home bias, we can think of Type O agents as intrinsically biased toward holding more domestic securities than a power investor. There are a number of possible justifications for this subgroup. Heaton and Lucas (2000) establish that "entrepreneurial risk" is faced by a large number of agents and is economically important. This risk is generally domestic. Furthermore, short-sales constraints may prevent these agents from diversifying away the tradeable, country-wide component of this risk. Also, some agents may have behavioral reasons for a domestic bias. We make the simplifying assumption that the assets held by Type O agents are perfect proxies for the domestic asset. Our model predicts that the home bias of entrepreneurs or behavioral agents will be subject to a feedback effect in the remainder of the population. It is important to note that the model makes the strong requirement that Type E agents must be able to invest in assets which span the assets held by Type O agents. If Type E agents cannot perfectly mimic the portfolios of Type O agents, their consumption may fall below habit in some state of the world. For example, if Type E agents represent entrepreneurs who hold proprietary businesses for agency reasons, the *aggregate* value of proprietary businesses must be spanned by tradeable assets.[5]

The consumption rule in Proposition 2 can be restated as

$$C_E = X + f^{-\frac{1}{\gamma}} \left(W_E - \frac{X + b\left[\beta W_O + (1-\beta)\, W_E\right]}{r+a} \right)$$

Note that surplus wealth,

$$W_S = W_E - \frac{X + b\left[\beta W_O + (1-\beta)\, W_E\right]}{r+a},$$

again plays an important role in the optimal policy.

If the Type E agents merely needed to hedge the changes in wealth of Type O agents, Proposition 1 shows that they would need to hold only

$$\alpha_O \frac{b\beta \frac{W_O}{W_E}}{r+a}.$$

In the presence of other Type E agents, however, the feedback mechanism (the need to mimic other Type E agents) described above further increases a Type E agent's holdings in α_O to

$$\alpha_O \frac{b\beta \frac{W_O}{W_E}}{r+a-b\,(1-\beta)}. \tag{22}$$

In this sense, we could view the multiplier, M,

$$M \equiv \frac{r+a}{r+a-b\,(1-\beta)} > 1$$

[5] More specifically, Type E agents must be able to buy an asset whose payoffs are weakly greater than that of the aggregate proprietary portfolio. The critical assumption is that Type E agents must be able to hold assets that prevent the aggregate wealth of Type O agents from exploding relative to Type E agents.

as the effect that the presence of other Type E agents has on the holdings of α_O by Type E agents. The Type E agent's portfolio weight in the Type O asset, α_O, cannot exceed the Type O agent's portfolio weight in the same asset, which is 1 by definition. For the problem to be well-defined, we must have

$$\frac{b\beta\frac{W_O}{W_E}}{r+a-b(1-\beta)} < 1.$$

Otherwise, surplus wealth would be negative, and agents would not be able to finance consumption above the habit level. Consequently, this places an upper bound of 1 on the fraction of wealth that the Type E agent devotes to mimicking the Type O agent. Put another way, Type E agents will not hold a levered version of the Type O portfolio.

3.2 A Simple Small Country Example

To illustrate more plainly the effect of external habit formation on home bias, we construct an example in which a power utility agent would never optimally invest in the domestic asset. Suppose there are two assets, one "domestic" asset held by Type O agents with $\alpha_O^D = 1$, and one "international" asset, not held by Type O agents. Assume that the return on the domestic asset is identical to the return on the international asset plus an additional uncorrelated shock, implying that the optimal power utility portfolio rule is to hold only the international asset and the risk-free asset. The price processes are

$$\begin{aligned}\frac{dP^I}{P^I} &= \mu dt + \sigma dz_I,\\ \frac{dP^D}{P^D} &= \mu dt + \sigma dz_I + \sigma_D dz_D,\\ \frac{dB}{B} &= r\,dt.\end{aligned}$$

The portfolio rule implied by Proposition 2 is

$$\alpha_E^I = \left(1 - \frac{\frac{X}{W_E} + b\beta\frac{W_O}{W_E}}{r+a-b(1-\beta)}\right)\frac{(\mu-r)}{\gamma\sigma^2} \tag{23}$$

$$\alpha_E^D = \frac{b\beta\frac{W_O}{W_E}}{r+a-b(1-\beta)}, \tag{24}$$

and the consumption rule is

$$C_E = X + \frac{1}{\gamma}\left[\delta - (1-\gamma)\left[r + \frac{1}{2}\frac{(\mu-r)^2}{\gamma\sigma^2}\right]\right] \times \frac{r+a-b(1-\beta)}{r+a}\left(W_E - \frac{X + b\beta W_O}{r+a-b(1-\beta)}\right). \tag{25}$$

While a power-utility agent would have no incentive to hold domestic assets, the Type E agent holds domestic assets in a proportion determined by the relative wealth of Type O agents and parameters of the habit process. Type E agents hold the domestic asset because their future habit is, in part, determined by consumption of Type O agents. The Type E agents are hedging their future habit risk by holding the domestic asset.

Equations (24) and (25) illustrate that, in small countries, home bias is completely insensitive to the return characteristics of the domestic asset. Because consumption must exceed habit in all states of the world, agents must mimic the asset holdings of Type O agents, regardless of the characteristics of the returns on this portfolio. In small counties, in which the domestic asset is simply the world asset with an additional noise term, consumption and portfolio choice are entirely insensitive to the volatility and expected return of the domestic asset.

3.3 General Equilibrium in an Exchange Economy

Our goal in this subsection is to establish the robustness of our result to general equilibrium considerations. We show that, in a model with difference-type external habit formation utility, portfolio choice implications similar to those derived in partial equilibrium persist in an exchange economy. In the above results, we take the price processes as given and solve for the optimal portfolio choice rule. When modeling a small economy, for which prices will be set by the rest of the world, this is an appropriate assumption. For a large economy, it is potentially important to consider the impact of changes in demand on asset prices.

In the previous sections, we have made some assumptions to solve the portfolio choice problem that, taken together, do not permit a solution for general equilibrium in an exchange economy. We make the assumption that utility is a power function of the difference between consumption and habit. Consequently, consumption must be greater than habit in all states for the problem to be well defined. In an exchange economy, dividends evolve exogenously and prices adjust so that optimal aggregate consumption is equal to aggregate dividends in each period. However, if aggregate dividends can fall below habit in any state of the world, expected utility is undefined. Since we also make the assumption that habit is a slow-moving, geometric average of past consumption, then aggregate dividends will fall below habit if dividends suffer a sufficiently bad shock.

We must therefore change either the form of the utility function or the process for habit. This could be achieved by making instantaneous utility a power function of the ratio of consumption to habit. This approach is taken by Abel (1990) in a homogeneous agent, discrete-time model. This framework leads to a countercyclical risk premium but an excessively variable risk free rate. Campbell and Cochrane (1999) use a difference utility framework but specify a non-linear habit process to guarantee that aggregate consumption always remains greater than habit. This model also generates a countercyclical risk premium and can be calibrated to have a constant interest rate.

Another option is to use a difference utility framework but set habit proportional to instantaneous consumption, instead of a backward-looking average. For the purposes of our illustration, we will pursue this option. Since preferences in this setting are a limiting case of the model we have characterized in partial equilibrium, we will be able to generate analytic representations of the equilibrium that maintain our intuition about the role of mimicking portfolios in generating home bias.

We assume that agents have pure comparison utility, as in (15). Recall that these preferences can be derived as a limiting case of the model we have used so far (17). External habit agents in country i compare themselves only to country i agents. As in our portfolio choice model, a fraction β_i of country i's population is assumed to be of Type O. These agents are constrained to hold a fraction ϕ_i of the asset which represents a claim on the domestic dividend D_i, and consume $\phi_i D_i$, the dividend from their share of the asset. The remainder of the agents in each economy are of Type E, have population $1-\beta_i$ and are assumed to have pure comparison utility.

Markets are assumed complete, so that marginal utility of any two agents is proportional in all states of the world. Consequently, in a two-country setting, we have

$$(C_{E1} - C_{A1})^{-\gamma} = \lambda (C_{E2} - C_{A2})^{-\gamma} \tag{26}$$

where C_{Ei} is per-capita consumption of Type E agents and C_{Ai} is aggregate per-capita consumption in country i. Imposing the market-clearing constraint, that agents consume the aggregate dividend in each period, we solve for the stochastic discount factor Λ in terms of the dividends in both economies:

$$\Lambda = e^{-\rho t} \left[k_1 D_1 + k_2 D_2\right]^{-\gamma}; \tag{27}$$

$$k_i = \frac{\psi(\beta_i - 1) + 1 - \beta_i \phi_i}{1 - \psi(1-\beta_i)}. \tag{28}$$

The stochastic discount factor is a function of a linear combination of the two dividends. This reflects the intuition that Type O agents consume some fraction of each dividend, that Type E agents must mimic them, and that remaining dividends are consumed by Type E agents as if they had power utility. Type O agents in country i consume a fraction $\beta_i \phi_i$ of the dividend D_i. Type E agents in country i must consume at least a fraction

$$\frac{\psi(1-\beta_i)}{1-\psi(1-\beta_i)} \beta_i \phi_i$$

of the dividend D_i at all times to meet the benchmark level of consumption. Consequently, regardless of the prices of the assets, a fraction

$$\beta_i \phi_i + \frac{\psi(1-\beta_i)}{1-\psi(1-\beta_i)} \beta_i \phi_i = 1 - k_i$$

of the dividend can be thought of as "set aside," either because it is consumed by Type O agents or because it must be used by Type E agents to mimic Type O consumption. We can think of the residual fraction as the "free dividend."

In a general equilibrium model with power utility, homogeneous risk aversion, and complete markets, assets are priced based on their covariance with the aggregate dividend, so that the stochastic discount factor is:

$$\Lambda^p = e^{-\rho t} \left[D_1 + D_2\right]^{-\gamma}. \tag{29}$$

In a pure comparison model, assets are priced based on their covariance with the aggregate "free dividend." When the importance of Type O agents is the same in the two economies, when $\beta_1 = \beta_2$ and $\phi_1 = \phi_2$, the stochastic discount factor above, (27), reduces to that of a power economy, (29). Since Type E agents must hold domestic assets to mimic Type E agents *regardless of asset prices*, home bias can be generated in a general equilibrium comparison economy even when the stochastic discount factor is the same as in a power economy.

