Strategic Trading and Manipulation with Spot Market Power*

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Abstract

When a spot market monopolist has a position in a corresponding futures market, he has an incentive to deviate from the spot market optimum to make this position more profitable. Rational futures market makers take this into account when setting prices. We show that the monopolist, by randomizing his futures market position, can strategically exploit his market power at the expense of other futures market participants. Furthermore, traders without market power can manipulate futures prices by hiding their orders behind the monopolist's strategic trades. The moral hazard problem stemming from spot market power thus provides a venue for strategic trading and manipulation that parallels the adverse selection problem stemming from inside information.

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1 Introduction

For many goods, spot markets with market power coexist with competitive futures markets. When a spot market monopolist participates in a futures market, this participation leads to a moral hazard problem in the spot market. He has an incentive to deviate from the monopoly optimum in order to make his futures market position more profitable. For example, if a monopolist producer of oil holds a short position in an oil futures contract, he will profit if the price of oil goes down. This gives him an incentive to produce more oil than he would otherwise. When rational futures market participants observe the monopolist's position, they will take the impact of this position on subsequent spot prices into account when setting futures prices. When they cannot observe the monopolist's position perfectly they must make rational inferences about the monopolist's position and take these into account when setting prices.[1]

In this paper, we explore strategic trading and manipulation in futures markets when market positions cannot be inferred perfectly. Spot market power allows the monopolist to trade strategically – randomly taking a position in the futures market and then moving spot prices to make that position profitable. This creates an opportunity for those without market power to engage in futures market manipulation – taking a position in a derivatives market and then mimicking the monopolist's futures trading to move futures market prices to make the derivatives position profitable. The literature on market microstructure deals extensively with the effects of asymmetric information when the positions of informed traders cannot be observed perfectly. This paper argues that the moral hazard problem created by spot market power parallels the adverse selection problem created by inside information.

In Section 2, we show how the monopolist can exploit his spot market power in the futures market. When he is able to hide his futures market position within the aggregate order flow, he will randomize his orders and then set spot prices to make his futures market position more profitable. This makes hedging more expensive for those who may be the monopolist's counterparty. Spot market power thus discourages futures market participation for agents without market power and provides a venue for a spot market monopolist to increase expected profits by trading strategically.[2]

Section 2 shows that results similar to those in Kyle's (1985) "noise trader" model are obtained when

[1] This paper takes Anderson's (1990) advice. He surveys the literature on futures trading with perfect inference when the underlying market is imperfectly competitive and suggests in his conclusion:

> "The theoretical development that would be most interesting would be to reconsider some of these models described above under conditions of asymmetric information. In particular, the models reviewed have made the assumption (at least implicitly) that the futures positions of powerful agents are observed so that forecasts of future cash prices can take this into account. In practice, futures positions of agents are likely to be imperfectly observable." (p. 246-247)

[2] Despite the importance of oil price risk faced by many industry sectors, the futures market on oil is relatively illiquid; in particular, the trading volume of longer-term futures contract is very low. Our paper suggests that this may stem from the imperfectly competitive nature of the spot market for oil.

there is spot market power instead of inside information. In our model, there are no informed traders with private information about future prices *at the time* trading takes place. Instead, the monopolist can set spot market prices *after* trading takes place. In contrast to the "noise traders" in the Kyle model who act mechanically, in our model agents without market power respond optimally to the monopolist's presence in the futures market by reducing their futures market participation (see Spiegel and Subrahmanyam (1992) for the analogous extension of the Kyle model). Pirrong (1995, 2001) shows that a trader who can buy or sell an arbitrarily large number of futures contracts is able to influence the price at liquidation by demanding or selling too many units of the commodity in the delivery market. He can profit in equilibrium from the artificially high or low spot market price if he randomizes his order flow to hide behind the order flow of "noise traders" and if the supply curve in the delivery market is upward sloping. While the randomized strategy in Pirrong parallels the one of the monopolist in this paper, we explicitly model decisions in the spot market, endogenize the initial futures position of the strategic agent and the response of hedgers.