3.4 Time Series of Portfolio Holdings

We will show that external habit formation implies a trading rule that is closer to a pure passive rule (a non-rebalancing rule) than the trading rule of a power utility investor. External habit formation predicts that, when domestic returns exceed world returns, the fraction of wealth devoted to domestic assets should increase, as Type O agents become relatively richer. While time series evidence on portfolio holdings is sparse and of poor quality, we use available data to show that this prediction is borne out by the data. Many existing models of the home bias do not have this prediction.

To understand the trading implications of our model more formally, we decompose assets held by Type E agents into three components and examine the trading rules associated with each. These components are: the portfolio invested in bonds to replicate riskless habit; the mimicking portfolio invested as Type O agents; and remaining wealth invested as the power agent. Since the quantity of riskless assets held can be characterized by

$$N_B = \frac{X}{B\left(r + a - b\left(1 - \beta\right)\right)},$$

the trading rule for the riskless asset is just

$$\frac{dN_B}{N_B} = \left[b\frac{C_A}{X} - \left(r + a - b\left(1 - \beta\right)\right)\right] dt.$$

Note that this trading rule is locally deterministic and merely reflects purchases in bonds proportional to consumption and coupon payments covering the consumption obligations incurred by habit.

The quantity of shares held to mimic Type O agents is

$$\frac{b\beta}{r + a - b\left(1 - \beta\right)} N_O,$$

where N_O is a vector characterizing the number of shares in each asset held by the Type O agent. Therefore, the trading rule in this portfolio merely reflects the Type O agents' trading rule:

$$\frac{b\beta}{r + a - b\left(1 - \beta\right)} dN_O.$$

Type E agents will only trade in the asset held by Type O agents to mimic their trading. If Type O agents sell one share of their asset, each Type E agent will sell $\frac{b\beta}{r+a-b(1-\beta)}$ shares. Note also that, if Type O agents sell shares to finance consumption, Type E agents will sell shares proportionally and use the proceeds to buy the riskless asset.

Remaining wealth is invested as a power agent. The quantity of asset i held by Type E agents in this remaining portion is

$$N_{P,i} = \frac{\left(W_E - \frac{X}{r+a-b(1-\beta)} - \frac{b\beta}{r+a-b(1-\beta)} W_O\right) \alpha_{P,i}}{P_i},$$

where $\alpha_{P,i}$ is the fraction of wealth held in asset i by a hypothetical agent with power utility. An agent with power utility holds a constant fraction of wealth in each asset,

$$\alpha_{P,i} = \frac{1}{\gamma} \left[\left(\Gamma\Gamma^T\right)^{-1} (\mu - r\iota) \right]_i .$$

In this portion of the portfolio, therefore, there is continuous rebalancing, as appreciating assets are sold and depreciating assets are purchased to keep α_P constant. In the power case, wealth follows a geometric Brownian motion; here

$$W_E - \frac{X}{r + a - b\,(1-\beta)} - \frac{b\beta}{r + a - b\,(1-\beta)} W_O$$

also follows a geometric Brownian motion. Therefore, continuous rebalancing is confined to wealth left over once habit and mimicking of Type O agents have been financed.

The complete trading rule of external habit agents in risky assets is

$$dN_{E,i} = d\left(\frac{\left(W_E - \frac{X}{r+a-b(1-\beta)} - \frac{b\beta}{r+a-b(1-\beta)} W_O\right) \alpha_{P,i}}{P_i} \right) + \frac{b\beta}{r + a - b\,(1-\beta)} dN_{O,i}.$$

The first term, which follows a geometric Brownian motion, is merely the continuous trading rule of an agent with power utility and wealth $\left(W_E - \frac{X}{r+a-b(1-\beta)} - \frac{b\beta}{r+a-b(1-\beta)} W_O\right)$. The second term is the trading rule required to mimic the trading of Type O agents. To the degree that Type O agents do not rebalance, we would expect Type E agents to mimic this. If we interpret Type O agents as proprietary business owners, it is plausible to expect that holdings in these businesses will not be rebalanced. Rebalancing is difficult because it is costly to shut down proprietary businesses, costly to start them, and difficult to sell them to foreign investors for agency reasons. In the international model outlined above, Type O agents invest all wealth in the domestic asset, and therefore do not rebalance. Type E agents will partially mimic this passivity.

This finding implies a negative correlation between domestic returns in excess of world returns and changes of the portfolio weight in the foreign asset. As the domestic asset appreciates relative to the foreign asset, the wealth of Type O agents increases relative to Type E agents. Consequently, Type E agents increase the fraction of their portfolio devoted to mimicking Type O agents.

To examine this prediction, we look at the time series data on the equity home bias assembled by Tessar and Werner (1998), with the caveat that this data may be measured with substantial error (Warnock, 2001). Tesar and Werner (1998) assemble annual data on portfolio holdings of foreign stocks in the U.K., Germany, Canada, and the U.S. from 1987 to 1996 (with the exception of the U.K., for which the series ends in 1995). We use MSCI country-specific annual returns during this period to calculate the correlation between returns and contemporaneous percentage changes in portfolio weights. Under a power utility model, these correlations would be zero. External habit formation implies that these correlations will be negative. In all four countries, we find large negative correlations ranging from -20% to -70%. The correlations are, for the U.S., -70.2%; for the U.K., -50.7%; for Canada, -20.4%; for Germany, -42.7%. Section 4 will demonstrate that similar correlations are found in our calibration of the model.

This evidence is only suggestive, as it cannot differentiate our model from other stories that imply passive adjustment. However, partial passivity of the portfolio is a prediction of external habit that distinguishes it from some other theories. If, as proposed by Hong, Kubick and Stein (2004), agents invest in domestic stocks because they learn about domestic opportunities from their neighbors, then investment behavior does not necessarily vary with asset returns. Similarly, if domestic agents have superior information about home country stocks (Coval and Moskowitz, 1999), it is not obvious why quality of information would increase as domestic stocks appreciate. Finally, models in which individuals have a raw familiarity bias towards own country stocks (Huberman, 2001) do not endogenously generate a time-varying home bias. Our time series evidence does not help to differentiate our hypothesis from goods hedging models (Stockman and Dellas, 1989; Obstfeld and Rogoff, 2000; DeMarzo, Kaniel, and Kremer, 2004), or models in which investors learn about asset return moments dynamically (Pástor, 2000). Also, passive adjustment is consistent with many general equilibrium models, so taking many of the existing explanations of home bias into a general equilibrium setting could make them consistent with passive adjustment under some circumstances.

We can use the trading implications of external habit formation to understand recent work by Benartzi (2001) on trading in 401(k) plans. Benartzi shows that agents hold too much of their 401(k) assets in the stock of their employer from a mean-variance efficient perspective. He also finds that employees buy own company stock when it appreciates and sell it when it declines, unlike power utility agents, who would sell winners and buy losers to rebalance their portfolio. Our model can explain both of these facts.

We can think of the set of company employees as a reference group in our model, in the sense that habit is generated as a backward-looking average of employees' consumption. Further, we can treat executives holding stock options for agency reasons as Type O agents. In this setting, Type E agents (other employees) will hold own company stock to track each others' performance and the performance of Type O agents (executives).

In addition, our model generates trading predictions that are consistent with the Benartzi findings. If Type O agents hold call options on company stock, as is the case with incentive stock options, then Type E agents will mimic Type O agents by holding options. If agents do not hold options directly (employees cannot hold own-company options in 401(k) portfolios), they can replicate them by holding a dynamically changing levered position in the stock. The strategy for replicating call options involves buying shares when the stock price rises, and selling shares when the stock price falls. This is exactly the pattern observed by Benartzi. Although Benartzi attributes observed trading behavior to excessive extrapolation, it is also consistent with rational behavior by external habit formation agents.

4 Calibration

4.1 Methodology

Our goal in calibrating the model is to examine the steady-state distribution of state variables in our model. In particular, we are interested in results on the home bias. Our model does not admit a closed-form representation of the unconditional distribution of state variables. Instead, we take a simulation approach to exploring the steady-state properties of the model. We choose parameter values to fit moments for consumption and asset prices, and then simulate the economy given these moments to explore the portfolio choice implications of external habit formation. We run simulations based on data from the U.S., U.K., and Sweden, chosen for the availability of high-quality, long-term consumption data.

Our approach is to choose parameters that are consistent with observed consumption and return moments assuming that there are no Type O agents, and then to evaluate the impact of a small number of Type O agents on portfolio allocation. The thrust of this simulation is not to match exactly the observed level of home bias, but rather to demonstrate the importance of portfolio mimicking for the level and time-series of the home bias. We set Type O agents to hold only the domestic asset and to constitute 5% or 10% of the population, both in wealth and number.

These numbers are chosen to reflect the results of Heaton and Lucas (2000), who find that between approximately 5% and 13% of wealth is held in (presumably domestic) proprietary businesses.[6] From Proposition 2, the portfolio choice and consumption rules are linear in the wealth of Type O agents. These rules are nearly linear in β, the fraction of agents of this type, except to the degree that β impacts the denominator of the rules as it gets larger, effectively reducing the amount of wealth controlled by Type E agents by reducing their number. Consequently, the reader can easily scale up or down estimates of home bias obtained in this exercise if they believe that our choice of the size of Type O agents is too aggressive or conservative.

To transfer the Heaton and Lucas result to our framework, we arbitrarily set $\frac{W_O}{W_E} \cong 1$, so that the per capita wealth of the two types of agents is the same, and run the calibration exercise for $\beta = 0.05$ and $\beta = 0.1$. Thus, the total wealth in the economy held by Type O agents in each case is $\beta\frac{W_O}{W_E} \cong 0.05$ and $\beta\frac{W_O}{W_E} \cong 0.1$, consistent with Heaton and Lucas. While the reader may prefer different choices of $\frac{W_O}{W_E}$, the calibration will not change substantially so long as $\beta\frac{W_O}{W_E}$ is held constant at the correct value. Due to the linearity of the consumption and portfolio choice rules with respect to W_O and the near linearity with respect to β when β is small, $\frac{W_O}{W_E}$ and β can be adjusted as long as $\beta\frac{W_O}{W_E}$ remains the same. Put another way, the model is nearly unchanged if small business owners make up 5% of the population or 10% of the population, so long as the fraction of aggregate wealth they hold stays constant.

[6] Heaton and Lucas (2000) impute from Survey of Consumer Finances data that, in 1992, non-elderly age groups held between 5.2% and 7.7% of wealth in proprietary businesses (Heaton and Lucas, Table IV). Based on statistics from the Federal Reserve Board and the National Income and Product Accounts, they report that, in 1989, 13.8% of wealth was held in "non-corporate equity" (Heaton and Lucas, Table VI). Certain sub-groups of the population, such as non-elderly, wealthy stockholders, hold a much greater proportion of their wealth in proprietary businesses.

We assume that prices follow a geomatric Brownian motion:

$$\frac{dP_I}{P_I} = \mu dt + \sigma_I dZ_I;$$
$$\frac{dP_D}{P_D} = \mu dt + \sigma_D dZ_D;$$
$$\frac{dB}{B} = r dt;$$
$$dZ_I dZ_D = \rho_{ID}.$$

We obtain estimates for the parameters on equities from MSCI indices. We estimate moments of the international asset return using the MSCI world index in real home currency, and moments of the local asset return using the MSCI domestic index in real home currency. We assume the domestic asset has the same drift as the international asset to facilitate cross-country comparisons. Historical world return may be a better proxy for expected return of domestic asset; this is consistent with an international CAPM, in which the domestic asset has a CAPM β of 1. From the perspective of explaining home bias, this is a conservative assumption in all three countries we examine, since the drift rate on the MSCI world index is lower than the drift rate for all three MSCI domestic indices. For the U.S., we estimate the short-term interest rate from 90-day T-bill returns, obtained from the CRSP U.S. Treasury and Inflation file. For the U.K. and Sweden, we use data from the International Monetary Fund's International Financial Statistics database. For the U.K., we use the rate at which 91-day bills are allotted (weighted average of Friday data). For Sweden, we use the rate on three month treasury discount notes. Our choice of these series follows Campbell (1999).

Since we are interested in real home currency asset returns, all returns are scaled by the domestic CPI, obtained from the CRSP U.S. Treasury and Inflation file for the U.S., and from the International Monetary Fund's International Financial Statistics database for the U.K. and Sweden. Exchange rate data is from MSCI. At times it will also be useful for us to refer to the foreign asset, which is the international asset less the fraction of the international asset held in the domestic asset. We obtain data on the fraction of domestic assets in the market-weighted world portfolio from Campbell (1999). Table 1 shows the moments we estimate for asset prices, together with the consumption moments and market capitalization obtained from Campbell (1999) that we use to calibrate the model.[7]

With price processes established, we turn to choosing parameters of the utility function, a, b, δ, and γ. Our goal is to choose parameters that are consistent with observed consumption and equity data. A closed-form, steady-state distribution is available for a special case of our model, the case in which there are no Type O agents, derived in Constantinides (1990). Although Constantinides' model is internal, it has identical consumption and portfolio rules to our external habit model when there are no Type O agents present (when $\beta = 0$).

[7] The relative market capitalization data is necessary because the domestic asset makes up a fraction of the world portfolio. We are interested in the holdings of domestic versus foreign (world less domestic) assets. Consequently, our measure of domestic assets includes the fraction of wealth held in the domestic asset directly, plus the fraction of wealth in the world asset, times the fraction of the world asset devoted to the domestic asset. Naturally, since both asset prices follow geometric Brownian motions, they will diverge in the long run. We show the fraction of the domestic asset held by the agent under the assumption that the domestic asset makes up the fraction of the world asset that it does currently.

Consequently, the two models will be observationally equivalent in this case (Shore, 2001). Constantinides uses this analytic representation to match model parameters to observed moments in U.S. data. We use a similar approach, exploiting the unconditional distribution function Constantinides has derived for this case, which is a function $p_z(z)$ of $z \equiv \frac{X}{C}$.[8] Given this distribution, we have expressions for instantaneous moments of consumption growth,

$$\frac{\mathrm{E}\left[\frac{dC}{C}\right]}{dt} = \frac{r-\delta}{\gamma} + \frac{(\mu - r)^2 (1+\gamma)}{2\gamma^2\sigma^2} + b - \left(\frac{r-\delta}{\gamma} + \frac{(\mu - r)^2 (1+\gamma)}{2\gamma^2\sigma^2} + a \right) \int_0^1 z p_z(z)\, dz,$$

and

$$\frac{\mathrm{var}\left[\frac{dC}{C}\right]}{dt} = \left(\frac{(\mu - r)}{\gamma\sigma} \right)^2 \int_0^1 (1-z)^2 p_z(z)\, dz.$$

We show results for a number of different (a, b) pairs. Our procedure is to choose a, then select b and γ under the assumption that $\beta = 0$ to match first and second instantaneous moments of consumption. We obtain annual frequency estimates of mean and variance of consumption growth from Campbell (1999). Our simulations, which take these moments as instantaneous, yield simulated annual moments that are close to the estimated annual moments. We set $\delta = 0.037$, the parameter value chosen by Constantinides.

In Appendix B, we derive sufficient conditions for the existence of a non-degenerate steady-state distribution of ratios of state variables. An important consideration is the relative wealth of Type E and Type O agents. Because we model price processes as a geometric Brownian motion, the surplus wealth of Type E agents follows a geometric Brownian motion. The simplest possible assumption about Type O agents is that their consumption-wealth ratio is constant (consistent with power utility). In this case, however, their wealth follows a different geometric Brownian motion, and the ratio of the wealths of these two groups will be degenerate. To give this ratio a non-degenerate steady-state distribution, we specify the consumption-wealth ratio of the Type O agents as a function of $\frac{W_S}{W_O}$:

$$\frac{C_O}{W_O} = \max(\phi - \kappa \frac{W_S}{W_O}, 0).$$

The consumption of Type O agents is relevant to Type E behavior only to the degree to

[8] Constantinides (1990) gives the following for $p_z(z)$:

$$p_z(z) = k e^{2b/m^2\sigma^2} (1-z)^{2\left(n+a-b-m^2\sigma^2\right)/m^2\sigma^2} z^{2(b-a-n)/m^2\sigma^2} e^{-2b/m^2\sigma^2 z},$$

where

$$n = \frac{r-\delta}{\gamma} + \frac{(\mu - r)^2 (1+\gamma)}{2\gamma^2\sigma^2}$$

and

$$m = \frac{\mu - r}{\gamma\sigma^2}.$$

In Constantinides, μ, σ, and r refer to moments of the single, closed economy.

which it affects Type O wealth, since

$$\alpha_E = \left(1 - \frac{\frac{X}{W_E} + b\beta \frac{W_O}{W_E}}{r + a - b(1-\beta)}\right) \frac{1}{\gamma} \left(\Gamma\Gamma^T\right)^{-1} (\mu - r\iota) + \alpha_O \frac{b\beta \frac{W_O}{W_E}}{r + a - b(1-\beta)}.$$

Note that the fraction of wealth invested as a power agent,

$$\left(1 - \frac{\frac{X}{W_E} + b\beta \frac{W_O}{W_E}}{r + a - b(1-\beta)}\right)$$

is invariant to the consumption policy of Type O agents, since expending one additional unit of wealth on consumption increases X by $b\beta$. The purpose of setting a consumption rule for Type O agents is to give the process a non-degenerate steady state, and to fix the relative importance of Type O agents in the long run. Therefore, we arbitrarily set $\phi = 0.15$ and select κ to match $\frac{W_O}{W_E} \approx 1$. Further, we impose that Type O agents hold only the domestic asset; that is, their portfolio weights are $\alpha = [0\ 1]'$. Other (ϕ, κ) pairs can be chosen to set $\frac{W_O}{W_E} \approx 1$, and our model is relatively insensitive to this modification, with the exception of differences in the variability of aggregate consumption due to differences in the variability of $\frac{C_O}{W_O}$.

We simulate the economy for 7500 years and discard the first 2500 years of data. We use a simple discrete approximation to the continuous-time price processes, with 10 discrete increments per year, and implement the continuous-time decision rules of the previous section within this framework. Our results are invariant to further improvements on the continuous-time approximation. We sample the simulated data annually. For each set of parameters, we run the simulation 15 times and present averaged results.

4.2 Calibration Results

Results from the calibration are presented in Tables 2 through 5. Table 2 summarizes the calibration results, reporting the range of home bias produced by the model for the parameter values we investigate. We find that a large fraction of observed home bias can be explained by our model, with allocations range from 64% to 75% for the U.S., 51% to 84% for the U.K., and 30% to 67% for Sweden. Mean-variance optimal domestic equity holdings, considering domestic equities and the MSCI EAFE index as investable assets, are approximately 57% for the U.S., 35% for the U.K., and 8% for Sweden.

In Tables 3 through 5, we investigate the calibration results in more detail. For each country, we present mean asset holdings for Type E agents, mean and standard deviation of consumption growth for the economy, and correlations between log excess domestic returns over international returns and percent changes in the fraction of the portfolio of risky assets held in foreign assets. Asset holdings are presented in terms of holding of foreign and domestic assets, rather than international and domestic, for easy comparison with commonly available benchmarks. (According to our earlier definition, the "international" asset includes the domestic asset, while the "foreign" asset does not.)

Each table presents results for $\beta = 0$, $\beta = 0.05$, and $\beta = 0.1$. The $\beta = 0.05$ and $\beta = 0.1$ cases illustrate the impact of introducing a small number of agents who hold only the domestic asset. While holdings of risky assets do not change very much as we add Type

O agents, the composition of risky asset holdings changes dramatically in all countries, but particularly in the U.K. and Sweden. In the $\beta = 0$ case, the composition of asset holdings in the risky portfolio $(\alpha_E^D / \left(\alpha_E^D + \alpha_F^D\right))$ is simply equal to the power utility allocation (reported as $\alpha_P^D/(\alpha_P^D + \alpha_P^F)$ in the Portfolio Benchmarks panel of the tables). Adding a small number of constrained agents boosts the fraction of wealth held in the domestic asset considerably. In the U.S., the domestic asset comprises 57% of the risky component of the mean-variance efficient portfolio. When we introduce a small number of Type O agents ($\beta = 0.05$ and $\beta = 0.1$ cases), the domestic asset comprises a larger fraction of the external habit agent's risky asset holdings (64% to 67%, and 69% to 75%, respectively). Holding relative risk aversion constant, equity home bias is obtained at the expense of consumption volatility. We conduct our calibrations at a fixed, low level of relative risk aversion, $\gamma = 2.5$. Consumption volatility rises as we add Type O agents to the model. Adding Type O agents increases aggregate consumption volatility as the more volatile consumption of Type O agents is averaged in to the aggregate. In addition, Type E agents hold more domestic equity, increasing the volatility of the Type E portfolio, causing Type E consumption to become more volatile.