In Section 3, we extend Section 2 by showing how traders can move (i.e. "manipulate") futures prices even when they do not have market power. If a futures market manipulator takes a long (short) position in the derivatives market, he has an incentive to move subsequent futures prices up (down) to make his initial position profitable. When this initial derivatives trade is hidden, the manipulator will take a hidden, long (short) position in the futures market while the monopolist will attempt to hide a random futures market order (as in Section 2) behind the manipulator's position. When market makers cannot separate the manipulator's trade from the monopolist's trade then the monopolist profits in the futures market (as in Section 2) at the expense of the manipulator. However, when the manipulator's trade reveals the monopolist's position to market makers then futures market prices move up (down) anticipating that the monopolist will set a higher (lower) spot market price. This, in turn, makes the manipulator's initial long (short) derivatives market position profitable. The manipulator's loses in the futures market when the monopolist's position is hidden are more than made up for by his profits in the derivatives market when the monopolist's trades are revealed.

Past research has shown that markets can be manipulated if some agents have private information about prices (see e.g. Hart (1977), Jarrow (1992), Allen and Gale (1992), Kumar and Seppi (1992)). For example, Kumar and Seppi develop a model in which uninformed manipulators are able to profit in the futures market because spot market makers are unable to differentiate the manipulator's order flow from the informed trader's order flow. We show that monopoly power serves a similar function.

2 Strategic Trading by Spot Market Monopolists

In this section, we show how a spot market monopolist can exploit his market power by trading strategically in the futures market. This trade discourages futures market participation since traders fear that the monopolist may be their counterparty or the counterparty of another trader with a similar position. The monopolist will exert spot market power to make his futures position more profitable, thereby reducing the profits of his counterparties.[3]

This section builds on the work of Kyle (1985), who shows that agents with inside information can profitably exploit their informational advantage by hiding behind the order flow of uninformed "noise traders". In our model, the aggregate hedging demand of agents without market power is stochastic just as the number of "noise traders" is stochastic in the Kyle model.

The monopolist can increase profit because his futures market position is not perfectly observable to the market makers. When setting the strike price, market makers are then unable to take fully into account the impact of the monopolist's (hidden) futures market position on the expected spot price. While the monopolist's expected spot market profit is reduced by deviating from the monopoly optimum, his expected profit in the futures market more than makes up for it. Since market makers earn zero expected profits, the monopolist's expected futures market profits imply expected futures market losses for other market participants. This increased cost deters these agents from hedging price risk as much as they otherwise might. While we consider only the case of monopoly – when there is exactly one agent with spot market power – to obtain greater analytic tractability, similar logic will apply in an oligopolistic setting.

2.1 Model Setup

We envision a model with one good and two periods, $t = 1, 2$. The good is produced only in the second period and sold in the spot market. The cost of production is normalized to zero. Demand is uncertain and realizes in between the two periods. There is a competitive futures market with futures market prices set in the first period, and futures market payoffs set in the second period given futures market prices.

There are three types of agents in this market. First, there is a spot market monopolist. The monopolist sets the spot price (and therefore quantity) to maximize expected profits. We assume that the monopolist is risk-neutral, so that he has no incentive to participate in the futures market unless he can increase expected profits by doing so. Second, there are competitive risk-neutral market makers who observe the aggregate

[3] Storage may reduce the ability of the monopolist to trade strategically. When storage is inexpensive, agents without market power may purchase and store the good in anticipation of higher prices in the future. This limits the ability of the monopolist to raise prices, as excess capacity will prevent prices from increasing. In this sense, storage serves the same function as durability in the durable-goods monopoly problem of Coase (1972). Like durability, storage erodes market power by providing a venue for the monopolist to compete against his future self. Here, we assume that storage costs are high enough that no storage takes place in equilibrium, so that monopoly power is not eroded.

order flow and set futures prices accordingly. Third, there are risk-averse agents whose payoff depends on the price realized in the spot market. They have an incentive to participate in the futures market because doing so allows them to hedge spot price risk. We assume that the number of these agents is stochastic and unobservable. $\tilde{N}$ denotes a random variable for the number of such agents; N denotes a realization of this random variable.[4]

The timing of events is as follows. First, nature chooses a number of risk-averse agents, N. Then in $t = 1$, the monopolist and each risk-averse agent simultaneously submit futures market orders. Observing the aggregate order flow, the sum of these orders, market makers set futures prices equal to expected spot prices. Next, demand is realized. In $t = 2$, the monopolist chooses spot market quantity to maximize profits. Figure 1 provides a timeline of these events.