In the U.K. and Sweden, the difference in portfolio holdings is much more dramatic because the domestic asset has a much smaller role in the mean-variance efficient portfolio (35% and 8%, respectively). Consequently, requiring investors to mimic agents whose holdings are very different from the mean-variance efficient portfolio leads to a larger change from the mean-variance efficient portfolio. In Sweden and the U.K., our model implies that, for $\beta = 0.1$, the fraction held in the domestic risky asset is between 43% and 67%, and between 60% and 83%, respectively. This is relatively close to the observed domestic holdings of 82% in the U.K. For Sweden, Cooper and Kaplanis (1994) report a home bias of 100% in 1989, based on OECD sources. This figure should probably be treated with some skepticism. Our main motivation for calibrating to Sweden is to better understand how our model functions in a very small economy.

Recall that for each country, we consider six different values for habit pairs. Moving across columns from left to right in Tables 3 through 5 increases the rate of decay of habit, a. To compensate, the calibration correspondingly increases the influence of aggregate consumption in creating new habit obligations, specified by b. Moving across the (a, b) pairs in the table impacts the overall holding of risky assets, but not the absolute level of domestic holdings, save for the fraction of the domestic asset held in the global portfolio. Recall that portfolio weights in equities can be decomposed into a mimicking component and a mean-variance efficient component. Since $\frac{b}{r+a-b(1-\beta)}$ is relatively constant across (a, b) pairs (a result of calibration to consumption moments), the fraction of wealth that must be held in domestic assets to mimic Type O agents is also relatively constant across (a, b) pairs. What changes is the fraction of wealth held in the mean-variance efficient portfolio; holdings of the domestic asset only vary across (a, b) pairs to the degree that the domestic asset is found in the mean-variance efficient portfolio. This fraction varies more across (a, b) pairs for the U.S., since the U.S. comprises a large fraction of the mean-variance efficient portfolio.

Remaining aware of the criticisms of the Tesar and Werner (1998) time series data discussed earlier, we compare time series implications from our calibration to moments found in the data. The results from our calibration are modestly encouraging. As in the data, our calibration produces large negative contemporaneous correlations between the percent change in the share of risky assets held in the foreign asset and the excess log return of the domestic asset over the international asset. In the U.S., we observe a correlation of -0.702 in the data; our calibration produces correlations from -0.513 to -0.808. In the U.K., we observe a correlation of -0.507 in the data; our calibration produces correlations from -0.573 to -0.847. We do not have data on the home bias for Sweden, but we note that correlations are more extreme, from -0.673 to -0.860. Since we do not have long-term consumption data for Canada and Germany, we do not run calibrations for these countries. However, we note that observed correlations in these countries are considerably closer to zero, at -0.204 and -0.427, respectively.

5 Conclusion

This paper shows that external habit formation preferences can help to explain the equity home bias. If agents measure utility relative to a benchmark that depends on aggregate consumption, they will mimic aggregate consumption to hedge changes in this benchmark. If a subset of agents in the economy is constrained to hold domestic equities, unconstrained agents will mimic this portfolio. Unconstrained agents also mimic one another, leading to a multiplier effect and generating a large equity home bias.

The particular form of preferences we use, difference-type external habit formation preferences, leads to a large equity home bias in small countries. As an illustration, when we calibrate the model, we are able to generate a large implied home bias for a small country, Sweden, in which mean-variance optimal holdings of the domestic asset are approximately 8%. This matches the observation that home bias in small countries tends to be substantial.

Our paper points to several directions for future research. One avenue for future work is theoretical. Our paper is framed in a partial equilibrium setting. Modifying the process followed by habit, in a manner akin to Campbell and Cochrane (1999), will allow us to assess the general equilbrium implications of external habit preferences. A general equilibrium model will allow us to simultaneously address both portfolio allocation and asset pricing implications of the model. Another direction for future work is empirical. The paper marshalls cross-country evidence on the home bias to provide suggestive support for our theory. In the future, better cross-country evidence on international equity holdings will become available. This cross-country evidence will serve as a valuable testing ground for home bias theories.

A Appendix: Proofs

Proof of Proposition 1. Consider the solution to the power utility problem. The Bellman equation for this problem is

$$\begin{aligned}0 = &\max_{C,\alpha} \mathrm{E}\left[u(C) - \delta J + J_W\left(W\left(r + \alpha^T(\mu - r\iota)\right) - C\right)\right.\\ &\left.+\frac{1}{2}W^2 J_{WW}\alpha^T\Gamma\Gamma^T\alpha + J_H\upsilon + \frac{1}{2}tr\left[\Phi^T J_{HH}\Phi\right] + W\alpha^T\Gamma\Phi^T J_{WH}^T\right].\end{aligned}$$

First-order conditions for α and C are

$$\begin{aligned}C &= J_W^{-\frac{1}{\gamma}};\\ \alpha &= -\left(\Gamma\Gamma^T\right)^{-1}(\mu - r\iota)\frac{J_W}{WJ_{WW}} - \left(\Gamma\Gamma^T\right)^{-1}\Gamma\Phi^T\frac{J_{WH}^T}{WJ_{WW}}.\end{aligned}$$

Finding a solution of the form

$$J = f(H)\frac{W^{1-\gamma}}{1-\gamma}$$

is equivalent to finding a function f consistent with the following differential equation, derived from the Bellman equation for the power utility problem:

$$\begin{aligned}0 = &\tfrac{\gamma}{1-\gamma}f^{-\frac{1}{\gamma}} - \delta\tfrac{1}{1-\gamma} + r\\ &+\frac{1}{2}\frac{1}{\gamma}\left[(\mu - r\iota)^T + \frac{\frac{df}{dH}}{f}\Phi\Gamma^T\right]\left(\Gamma\Gamma^T\right)^{-1}\left[(\mu - r\iota) + \Gamma\Phi\frac{\frac{df^T}{dH}}{f}\right]\\ &+\tfrac{1}{1-\gamma}\frac{\frac{df}{dH}}{f}\upsilon + \tfrac{1}{2}\tfrac{1}{1-\gamma}tr\left[\Phi^T\frac{\frac{d^2f}{dHdH^T}}{f}\Phi\right].\end{aligned} \tag{30}$$

The Bellman equation for the habit problem is

$$\begin{aligned}0 = &\max_{C,\alpha}\mathrm{E}\left[u(C_t, X_t) - \delta I + I_W\left(W\left(r + \alpha^T(\mu - r\iota)\right) - C\right)\right.\\ &+\frac{1}{2}W^2 I_{WW}\alpha^T\Gamma\Gamma^T\alpha + I_H\upsilon + \frac{1}{2}tr\left[\Phi^T I_{HH}\Phi\right]\\ &+W\alpha^T\Gamma\Phi^T I_{WH}^T + I_X(bC_A - Xa) + I_{W_A}\left(W_A\left(r + \alpha_A^T(\mu - r\iota)\right) - C_A\right)\\ &\left.+\frac{1}{2}W_A^2 I_{W_AW_A}\alpha_A^T\Gamma\Gamma\alpha_A + I_{WW_A}W_AW\alpha^T\Gamma\Gamma^T\alpha_A + W_A\alpha_A^T\Gamma\Phi^T I_{W_AH}^T\right]\end{aligned}$$

The first order conditions for the maximization are

$$\begin{aligned}C_t &= X + I_W^{-\frac{1}{\gamma}},\\ \alpha &= -\left(\Gamma\Gamma^T\right)^{-1}(\mu - r\iota)\frac{I_W}{WI_{WW}} - \left(\Gamma\Gamma^T\right)^{-1}\Gamma\Phi^T\frac{I_{WH}^T}{WI_{WW}} - \alpha_A\frac{W_AI_{WW_A}}{WI_{WW}}.\end{aligned}$$

Now guess a solution to the habit problem of the form

$$I = f(H)\frac{\left(W - \frac{X + bW_A}{r+a}\right)^{1-\gamma}}{1-\gamma}.$$

Plugging in to the Bellman equation, with optimal C and α, we obtain

$$
\begin{aligned}
0 = & \frac{\gamma}{1-\gamma} f^{-\frac{1}{\gamma}} - \delta \frac{1}{1-\gamma} + r \\
& + \frac{1}{2} \frac{1}{\gamma} \left[(\mu - r\iota)^T + \frac{\frac{df}{dH}}{f} \Phi \Gamma^T \right] \left(\Gamma \Gamma^T \right)^{-1} \left[(\mu - r\iota) + \Gamma \Phi \frac{\frac{df^T}{dH}}{f} \right] \\
& + \frac{1}{1-\gamma} \frac{\frac{df}{dH}}{f} \upsilon + \frac{1}{2} \frac{1}{1-\gamma} tr \left[\Phi^T \frac{\frac{df^2}{dH dH^T}}{f} \Phi \right].
\end{aligned}
$$

which is a differential equation in $f(H)$, corresponding to equation (30). Therefore, whenever a solution of the standard form exists for the power utility problem, we can obtain a solution to the analogous external habit formation problem. Moreover, the optimal consumption and portfolio rules are

$$
\begin{aligned}
C = & X + f^{-\frac{1}{\gamma}} \left(W - \frac{X + bW_A}{r+a} \right); \\
\alpha = & \left(1 - \frac{\frac{X}{W} + b \frac{W_A}{W}}{r+a} \right) \frac{1}{\gamma} \left(\Gamma \Gamma^T \right)^{-1} \left[(\mu - r\iota) + \Gamma \Phi^T \frac{1}{f} \frac{df^T}{dH} \right] \\
& + \alpha_A \frac{b \frac{W_A}{W}}{r+a}.
\end{aligned}
$$

■

Proof of Proposition 2. Note that

$$
\begin{aligned}
\alpha_A = & \frac{\beta W_O \alpha_O + (1-\beta) W_E \alpha_E}{\beta W_O + (1-\beta) W_E}; && (31) \\
W_A = & \beta W_O + (1-\beta) W_E. && (32)
\end{aligned}
$$

From Proposition (1), for the external habit agent,

$$
\begin{aligned}
C_E = & X + f^{-\frac{1}{\gamma}} \left(W_E - \frac{X + bW_A}{r+a} \right); && (33) \\
\alpha_E = & \frac{1}{\gamma} \left(1 - \frac{\frac{X}{W_E} + b \frac{W_A}{W_E}}{r+a} \right) \left(\Gamma \Gamma^T \right)^{-1} \left[(\mu - r\iota) + \Gamma \Phi^T \frac{1}{f} \frac{df^T}{dH} \right] && (34) \\
& + \alpha_A \frac{b \frac{W_A}{W_E}}{r+a}.
\end{aligned}
$$

Substituting (31) and (32) into (33) and (34) yields,

$$
\begin{aligned}
C_E = & X + f^{-\frac{1}{\gamma}} \left(W_E - \frac{X + b \left[\beta W_O + (1-\beta) W_E \right]}{r+a} \right); \\
\alpha_E = & \frac{1}{\gamma} \left(1 - \frac{\frac{X}{W_E} + b \frac{\beta W_O + (1-\beta) W_E}{W_E}}{r+a} \right) \left(\Gamma \Gamma^T \right)^{-1} \left[(\mu - r\iota) + \Gamma \Phi^T \frac{1}{f} \frac{df^T}{dH} \right] \\
& + \left(\frac{\beta W_O \alpha_O + (1-\beta) W_E \alpha_E}{\beta W_O + (1-\beta) W_E} \right) \frac{b \frac{\beta W_O + (1-\beta) W_E}{W_E}}{r+a}.
\end{aligned}
$$

Rearranging terms and solving for α_E gives

$$\alpha_E = \frac{1}{\gamma}\left(1-\frac{\frac{X}{W_E}+b\beta\frac{W_O}{W_E}}{r+a-b\left(1-\beta\right)}\right)\left(\Gamma\Gamma^T\right)^{-1}\left[\left(\mu-r\iota\right)+\Gamma\Phi^T\frac{1}{f}\frac{df^T}{dH}\right]$$
$$+\alpha_O\frac{b\beta}{r+a-b\left(1-\beta\right)}\frac{W_O}{W_E};$$

$$C_E = X+f^{-\frac{1}{\gamma}}\left(W_E-\frac{X+b\left[\beta W_O+\left(1-\beta\right)W_E\right]}{r+a}\right)$$
$$= X+f^{-\frac{1}{\gamma}}\frac{r+a-b\left(1-\beta\right)}{r+a}\left(W_E-\frac{X+b\beta W_O}{r+a-b\left(1-\beta\right)}\right).$$

■

B Appendix: Evolution of the State Variables

This appendix considers the steady-state properties of processes for the state variables X, W_O, and W_E when the investment opportunity set is constant. We assume that prices follow the simple processes

$$\frac{dP_i}{P_i} = \mu_i dt+\Gamma_i dZ, \forall i=1,...,N;$$
$$dZ_i dZ_j = 0, i\neq j;\quad dZ_i dZ_j = dt, i=j.$$

Here, μ and Γ are assumed constant. A natural concern in infinite horizon, heterogeneous agent models is the stability of the distribution of wealth among types in the economy. Additionally, in habit formation models, an analogous concern is the stability of the relationship between habit and consumption. While agents are guaranteed to be able to consume at least habit, models in which consumption converges to habit or in which habit becomes insignificant relative to consumption are less useful than models in which some type of non-degenerate steady-state relationship holds.

To establish stability of relationships among the three state variables, we reuse the concept of "surplus wealth" introduced above:

$$W_S = W_E-\frac{X+b\left[\beta W_O+\left(1-\beta\right)W_E\right]}{r+a}.$$

Our reason for using surplus wealth is expositional: under constant investment opportunities, W_S follows a geometric Brownian motion (35). For stability of relationships among the state variables, it is equivalent to establish stable relationships among W_O, W_S, and X, since W_S is a linear combination of W_O, W_E, and X. In the spirit of Constantinides (1990), we examine $\frac{W_O}{W_S}$ and $\frac{X}{W_S}$. The first ratio compares the wealth of Type O agents to the surplus wealth of Type E agents. The second ratio is proportional to $\frac{X}{C-X}$ and establishes a steady state relationship between consumption and habit.

Given optimal investment and consumption by the Type E agents, the process for surplus wealth is

$$\frac{dW_S}{W_S} = \left[\begin{array}{c} r + \frac{1}{\gamma}(\mu - r\iota)^T \left(\Gamma\Gamma^T\right)^{-1}(\mu - r\iota) \\ -\frac{1}{\gamma}\left[\delta - (1-\gamma)\left(r + \frac{1}{2}\frac{1}{\gamma}(\mu - r\iota)^T \left(\Gamma\Gamma^T\right)^{-1}(\mu - r\iota)\right)\right] \end{array} \right] dt$$
$$+\frac{1}{\gamma}(\mu - r\iota)^T \left(\Gamma\Gamma^T\right)^{-1}\Gamma dZ. \qquad (35)$$

This process is identical to the process for optimally invested and consumed wealth for the dual power agent described in Proposition 1, consistent with the intuition that external habit agents behave like power agents once they hold an asset paying habit.

Using Ito's lemma, we can obtain a process for $\frac{W_O}{W_S}$:

$$d\frac{W_O}{W_S} = \frac{W_O}{W_s}\left\{ \begin{array}{c} -\frac{C_O}{W_O} + \frac{1}{\gamma}\left[\rho - (1-\gamma)\left(r + \frac{1}{2}\frac{1}{\gamma}(\mu - r\iota)^T \left(\Gamma\Gamma^T\right)^{-1}(\mu - r\iota)\right)\right] \\ +\left(\alpha_O^T - \frac{1}{\gamma}(\mu - r\iota)^T \left(\Gamma\Gamma^T\right)^{-1}\right)(\mu - r\iota) \\ +\frac{1}{\gamma^2}(\mu - r\iota)^T \left(\Gamma\Gamma^T\right)^{-1}(\mu - r\iota) \end{array} \right\} dt$$
$$+\frac{W_O}{W_S}\left(\alpha_O^T - \frac{1}{\gamma}(\mu - r\iota)^T \left(\Gamma\Gamma^T\right)^{-1}\right)\Gamma dZ$$

This process is a geometric Brownian motion for $\frac{W_O}{W_S}$ if $\frac{C_O}{W_O}$ is constant. Under this assumption, therefore, the process is not stationary. However, if we assume that the consumption rule for Type O agents is a function of $\frac{W_O}{W_S}$, we can find conditions under which the $\frac{W_O}{W_S}$ process is stationary. Generally, we require that the consumption wealth ratio of Type O agents be increasing in $\frac{W_O}{W_S}$. This is a feature of the external habit agents described in this paper, so it is not an unreasonable assumption if the Type O agents have a habit flavor to their utility. In particular, for small values of $\frac{W_O}{W_S}$, the drift term must be positive; as $\frac{W_O}{W_S}$ approaches infinity, the drift term must be negative. One simple form for the Type O consumption rule $\frac{C_O}{W_O}$ that can implement these conditions is

$$\frac{C_O}{W_O} = \max(\phi - \kappa\frac{W_S}{W_O}, 0).$$

Under this rule,

$$d\frac{W_O}{W_S} = \left(\begin{array}{c} \left\{ \begin{array}{c} \left\{ \begin{array}{c} \frac{1}{\gamma}\left[\delta - (1-\gamma)\left(r + \frac{1}{2}\frac{1}{\gamma}(\mu - r\iota)^T \left(\Gamma\Gamma^T\right)^{-1}(\mu - r\iota)\right)\right] \\ +\left(\alpha_O^T - \frac{1}{\gamma}(\mu - r\iota)^T \left(\Gamma\Gamma^T\right)^{-1}\right)(\mu - r\iota) \\ +\frac{1}{\gamma^2}(\mu - r\iota)^T \left(\Gamma\Gamma^T\right)^{-1}(\mu - r\iota) \end{array} \right\} \frac{W_O}{W_S} \\ +\min\left(-\frac{W_O}{W_S}\phi + \kappa, 0\right) \end{array} \right\} dt \\ +\frac{W_O}{W_S}\left(\alpha_O^T - \frac{1}{\gamma}(\mu - r\iota)^T \left(\Gamma\Gamma^T\right)^{-1}\right)\Gamma dZ \end{array} \right).$$

Provided that $\frac{C_O}{W_O}$ is strictly positive, this is a simple mean-reverting process. The requirements for stability are therefore

$$\left\{\begin{array}{c} \frac{1}{\gamma}\left[\delta-(1-\gamma)\left(r+\frac{1}{2}\frac{1}{\gamma}(\mu-r\iota)^T\left(\Gamma\Gamma^T\right)^{-1}(\mu-r\iota)\right)\right] \\ +\left(\alpha_O^T-\frac{1}{\gamma}(\mu-r\iota)^T\left(\Gamma\Gamma^T\right)^{-1}\right)(\mu-r\iota) \\ +\frac{1}{\gamma^2}(\mu-r\iota)^T\left(\Gamma\Gamma^T\right)^{-1}(\mu-r\iota)-\phi \end{array}\right\}<0$$

$$\kappa>0.$$

The possibility that the $\frac{C_O}{W_O}=0$ constraint could bind imposes an additional restriction,

$$\left\{\begin{array}{c} \frac{1}{\gamma}\left[\delta-(1-\gamma)\left(r+\frac{1}{2}\frac{1}{\gamma}(\mu-r\iota)^T\left(\Gamma\Gamma^T\right)^{-1}(\mu-r\iota)\right)\right] \\ +\left(\alpha_O^T-\frac{1}{\gamma}(\mu-r\iota)^T\left(\Gamma\Gamma^T\right)^{-1}\right)(\mu-r\iota) \\ +\frac{1}{\gamma^2}(\mu-r\iota)^T\left(\Gamma\Gamma^T\right)^{-1}(\mu-r\iota) \end{array}\right\}>0.$$