[Insert Figure 1 here]

We make the following assumptions:

- The demand curve is linear, so that spot prices are given by $P = a - bQ$, where a is the realization of random variable $\tilde{a}$ and $b > 0$.[5]

- All risk-averse agents are atomistic and identical. The (unobservable) number of such agents is N, a realization of random variable $\tilde{N}$. We assume that $\tilde{N}$ is uniformly distributed on $[0, 1]$. Any risk-averse agent is too small to affect aggregate order flow and thus takes prices as given. Risk-averse agents are indexed by n on $[0, N]$.

- Each risk-averse agent has profits that are linear in the spot market price, i.e. $\pi^n(P) = \alpha - \beta P$, with $\beta > 0$, so that higher spot prices imply lower profits.[6]

Monopolist In the initial period, the monopolist chooses a number of futures contracts, C^M. The payoff per contract is $P - k$, where k is the futures price. Given the number of contracts chosen (C^M) and the realization of demand (a), the monopolist sets the spot market price and quantity to maximize profits

$$\begin{aligned} \pi &= C^M (P - k) + PQ \\ &= C^M (a - bQ - k) + (a - bQ) Q. \end{aligned} \tag{1}$$

[4] Throughout this paper, we use superscript "~" to denote random variables and omit superscript "~" to indicate their realizations (i.e., y refers to a specific realization of random variable $\tilde{y}$).

[5] The choice of a linear demand function is for analytic tractability. While a much broader class of functions will obtain similar results, not all demand functions will obtain the same results. In particular, convex demand curves will provide an even stronger incentive for the monopolist to strategically trade in the futures market as large changes in the spot price lead to relatively small changes in monopoly profits. Concave demand curves provide a weaker incentive for strategic trading.

[6] The linear functional form is used for tractability. Any profit function that provides hedging motives will yield similar results. In particular, an upward sloping profit function, $\beta < 0$, would imply risk-averse agents taking a short instead of a long position, but to a lesser extend than they would in the absence of strategic trading by the spot market monopolist.

The spot market first-order condition (FOC) is

$$\frac{\partial \pi}{\partial Q} = -bC^M + a - 2bQ = 0.$$

Note that the second-order condition (SOC) is satisfied, yielding an optimal quantity and price

$$\begin{aligned} Q^* &= \frac{1}{2b}\left(a - bC^M\right); \\ P^* &= \frac{1}{2}\left(a + bC^M\right). \end{aligned} \tag{2}$$

Plugging the solution from equation (2) into equation (1) yields the monopolist profit, the sum of profit in the futures and spot markets:

$$\pi = -C^M k + \frac{1}{4b}\left(a + bC^M\right)^2 \tag{3}$$

We will present an equilibrium in which the monopolist randomizes the number of futures market contracts he submits; this order is a random variable, $\tilde{C}^M$.

Risk-Averse Agents In the initial period, each atomistic risk-averse agent n chooses a number of futures contracts, c^n, to maximize expected utility. The total number of contracts submitted by these agents, $\tilde{C}^N \equiv \int_{n=0}^{\tilde{N}} c^n dn$, is stochastic only because the number of these agents, $\tilde{N}$, is stochastic; $C^N = \int_{n=0}^{N} c^n dn$ is the total number of contracts they submit given N. Since all risk-averse agents are identical, each one submits the same order in equilibrium; $c^n = c$ for all $0 \le n \le N$ so that $C^N = cN$.

Market Makers Market makers only observe the realized aggregate order flow, $\theta \equiv C^N + C^M$. They have beliefs about the order flow submitted by the monopolist and the risk-averse agents and set the futures price accordingly, $k = k(\theta)$.

2.2 Equilibrium with Strategic Trading

In this setup, we look for perfect Bayesian equilibria in the futures market given optimal subsequent behavior in the spot market. We assume a set of actions and beliefs for all agents and explore whether any agent has an incentive to deviate. This section explores equilibria in which the monopolist hides his futures market participation by randomizing the order flow he submits. When the monopolist submits a positive (negative) order flow – with plans to drive up (down) spot prices to make this position profitable – market makers cannot identify perfectly the order submitted by the monopolist; it is hidden behind the orders submitted by risk-averse agents. This imperfect inference allows the monopolist to receive favorable futures market prices at the expense of the rational risk-averse agents.

In this setting, a perfect Bayesian equilibrium consists of the following triple, $\left\{\tilde{C}^M, c, k(.)\right\}$. The first element is the random futures market order submitted by the monopolist, $\tilde{C}^M$; the second element is the futures market order, c, submitted by each atomistic, risk-averse agent; the third element is the futures market price schedule set by market makers given the observed aggregate order flow, $k(.)$. *In equilibrium, all agents believe that others' actions conform to this triple; each agent's optimal action given these beliefs also conforms to this triple.* The perfect Bayesian equilibrium is then characterized by the following conditions:

1. market makers: If the monopolist submits random order $\tilde{C}^M$, each risk-averse agent submits order c, and given the price schedule $k(.)$, no market maker has an incentive to offer another price schedule,

2. risk-averse agents: If the monopolist submits random order $\tilde{C}^M$, all other risk-averse agents submit order c, and given the price schedule $k(.)$, no risk-averse agent could increase expected utility by submitting an order other than c,

3. monopolist: If each risk-averse agent submits order c and given the price schedule $k(.)$, the monopolist receives the same expected wealth for each value of C^M with strictly positive probability in the distribution of $\tilde{C}^M$; this expected wealth is at least as high as it would be for any other action without strictly positive probability in the distribution of $\tilde{C}^M$.

There are many possible equilibria. Here, we consider the simplest set of such equilibria, which we index by x and denote $\left\{\tilde{C}_x^M, c_x, k_x(.)\right\}$. In equilibrium x, $\tilde{C}_x^M$ takes the values $+x$ and $-x$ with equal probability; the monopolist finds it optimal to randomize between orders of $+x$ and $-x$ with equal probability. (x is normalized to be non-negative.) Note that there will be a continuum of such equilibria, each corresponding to a different value of x.

In the subsections to follow, we show that it is optimal for individual market makers (Section 2.2.1), the monopolist (Section 2.2.2), and individual risk-averse agents (Section 2.2.3) to act consistently with $\left\{\tilde{C}_x^M, c_x, k_x(.)\right\}$ when they believe that other market participants are doing so. We make the standard assumption that all market participants know which equilibrium they are in.

2.2.1 Beliefs and Prices of Market Makers

Consider equilibrium x, in which market participants' actions are described by the triple $\left\{\tilde{C}_x^M, c_x, k_x(.)\right\}$. Market makers believe that each risk-averse agent n submits an order $c^n = c_x$ and that the monopolist submits a random order $\tilde{C}^M = \tilde{C}_x^M$, randomizing between $+x$ and $-x$ with equal probability. Based on these beliefs, market makers set actuarially fair prices. Off the equilibrium path, we assume that market makers set prices based on the most punitive beliefs they could hold about the monopolist's actions.

Market makers set strike prices competitively, namely at the expected spot price given the observed order flow and given the assumed actions of other market participants in the equilibrium, $k_x(\theta) = E\left[P|\theta, \left\{\tilde{C}_x^M, c_x\right\}\right]$. The aggregate order flow (θ) is a noisy signal of the monopolist's futures market order (C^M), which in turn affects the spot price following equation (2). The binomial distribution of $\tilde{C}_x^M$ in equilibrium x and the uniform distribution of $\tilde{N}$ makes inference about C^M given θ straightforward. In equilibrium x, we can partition the real line into five ranges for aggregate order flow (θ). Here, we show expected spot prices (and therefore futures strike prices) in each range, presenting the piece-wise futures price schedule $k_x(.)$ in equilibrium x:[7]

$$
\begin{aligned}
A1.\ k_x(\theta) &= \frac{1}{2}E[\tilde{a}] + \frac{1}{2}b\theta \text{ if } \theta > x + c_x \\
A2.\ k_x(\theta) &= \frac{1}{2}E[\tilde{a}] + \frac{1}{2}bx \text{ if } -x + c_x < \theta \leq x + c_x \\
A3.\ k_x(\theta) &= \frac{1}{2}E[\tilde{a}] \text{ if } x \leq \theta \leq -x + c_x \\
A4.\ k_x(\theta) &= \frac{1}{2}E[\tilde{a}] - \frac{1}{2}bx \text{ if } -x \leq \theta < x \\
A5.\ k_x(\theta) &= \frac{1}{2}E[\tilde{a}] + \frac{1}{2}b(\theta - c_x) \text{ if } \theta < -x
\end{aligned}
\tag{4}
$$