We have established conditions for the stability of the ratio between the wealth of Type O agents and surplus wealth of Type E agents. In combination with stability of $\frac{X}{W_S}$, this condition assures that Type O and Type E wealth have a non-degenerate, stable relationship. This implies that the level of home bias will be non-degenerate in the steady state. We now turn to the process for $\frac{X}{W_S}$:

$$d\frac{X}{W_S}=\left\{\begin{array}{c} b\beta\frac{C_O}{W_O}\frac{W_O}{W_S}+\left\{\begin{array}{c} b(1-\beta)-a-r \\ +\frac{1}{\gamma}\left[\delta-(1-\gamma)\left(r+\frac{1}{2}\frac{1}{\gamma}(\mu-r\iota)^T\left(\Gamma\Gamma^T\right)^{-1}(\mu-r\iota)\right)\right] \\ -\frac{1}{\gamma}(\mu-r\iota)^T\left(\Gamma\Gamma^T\right)^{-1}(\mu-r\iota) \\ +\frac{1}{\gamma^2}(\mu-r\iota)^T\left(\Gamma\Gamma^T\right)^{-1}(\mu-r\iota) \end{array}\right\}\frac{X}{W_S} \\ +b(1-\beta)\frac{1}{\gamma}\left[\rho-(1-\gamma)\left(r+\frac{1}{2}\frac{1}{\gamma}(\mu-r\iota)^T\left(\Gamma\Gamma^T\right)^{-1}(\mu-r\iota)\right)\right] \end{array}\right\}dt$$

$$-\frac{1}{\gamma}\frac{X}{T}(\mu-r\iota)^T\left(\Gamma\Gamma^T\right)^{-1}\Gamma dZ.$$

Provided that $\frac{C_O}{W_O}\frac{W_O}{W_S}$ is stationary and positive, as it is in the consumption rule proposed above, and that

$$\left\{\begin{array}{c} b(1-\beta)-a-r \\ +\frac{1}{\gamma}\left[\delta-(1-\gamma)\left(r+\frac{1}{2}\frac{1}{\gamma}(\mu-r\iota)^T\left(\Gamma\Gamma^T\right)^{-1}(\mu-r\iota)\right)\right] \\ -\frac{1}{\gamma}(\mu-r\iota)^T\left(\Gamma\Gamma^T\right)^{-1}(\mu-r\iota) \\ +\frac{1}{\gamma^2}(\mu-r\iota)^T\left(\Gamma\Gamma^T\right)^{-1}(\mu-r\iota) \end{array}\right\}<0 \text{ and}$$

$$b(1-\beta)\frac{1}{\gamma}\left[\delta-(1-\gamma)\left(r+\frac{1}{2}\frac{1}{\gamma}(\mu-r\iota)^T\left(\Gamma\Gamma^T\right)^{-1}(\mu-r\iota)\right)\right]>0,$$

$\frac{X}{W_S}$ is stationary, since it is a mean-reverting process with an additional positive stationary component in the drift. This assures that portfolio choice will not be dominated either by the power utility solution or by the asset paying habit as its dividend. It also assures stability of $\frac{X}{C-X}$, since $(C-X)$ is proportional to W_S.

References

[1] Abel, Andrew B. "Asset Prices under Habit Formation and Catching Up with the Joneses," *American Economic Review*, May 1990, 80(2), pp. 38-42.

[2] Baxter, Marianne; Jermann, Urban J. "The International Diversification Puzzle Is Worse Than You Think," *American Economic Review*, March 1997, 87(1), pp. 170-180.

[3] Bekaert, Geert; Urias, Michael S. "Diversification, Integration, and Emerging Market Closed-End Funds," *Journal of Finance*, July 1996, 51(3), pp. 835-69.

[4] Benartzi, Shlomo, "Excessive Extrapolation and the Allocation of 401(k) Accounts to Company Stock," *Journal of Finance*, October 2001, 56(5).

[5] Bottazzi, Laura; Pesenti, Paolo; van Wincoop, Eric. "Wages, Profits and the International Portfolio Puzzle," *European Economic Review*, 1996, 40(6), pp. 219-254.

[6] Campbell, John Y. "Asset Prices, Consumption, and the Business Cycle", in John B. Taylor and Michael Woodford, eds., *Handbook of Macroeconomics*, Volume 1, Amsterdam: North-Holland, 1999, pp. 1231-1303.

[7] Campbell, John Y; Cochrane, John H. "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, April 1999, 107(2), pp. 205-51.

[8] Chan, Y. Lewis; Kogan, Leonid. "Catching Up with the Joneses: Heterogeneous Preferences and the Dynamics of Asset Prices," *Journal of Political Economy*, December 2002, 110(6), pp. 1255-1285.

[9] Chue, Timothy K. "Local Envy and Global Diversification," Working Paper, Hong Kong University of Science and Technology, 2002.

[10] Constantinides, George M. "Habit Formation: A Resolution of the Equity Premium Puzzle," *Journal of Political Economy*, June 1990, 98(3), pp. 519-543.

[11] Cooper, Ian A; Kaplanis, Evi. "Home Bias in Equity Portfolios, Inflation Hedging, and International Capital Market Equilibrium," *Review of Financial Studies*, Spring 1994, 7(1), pp. 45-60.

[12] Coval, Joshua D.; Moskowitz, Tobias J. "Home Bias at Home: Local Equity Preferences in Domestic Portfolios," *Journal of Finance*, December 1999, 54(6), pp. 2045-2073.

[13] Coval, Joshua D.; Moskowitz, Tobias J. "The Geography of Investment: Informed Trading and Asset Prices," *Journal of Political Economy*, August 2001, 109(4), pp. 811-841.

[14] DeMarzo, Peter M.; Kaniel, Ron; Kremer, Ilan, "Diversification as a Public Good: Community Effects and Externalities in Portfolio Choice," *Journal of Finance*, August 2004, 59(4), pp. 1677-1715.

[15] Duesenberry, James S. *Income, Saving, and the Theory of Consumer Behavior*, Cambridge, MA: Harvard University Press, 1949.

[16] Easterlin, Richard A. "Does Economic Growth Improve the Human Lot? Some Empirical Evidence," in David, Paul A.; Reder, Melvin W. eds. *Nations and Households in Economic Growth*, New York: Academic Press, 1974, pp. 89-126.

[17] Easterlin, Richard A. "Will Raising the Incomes of All Increase the Happiness of All?" *Journal of Economic Behavior & Organization*, June 1995, 27(1), pp. 35-47.

[18] French, Kenneth R.; Poterba, James M. "International Diversification and International Equity Markets," *American Economic Review*, May 1991, 81(2), pp. 222-226.

[19] Froot, Kenneth A.; O'Connell, Paul G. J.; Seasholes, Mark S. "The Portfolio Flows of International Investors," *Journal of Financial Economics*, February 2001, 59(2), pp. 151-193.

[20] Froot, Kenneth A.; Ramadorai, Tarun. "The Information Content of International Portfolio Flows," NBER Working Paper No. 8472, 2001.

[21] Gali, Jordi. "Keeping Up with the Joneses: Consumption Externalities, Portfolio Choice, and Asset Prices," *Journal of Money, Credit & Banking,* February 1994, 26 (1), pp. 1-8.

[22] Gollier, Christian. "Misery Loves Company: Equilibrium Portfolios with Heterogeneous Consumption Externalities," *International Economic Review*, November 2004, 45(4), pp. 1169-1192.

[23] Heaton, John; Lucas, Deborah. "Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk," *Journal of Finance*, 55(3), pp. 1163-1198.

[24] Hentschel, Ludger; Long, John B. "Numeraire Portfolio Measures of the Size and Source of Gains from International Diversification," Working Paper, University of Rochester, 2004.

[25] Hong, Harrison; Kubik, Jeffrey; Stein, Jeremy. "Social Interaction and Stock Market Participation," *Journal of Finance*, February 2004, 59(1), pp. 137-163.

[26] Huberman, Gur. "Familiarity Breeds Investment," *Review of Financial Studies*, July 2001, 14(3), pp. 659-680.

[27] Jacquillat, Bertrand; Solnik, Bruno H. "Multinationals are Poor Tools for Diversification," *Journal of Portfolio Management*, Winter 1987, 4(2), pp. 8-12.

[28] Lauterbach, Beni; Reisman, Haim. "Preferences on Relative Return: A Potential Explanation for Some Pricing Anomalies," *Advances in Financial Economics*, 2002, Vol. 7, pp. 95-103.

[29] Lauterbach, Beni; Reisman, Haim. "Keeping up with the Joneses and the Home Bias," *European Financial Management*, 2004, 10(2), pp. 251-259.

[30] Lax, Yoel. "Optimal Consumption and Portfolio Policies with Habit Formation and Subsistence Levels," Working Paper, Goldman Sachs, 2001.

[31] Lax, Yoel. "Habit Formation and Lifetime Portfolio Selection," Working Paper, Goldman Sachs, 2002.

[32] Lewis, Karen K. "Trying to Explain Home Bias in Equities and Consumption," *Journal of Economic Literature*, June 1999, 37 (2), pp. 571-608.

[33] Lewis, Karen K. "Why Do Stocks and Consumption Imply Such Different Gains From International Risk Sharing?" *Journal of International Economics*, October 2000, 52(1), pp. 1-35.

[34] Levy, Haim; Sarnat, Marshall. "International Diversification of Investment Portfolios," *American Economic Review*, September 1970, 60(4), pp. 668-75.

[35] Lynch, Peter; Rothchild, John. *One Up on Wall Street : How to Use What You Already Know to Make Money in the Market*, New York: Simon & Schuster, April 2000.

[36] Obstfeld, Maurice; Rogoff, Kenneth, *Foundations of International Macroeconomics*, Cambridge, MA: MIT Press, 1998.

[37] Obstfeld, Maurice; Rogoff, Kenneth. "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?" in Ben Bernanke and Kenneth Rogoff (eds.), *NBER Macroeconomics Annual 2000* (Cambridge: MIT Press), pp. 339-390.

[38] Pástor, Ľuboš, "Portfolio Selection and Asset Pricing Models," *Journal of Finance*, February 2000, 55(1), pp. 179-224.

[39] Rubinstein, Mark. "The Strong Case for the Generalized Logarithmic Utility Model as the Premier Model of Financial Markets," *Journal of Finance*, May 1976, 31, pp. 551-571.

[40] Rubinstein, Mark. "The Valuation of Uncertain Income Streams and the Pricing of Options," *Bell Journal of Economics*, Autumn 1976, 7, pp. 407-25.

[41] Scholl, Russell B. "The International Investment Position of the United States in 1996," *Survey of Current Business*, July 1997, pp. 24-33.

[42] Schroder, Mark; Skiadas, Costis. "An Isomorphism Between Asset Pricing Models with and without Linear Habit Formation," *Review of Financial Studies*, 2002, 15(4), pp. 1189-1221.

[43] Seasholes, Mark S. "Re-examining Information Asymmetries in Emerging Stock Markets," Working Paper, University of California, Berkeley, 2004.