In ranges $A1$ and $A5$, aggregate order flow (θ) is either too high ($A1 : \theta > x + c_x$) or too low ($A5 : \theta < -x$) to be consistent with equilibrium x. In equilibrium x, the risk-averse agents' total order flow will be at least 0 and at most c_x; the monopolist's order flow will be at least $-x$ and at most $+x$. Therefore, an observed aggregate order flow $\theta > x + c_x$ (in range $A1$) is too large to be consistent with equilibrium x actions; an observed order flow $\theta < -x$ (in range $A5$) is too small to be consistent with equilibrium x actions. In a perfect Bayesian equilibrium, market makers are free to hold any possible beliefs in off-equilibrium-path situations like the ones in ranges $A1$ and $A5$. We choose the most punitive possible beliefs market makers could hold about the monopolist's actions. In range $A1$, we assume that $N = 0$ and therefore that $C^M = \theta$; in range $A5$, we assume that $N = 1$ and $c^n = c_x$ and therefore that $C^M = \theta - c_x$. Prices are set accordingly.

In ranges $A2$ and $A4$, aggregate order flow (θ) is high enough and low enough, respectively, to infer the monopolist's trade (C^M) perfectly given aggregate order flow (θ). In range $A2$, the aggregate order flow ($\theta > -x + c_x$) is too high to be consistent with $C^M = -x$ even if the number of risk-averse agents were as low as $N = 0$; market makers then know that the monopolist set $C^M = +x$ and they set the strike price accordingly. In range $A4$, the aggregate order flow ($\theta < x$) is too low to be consistent with $C^M = +x$ even

[7] Note that the ranges for θ in $A2$ and $A4$ will be empty when $x = 0$; the range for θ in $A3$ will be empty when $x \geq \frac{1}{2}c_x$.

if the number of risk-averse agents were as high as $N = 1$; market makers then know that the monopolist set $C^M = -x$ and they set the strike price accordingly.

In range $A3$, aggregate order flow is neither too high nor too low so that the monopolist hides successfully within the aggregate order flow. Market makers have no way of knowing whether $C^M = +x$ (in which case N would be $(\theta - x)/c_x$) or $C^M = -x$ (in which case N would be $(\theta + x)/c_x$); $C^M = +x$ and $C^M = -x$ are equally likely and market makers set prices accordingly.

2.2.2 Beliefs and Actions of Monopolist

In this subsection, we show that the monopolist finds it optimal to randomize between $C^M = +x$ and $C^M = -x$ in equilibrium x. Before doing so, we provide intuition behind such a mixed strategy by ruling out equilibria without randomization, where the monopolist submits a fixed non-zero futures market order.

Consider a candidate equilibrium ($\left\{C^M \neq 0, c, k(.)\right\}$, which we will rule out here) in which the monopolist submits a fixed non-zero order, C^M. Since the market makers are rational and know the (candidate) equilibrium they are in, they know C^M. Since the futures market is competitive, market makers set strike prices equal to expected spot prices given this order, $k = E\left[P|C^M\right] = \frac{1}{2}\left(E\left[\tilde{a}\right] + bC^M\right)$ from equation (2). The monopolist then earns zero expected profits in the futures market. However, this break-even futures market position would reduce the monopolist's expected profits by inducing him to deviate from the spot market monopoly optimum. The monopolist's expected profits are then:

$$\begin{aligned} E\left[\pi|C^M\right] &= E\left[C^M(P-k) + PQ\right] \\ &= C^M E\left[P - E\left[P\right]\right] + \frac{1}{4b}E\left[\tilde{a}^2 - b^2\left(C^M\right)^2\right] \\ &= \frac{1}{4b}E\left[\tilde{a}^2 - b^2\left(C^M\right)^2\right] \end{aligned}$$

If the monopolist cannot earn positive expected profits in the spot market (as in the case where he submits a fixed futures market order), then he is better of not participating in the futures market. When he does not participate, his expected profits are

$$E\left[\pi|C^M = 0\right] = \frac{1}{4b}E\left[\tilde{a}^2\right].$$

If the monopolist does participate in the futures market, he must play a mixed strategy. This allows him to hide the order he submits within the aggregate order flow. Consider equilibrium x (characterized by the triple $\left\{\tilde{C}_x^M, c_x, k_x(.)\right\}$) in which the monopolist randomizes between $C^M = +x$ and $C^M = -x$. When the monopolist submits an order of $C^M = +x$, the aggregate order flow (θ) will fall in range $A2$ from equation

(4) if $N > 1 - \frac{2x}{c_x}$ and fall in range $A3$ if $N \leq 1 - \frac{2x}{c_x}$.

In the former case (range $A2$), the number of risk-averse agents – and therefore the aggregate order flow – is large enough that the monopolist's futures market order is identified by the market makers. They set futures prices that take into account the impact of the monopolist's futures market position on the strike price he will choose. The monopolist will then break-even in the futures market and earn a spot market profit that is $\frac{1}{4}bx^2$ lower than it would be without futures market participation.

In the latter case (range $A3$), the aggregate order flow is neither too high nor too low, so the market makers have no way of knowing if $C^M = +x$ or $C^M = -x$. Because the problem is linear and both are equally likely, market makers set prices as if $C^M = 0$. The monopolist can then earn an expected profit of $\frac{1}{2}bx^2$ in the futures market (but again an expected loss of $\frac{1}{4}bx^2$ relative to the monopoly optimum in the spot market) by increasing the spot price to make his futures market position more profitable. The net gain in range $A3$ is then $\frac{1}{2}bx^2 - \frac{1}{4}bx^2 = \frac{1}{4}bx^2$.

The same cases can be considered for $C^M = -x$, when the monopolist's order flow is discovered (range $A4$) when the number of risk-averse agents is small but hidden (range $A3$) when the number of risk-averse agents is large. Note that for the expected benefits of hiding ($\frac{1}{4}bx^2$ × probability of being in range $A3$) to exceed the expected costs of being discovered ($\frac{1}{4}bx^2$ × probability of being in either range $A2$ or range $A4$), the probability of hiding successfully, $prob\left(\tilde{N} \leq 1 - \frac{2x}{c_x}\right)$, must be greater than 0.5. Given the uniform distribution of $\tilde{N}$, this will happen if and only if $x < \frac{1}{4}c_x$. This places an upper bound on the range of possible values for x for which there can be an equilibrium.

In equilibrium x, $\left|C^M\right| > x$ will not be optimal because of the punitive futures market prices set by the market makers when the aggregate order is so extreme that it must reflect off-equilibrium-path behavior (A1 or A5). In this setting, the monopolist is indifferent between submitting order of $+x$ and $-x$, and strictly prefers these to other possible orders. This is presented more formally here:

Proposition 1 *Given that market makers set $k_x(.)$ as in (4) and each risk-averse agent submits an order c_x, the monopolist maximizes expected profits by submitting either $C^M = +x$ or $-x$ $(x > 0)$ as long as $x < \frac{1}{4}c_x$. The monopolist's expected profits are*

$$\begin{aligned} E[\pi|x] &= E[\pi|-x] = \frac{1}{4b}E\left[\tilde{a}^2\right] + bx^2\left(\frac{1}{4} - \frac{x}{c_x}\right) \\ &> E\left[\pi|C^M = 0\right]. \end{aligned}$$

Proof. See Appendix A.1. ■

2.2.3 Beliefs and Actions of Risk-Averse Agents

In equilibrium x, the risk-averse agents know that it is optimal for the monopolist to hide his futures position within the aggregate order flow by randomizing C^M between $+x$ and $-x$ and then setting spot market prices optimally given this futures position. The preferences of each of the risk-averse agents are represented by a concave utility function, u. To reduce their exposure to spot market price risk in equilibrium x, risk-averse agent n will participate in the futures market by purchasing $c^n = c_x^n$ units of the futures contract. In equilibrium x, each risk-averse agent n chooses their order c_x^n taking as given the futures market price schedule $k_x(.)$ (as in Section 2.2.1), the mixed strategy of the monopolist $\tilde{C}_x^M$ (as in Section 2.2.2), and the futures market position of all other risk-averse agents, c_x:

$$\begin{aligned} c_x^n &= \arg\max_c E\left[u\left(c\left(P^* - k_x\left(\tilde{N}c_x + \tilde{C}_x^M\right)\right) + \alpha - \beta P^*\right)\right] \qquad (5) \\ &= \arg\max_c E\left[u\left(c\left(\frac{1}{2}\left(\tilde{a} + b\tilde{C}_x^M\right) - k_x\left(\tilde{N}c_x + \tilde{C}_x^M\right)\right) + \alpha - \beta\frac{1}{2}\left(\tilde{a} + b\tilde{C}_x^M\right)\right)\right] \end{aligned}$$

Note that we assumed that any risk-averse agent is too small to affect aggregate order flow and thus takes prices as given. In equilibrium, the optimal actions of risk-averse agent n will coincide with the anticipated actions of all other risk-averse agents, $c_x^n = c_x$.