[44] Shore, Stephen H. "Taxing the Joneses: Optimal Taxation Under Multiple-Good External Habit Formation," Working Paper, Wharton Business School, 2001.

[45] Solnik, Bruno. "The International Price of Risk: An Empirical Investigation of the World Capital Market Structure," *Journal of Finance*, May 1974, 29(2), pp. 365-78.

[46] Stockman, Alan C.; Dellas, Harris. "International Portfolio Nondiversification and Exchange Rate Variability," *Journal of International Economics*, 1989, 26(3/4), pp. 271-90.

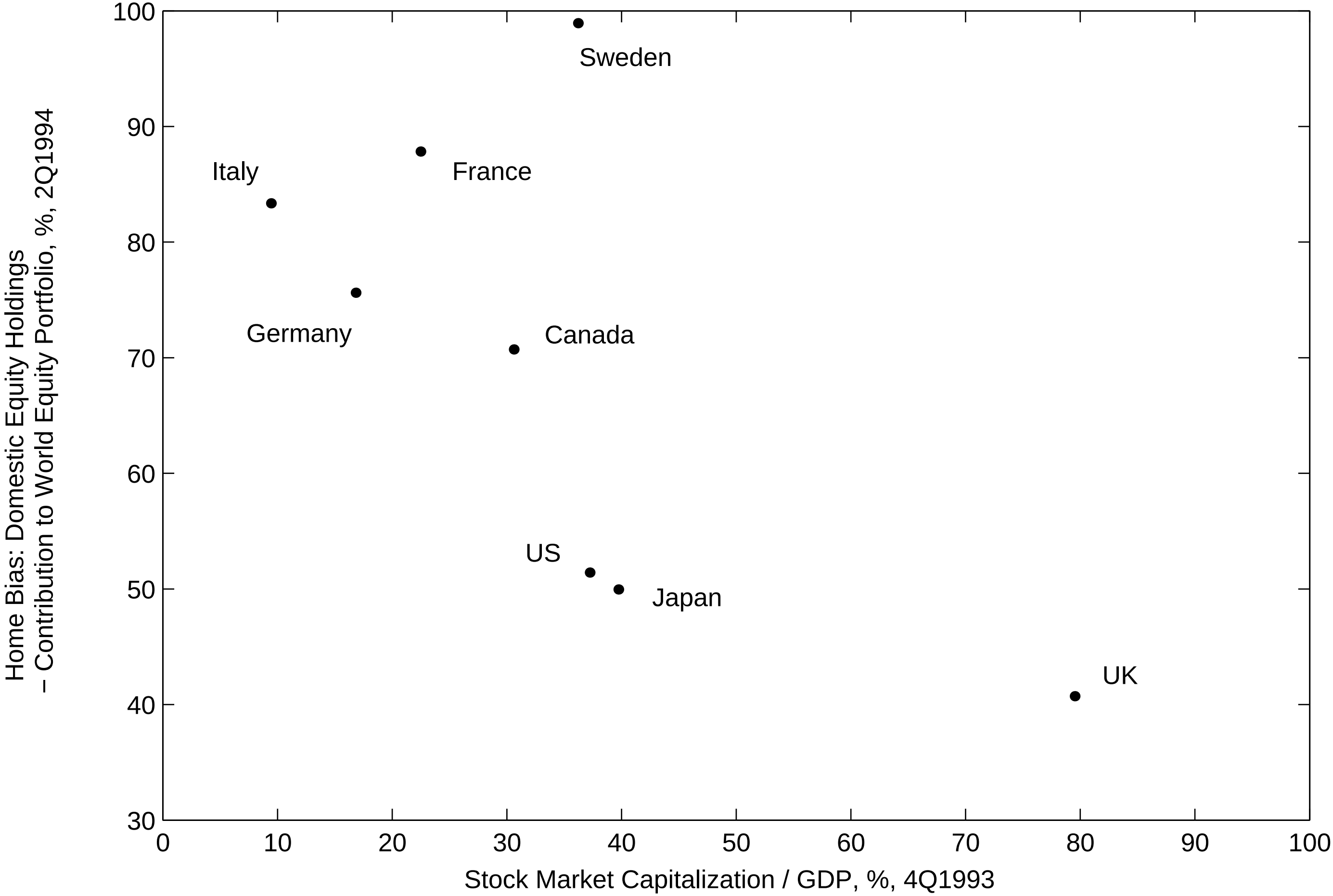
[47] Tesar, Linda; Werner, Ingrid M. "Home Bias and High Turnover," *Journal of International Money and Finance*, August 1995, 14(4), pp. 467-92.

[48] Tesar, Linda; Werner, Ingrid M. "The Internationalization of Securities Markets since the 1987 Crash," in Litan, Robert E.; Santomero, Anthony M., eds. *Brookings-Wharton Papers on Financial Services*. Washington, D.C.: Brookings, 1998, pp. 281-349.

[49] Warnock, Francis E. "Home Bias and High Turnover Reconsidered," forthcoming, *Journal of International Money and Finance*.

[50] Wheatley, Simon. "Keeping up with the Joneses, Human Capital, and the Home-Equity Bias," Working Paper, University of Melbourne, 2000.

Figure 1: Home Bias and MSCI Stock Market Capitalization / GDP



Note: Holdings data from Coen (2002), as of December 31, 1994. Source: OECD and Financial Times Database. Market Capitalization / GDP data from Campbell, 1998, as of Dec. 31, 1993. Market capitalization is MSCI index market capitalization.

TABLE 1
Summary of Data Used in Calibration Exercise

	U.S.	U.K.	Sweden
Estimated Asset Return Moments[1]			
$\bar{r}$	0.021	0.013	0.020
$\bar{r}_I - \bar{r} + \frac{1}{2}\sigma^2 [r_I - r]$	0.052	0.059	0.064
$\sigma [r_I - r]$	0.144	0.178	0.176
$\sigma [r_D - r]$	0.154	0.227	0.253
$\rho [r_I - r, r_D - r]$	0.845	0.660	0.709
Consumption Moments[2]			
$\mathrm{E} [\Delta \ln C]$	0.018	0.014	0.018
$\sigma [\Delta \ln C]$	0.033	0.029	0.029
Market Capitalization Relative to World[3]			
$V_i / \left(\sum_j V_j \right)$	0.399	0.119	0.010

(1)Equity data from MSCI. CPI and short rate data from CRSP Treasury and Inflation file for U.S., from the International Monetary Fund's International Financial Statistics database for the U.K and Sweden. Estimates are of real home-currency log returns. All financial data quarterly, 1970Q1-1999Q2. (2)Source: Campbell (1999), Table 3. (3)Source: Campbell (1999), Table 1.

TABLE 2
Summary of Calibration Results

	U.S.	U.K.	Sweden
Portfolio Benchmarks[1]			
$\alpha_P^D/(\alpha_P^D+\alpha_P^F)$	0.569	0.351	0.076
$\alpha_{obs}^D/(\alpha_{obs}^D+\alpha_{obs}^F)$	0.938	0.820	1.000
Calibrated Range of Domestic Equity Holdings			
maximum $\mathrm{E}\left[\alpha_E^D\right]/\mathrm{E}\left[\alpha_E^D+\alpha_E^F\right]$	0.638	0.509	0.304
minimum $\mathrm{E}\left[\alpha_E^D\right]/\mathrm{E}\left[\alpha_E^D+\alpha_E^F\right]$	0.751	0.835	0.672

[1] α_P refers to the portfolio allocation of a hypothetical power-utility agent. α_{obs} refers to observed portfolio holdings. Equity data from MSCI. CPI and short rate data from CRSP Treasury and Inflation file for U.S., from the International Monetary Fund's International Financial Statistics database for the U.K and Sweden. Estimates are of real home-currency log returns. All financial data quarterly, 1970Q1-1999Q2. Source for home bias estimates is French and Poterba (1991) for U.S. and U.K., Cooper and Kaplanis (1994) for Sweden.

TABLE 3
Moments from Calibrated Model, U.S. Data[1]

Habit Parameters						
a	0.100	0.200	0.300	0.400	0.500	0.600
b	0.087	0.160	0.234	0.307	0.381	0.454
Portfolio Benchmarks[2]						
$\alpha_P^D/(\alpha_P^D+\alpha_P^F)$	0.569					
$\alpha_{obs}^D/(\alpha_{obs}^D+\alpha_{obs}^F)$	0.938					
$\rho\left[\%\Delta\alpha, er_{DI}\right]_{obs}$	−0.702					
Annual Frequency Moments, $\beta = 0.0$						
$\mathrm{E}\left[\alpha_E^D\right]$	0.238	0.314	0.355	0.381	0.399	0.412
$\mathrm{E}\left[\alpha_E^F\right]$	0.180	0.238	0.270	0.290	0.303	0.313
$\alpha_E^D/\alpha_E^D+\alpha_E^F$	0.569	0.569	0.569	0.569	0.569	0.569
$\mathrm{E}\left[\log\left(\Delta C_A\right)\right]$	0.018	0.018	0.018	0.018	0.018	0.018
$\sigma\left[\log\left(\Delta C_A\right)\right]$	0.036	0.038	0.040	0.042	0.044	0.046
$\rho\left[\%\Delta\alpha, er_{DI}\right]$	0.000	0.000	0.000	0.000	0.000	0.000
Annual Frequency Moments, $\beta = 0.05$						
$\mathrm{E}\left[\alpha_E^D\right]$	0.298	0.381	0.424	0.451	0.468	0.481
$\mathrm{E}\left[\alpha_E^F\right]$	0.148	0.203	0.233	0.251	0.263	0.272
$\mathrm{E}\left[\alpha_E^D\right]/\mathrm{E}\left[\alpha_E^D+\alpha_E^F\right]$	0.669	0.653	0.646	0.642	0.640	0.638
$\mathrm{E}\left[\log\left(\Delta C_A\right)\right]$	0.018	0.018	0.018	0.018	0.018	0.018
$\sigma\left[\log\left(\Delta C_A\right)\right]$	0.049	0.049	0.051	0.053	0.055	0.057
$\rho\left[\%\Delta\alpha, er_{DI}\right]$	−0.538	−0.648	−0.713	−0.756	−0.786	−0.808
Annual Frequency Moments, $\beta = 0.1$						
$\mathrm{E}\left[\alpha_E^D\right]$	0.352	0.438	0.481	0.506	0.522	0.534
$\mathrm{E}\left[\alpha_E^F\right]$	0.117	0.173	0.203	0.221	0.233	0.242
$\mathrm{E}\left[\alpha_E^D\right]/\mathrm{E}\left[\alpha_E^D+\alpha_E^F\right]$	0.751	0.716	0.703	0.696	0.691	0.689
$\mathrm{E}\left[\log\left(\Delta C_A\right)\right]$	0.018	0.018	0.018	0.018	0.018	0.018
$\sigma\left[\log\left(\Delta C_A\right)\right]$	0.064	0.062	0.063	0.065	0.067	0.069
$\rho\left[\%\Delta\alpha, er_{DI}\right]$	−0.513	−0.627	−0.698	−0.745	−0.778	−0.802