When a risk-averse agent wants to hedge, this provides him with information that the expected aggregate hedging demand is high. He then acts rationally taking this information into account. Since risk-averse agents are identical, none is more or less likely to hedge than any other. Therefore, when the unconditional density function for $\tilde{N}$ uniformly distributed on $[0,1]$ is $f(N) = 1$, the conditional density for $\tilde{N}$ – given that a particular risk-averse agent wants to hedge – is $f(N) = 2N$.[8]

First, we examine optimal hedging in the absence of the monopolist, i.e. if $x = 0$ so that $\tilde{C}_x^M = 0$. In this case,

$$c_0^n = \arg\max_c E\left[u\left(c\left(\frac{1}{2}\tilde{a} - \frac{1}{2}E[\tilde{a}]\right) + \alpha - \frac{1}{2}\beta\tilde{a}\right)\right].$$

[8] This distribution is calculated using Bayes' rule. The unconditional distribution of $\tilde{N}$ has the density function $f(N) = 1$. Now, consider a randomly drawn risk-averse agent n. If his index n (unknown to him and drawn from the random variable $\tilde{n}$, uniformly distributed on $[0,1]$) is less than the number of agents with hedging demands, N, then he has a hedging demand. Therefore, knowing that he is one of these agents – namely, $n < N$ – he updates his beliefs about the conditional distribution of $\tilde{N}$ according to Bayes' rule:

$$f(N|n < N) = \frac{f(n < N|N) \cdot f(N)}{f(n < N)} = 2N.$$

Similarly, agents without hedging needs would update their beliefs about the expected number of hedgers accordingly. These agents will participate in the futures market to exploit their information. This will mitigate but not eliminate the effect we discuss. If all agents do not take into account the information contained in their own hedging demand, these agents will not believe that they face unfavorable prices on average, and will not reduce their hedging demand. However, their expected profits will be lower if they hold these naïve beliefs.

The FOC is then

$$E\left[\left(\frac{1}{2}\tilde{a}-\frac{1}{2}E\left[\tilde{a}\right]\right)u'\left(c\left(\frac{1}{2}\tilde{a}-\frac{1}{2}E\left[\tilde{a}\right]\right)+\alpha-\frac{1}{2}\beta\tilde{a}\right)\right]=0.$$

Note that the SOC is satisfied. For $c_0^n=\beta$, the FOC is satisfied and it is a global maximum. Since this will be true for all agents, we also have $c_0^n=c_0$. Without the monopolist's participation in the futures market, it is optimal for risk-averse agents to eliminate all risk. We now examine optimal hedging when the monopolist participates in the futures market, i.e. when $x>0$.

Proposition 2 *Given that market makers set $k_x(.)$ as in (4), that the monopolist participates in the futures market with order flow $+x$ and $-x$ ($x>0$) with equal probability, and that all other risk-averse agents submit the futures market order $c_x>2x$, then*

- *a) the optimal futures market order submitted by risk-averse agent n, c_x^n – the solution to equation (5) – is strictly less than it would be if the monopolist did not participate, i.e. $c_x^n<\beta=c_0$;*
- *b) for sufficiently small values of x, $c_x^n>0$; and,*
- *c) for sufficiently small values of x, there exist c_x with $0<c_x<\beta$ such that $c_x^n=c_x$.*

Proof. See Appendix A.2. ■

If risk-averse agents believe that the monopolist trades strategically in the futures market, they are concerned that the monopolist will hold a position opposite to their own and will move spot prices against them. A given risk-averse agent knows that he is more likely to want to hedge precisely at the wrong times as he is more likely to hedge when aggregate order flow from risk-averse agents is large. In this case, either the monopolist also submits a large order flow and is spotted – in which case futures prices are set fairly – or the monopolist submits a small order flow and hides successfully – in which case the monopolist gains at the risk-averse agents' expense. This makes hedging more expensive for agents without market power and thus discourages their participation in the futures market.