[1] Parameters: $r = 0.021$, $\mu - r = 0.052$, $\sigma_I = 0.144$, $\sigma_D = 0.154$, $\rho_{ID} = 0.845$, $\delta = .037$, $\phi = .15$. (γ, b) chosen to match instantaneous consumption moments; $\gamma \approx 3$ for all (a, b) pairs. [2] α_P denotes portfolio chosen by hypothetical power agent; α_{obs} are observed portfolio weights (Source: French and Poterba (1991), Table 1). $\rho\left[\%\Delta\alpha, er_{DI}\right]$ denotes $\rho\left[\%\Delta\left(\frac{\alpha_{F,obs}}{\alpha_{F,obs}+\alpha_{D,obs}}\right), r_D - r_I\right]$, the correlation between the percent change in the risky weight in the foreign asset and the excess log return of the domestic asset over the international asset. $\rho\left[\%\Delta\alpha, er_{DI}\right]_{obs}$ denotes the observed moment from the data. Source for time series data on portfolio allocations is Tesar and Werner (1998). Data is annual, from 1987 through 1996.

TABLE 4
Moments from Calibrated Model, U.K. Data(1)

Habit Parameters						
a	0.100	0.200	0.300	0.400	0.500	0.600
b	0.092	0.171	0.250	0.329	0.408	0.488
Portfolio Benchmarks(2)						
$\alpha_P^D/(\alpha_P^D+\alpha_P^F)$	0.351					
$\alpha_{obs}^D/(\alpha_{obs}^D+\alpha_{obs}^F)$	0.820					
$\rho\left[\%\Delta\alpha, er_{DI}\right]_{obs}$	−0.507					
Annual Frequency Moments, $\beta = 0.0$						
$\mathrm{E}\left[\alpha_E^D\right]$	0.112	0.160	0.188	0.206	0.218	0.227
$\mathrm{E}\left[\alpha_E^F\right]$	0.207	0.296	0.348	0.380	0.403	0.420
$\alpha_E^D/\alpha_E^D+\alpha_E^F$	0.351	0.351	0.351	0.351	0.351	0.351
$\mathrm{E}\left[\log\left(\Delta C_A\right)\right]$	0.014	0.014	0.014	0.014	0.014	0.014
$\sigma\left[\log\left(\Delta C_A\right)\right]$	0.032	0.035	0.037	0.040	0.042	0.044
$\rho\left[\%\Delta\alpha, er_{DI}\right]$	0.000	0.000	0.000	0.000	0.000	0.000
Annual Frequency Moments, $\beta = 0.05$						
$\mathrm{E}\left[\alpha_E^D\right]$	0.224	0.283	0.314	0.332	0.345	0.354
$\mathrm{E}\left[\alpha_E^F\right]$	0.137	0.225	0.274	0.305	0.326	0.342
$\mathrm{E}\left[\alpha_E^D\right]/\mathrm{E}\left[\alpha_E^D+\alpha_E^F\right]$	0.621	0.557	0.534	0.521	0.514	0.509
$\mathrm{E}\left[\log\left(\Delta C_A\right)\right]$	0.015	0.015	0.015	0.015	0.015	0.015
$\sigma\left[\log\left(\Delta C_A\right)\right]$	0.067	0.062	0.063	0.064	0.066	0.068
$\rho\left[\%\Delta\alpha, er_{DI}\right]$	−0.605	−0.722	−0.777	−0.810	−0.832	−0.847
Annual Frequency Moments, $\beta = 0.1$						
$\mathrm{E}\left[\alpha_E^D\right]$	0.334	0.380	0.405	0.423	0.434	0.441
$\mathrm{E}\left[\alpha_E^F\right]$	0.066	0.172	0.226	0.256	0.276	0.291
$\mathrm{E}\left[\alpha_E^D\right]/\mathrm{E}\left[\alpha_E^D+\alpha_E^F\right]$	0.835	0.688	0.642	0.623	0.611	0.603
$\mathrm{E}\left[\log\left(\Delta C_A\right)\right]$	0.014	0.014	0.014	0.014	0.014	0.014
$\sigma\left[\log\left(\Delta C_A\right)\right]$	0.111	0.093	0.090	0.091	0.092	0.094
$\rho\left[\%\Delta\alpha, er_{DI}\right]$	−0.573	−0.703	−0.766	−0.802	−0.827	−0.844

(1) Parameters: $r = 0.013$, $\mu - r = 0.059$, $\sigma_I = 0.178$, $\sigma_D = 0.227$, $\rho_{ID} = 0.660$, $\delta = .037$, $\phi = .15$. (γ, b) chosen to match instantaneous consumption moments; $\gamma \approx 2.5$ for all (a, b) pairs. (2) α_P denotes portfolio chosen by hypothetical power agent; α_{obs} are observed portfolio weights (Source: French and Poterba (1991), Table 1). $\rho\left[\%\Delta\alpha, er_{DI}\right]$ denotes $\rho\left[\%\Delta\left(\frac{\alpha_{F,obs}}{\alpha_{F,obs}+\alpha_{D,obs}}\right), r_D - r_I\right]$, the correlation between the percent change in the risky weight in the foreign asset and the excess log return of the domestic asset over the international asset. $\rho\left[\%\Delta\alpha, er_{DI}\right]_{obs}$ denotes the observed moment from the data. Source for time series data on portfolio allocations is Tesar and Werner (1998). Data is annual, from 1987 through 1995.

TABLE 5
Moments from Calibrated Model, Sweden Data[1]

Habit Parameters						
a	0.100	0.200	0.300	0.400	0.500	0.600
b	0.092	0.170	0.248	0.325	0.403	0.481
Portfolio Benchmarks[3]						
$\alpha_P^D/(\alpha_P^D+\alpha_P^F)$	0.076					
$\alpha_{obs}^D/(\alpha_{obs}^D+\alpha_{obs}^F)$	1.000					
$\rho\left[\%\Delta\alpha, er_{DI}\right]_{obs}$	NA					
Annual Frequency Moments, $\beta=0.0$						
$\mathrm{E}\left[\alpha_E^D\right]$	0.023	0.032	0.037	0.041	0.043	0.045
$\mathrm{E}\left[\alpha_E^F\right]$	0.282	0.391	0.454	0.495	0.524	0.544
$\alpha_E^D/\alpha_E^D+\alpha_E^F$	0.076	0.076	0.076	0.076	0.076	0.076
$\mathrm{E}\left[\log\left(\Delta C_A\right)\right]$	0.018	0.018	0.018	0.018	0.018	0.018
$\sigma\left[\log\left(\Delta C_A\right)\right]$	0.032	0.034	0.036	0.039	0.041	0.042
$\rho\left[\%\Delta\alpha, er_{DI}\right]$	0.000	0.000	0.000	0.000	0.000	0.000
Annual Frequency Moments, $\beta=0.05$						
$\mathrm{E}\left[\alpha_E^D\right]$	0.143	0.169	0.181	0.188	0.192	0.195
$\mathrm{E}\left[\alpha_E^F\right]$	0.201	0.303	0.363	0.401	0.428	0.447
$\mathrm{E}\left[\alpha_E^D\right]/\mathrm{E}\left[\alpha_E^D+\alpha_E^F\right]$	0.416	0.358	0.333	0.319	0.310	0.304
$\mathrm{E}\left[\log\left(\Delta C_A\right)\right]$	0.018	0.018	0.018	0.018	0.018	0.018
$\sigma\left[\log\left(\Delta C_A\right)\right]$	0.069	0.066	0.066	0.067	0.069	0.071
$\rho\left[\%\Delta\alpha, er_{DI}\right]$	−0.682	−0.766	−0.805	−0.828	−0.843	−0.853
Annual Frequency Moments, $\beta=0.1$						
$\mathrm{E}\left[\alpha_E^D\right]$	0.256	0.274	0.283	0.288	0.291	0.293
$\mathrm{E}\left[\alpha_E^F\right]$	0.125	0.244	0.306	0.344	0.370	0.388
$\mathrm{E}\left[\alpha_E^D\right]/\mathrm{E}\left[\alpha_E^D+\alpha_E^F\right]$	0.672	0.529	0.480	0.456	0.441	0.431
$\mathrm{E}\left[\log\left(\Delta C_A\right)\right]$	0.018	0.018	0.018	0.018	0.018	0.018
$\sigma\left[\log\left(\Delta C_A\right)\right]$	0.112	0.099	0.096	0.096	0.098	0.100
$\rho\left[\%\Delta\alpha, er_{DI}\right]$	−0.673	−0.763	−0.805	−0.831	−0.848	−0.860

(1) Parameters: $r=0.020$, $\mu-r=0.064$, $\sigma_I=0.176$, $\sigma_D=0.253$, $\rho_{ID}=0.709$, $\delta=.037$, $\phi=.15$. (γ,b) chosen to match instantaneous consumption moments; $\gamma\approx 2.85$ for all (a,b) pairs. (2) α_P denotes portfolio chosen by hypothetical power agent; α_{obs} are observed portfolio weights (Source: French and Poterba (1991), Table 1). $\rho\left[\%\Delta\alpha, er_{DI}\right]$ denotes $\rho\left[\%\Delta\left(\frac{\alpha_{F,obs}}{\alpha_{F,obs}+\alpha_{D,obs}}\right), r_D-r_I\right]$, the correlation between the percent change in the risky weight in the foreign asset and the excess log return of the domestic asset over the international asset. $\rho\left[\%\Delta\alpha, er_{DI}\right]_{obs}$ denotes the observed moment from the data. Data on portfolio holdings for Sweden is from Cooper and Kaplanis (1994), constructed from OECD National Accounts data.