The following proposition shows that a perfect Bayesian equilibrium exists with beliefs and actions as specified above.

Proposition 3 *Given the market structure described in Subsection 2.1, there is a perfect Bayesian equilibrium x described by the triple $\left\{\tilde{C}_x^M, c_x, k_x(.)\right\}$ for any sufficiently small value of x. In each equilibrium x, futures market prices $k_x(.)$ are set as in (4), risk-averse agents each submit an order of c_x (with c_x satisfying $x<\frac{1}{4}c_x$ and the properties laid out in Proposition 2), and the monopolist submits an order flow of either $+x$ or $-x$ with equal probability.*

Proof. *Proposition 1 shows that the monopolist has no incentive to deviate from this equilibrium for* $x < \frac{1}{4}c_x$. *Proposition 2 shows that risk-averse agents have no incentive to deviate from this equilibrium for sufficiently small* x. *Market makers earn zero profits and none has an incentive to offer another price schedule. Since* $x < \frac{1}{4}c_x$ *is satisfied for* $x = 0$ $(c_0 = \beta > 0)$ *and since* c_x *is smooth in* x, *the condition* $x < \frac{1}{4}c_x$ *is satisfied for small, but strictly positive values of* x. ■

Here, a spot market monopolist is able to increase profits by trading strategically in the futures market. The monopolist takes a futures market position randomly, then deviates from the spot market monopoly optimum to move spot market prices and make this position more profitable. If the monopolists futures market position were perfectly observable, market makers would set futures market prices anticipating these actions. In this case, the monopolist would not want to participate in the futures market since doing so would decrease expected spot market profits without increasing expected futures market profits. However, when there are other traders in the market, the futures market position submitted by the monopolist cannot be perfectly inferred by observing the aggregate order flow. In this case, market makers set prices based on the rational belief that the orders they receive could have come from either the monopolist or from other agents without market power. As a result, trades submitted by the monopolist move prices less than they would had they been observable. Just as an informed trader in the Kyle model profits at the expense of "noise traders", the monopolist earns positive expected profits in the futures market at the expense of the other market participants. This makes futures market participation expensive, and reduces the optimal participation of risk-averse agents.

3 Futures Market Manipulation under Spot Market Power

The last section showed that a spot market monopolist can profitably exploit spot market power in the futures market. This section documents that even those without market power can profit in the derivatives market when another agent has spot market power. This section is related to the insight of Kumar and Seppi (1992) that agents without inside information can manipulate a market if they are mistaken for agents with inside information. Here, we show that the same can be said of market power: agents without market power who hold derivatives market positions can use subsequent trading to manipulate prices to make the original position profitable. This happens when the manipulator's subsequent trades reveal to market makers the similar trades made by the monopolist. For example, a positive futures market position by the manipulator increases aggregate order flow to the point where the positive futures market position of the monopolist is spotted by market makers. Market makers then set high futures market strike prices anticipating that the monopolist will set high spot prices; these high futures market strike prices make the manipulator's positive derivatives market position profitable. In case the manipulator is unsuccessful in revealing the monopolist's position to market makers, the monopolist profits in the futures market (as in Section 2) at the expense of the manipulator.

3.1 Model Setup with Manipulators

3.1.1 Timing and Markets

While the model developed Section 2 has markets in only two periods, the model in this section requires trade in three periods. $t = 1, 2$ mirror our earlier setup; here, we add an initial period, $t = 0$, in which agents trade contracts whose payoffs are contingent on futures prices in the next period. Presenting the markets in reverse chronological order:

$t = 2$: There is a spot market at time $t = 2$. As before, production in this period is controlled by a monopolist, who faces a linear demand curve, i.e. spot prices are given by $P = a - bQ$, where a is the realization of random variable $\tilde{a}$ and $b > 0$.[9] The cost of production is zero.

$t = 1$: There is a futures market at time $t = 1$, characterized by linear cash-settled contracts based on the spot price in the next period, with payoff $P - k_1$ per contract.

$t = 0$: There is a derivatives market at time $t = 0$, characterized by linear cash-settled contracts based on the futures strike price in the next period, with payoff $k_1 - k_0$ per contract.[10]

3.1.2 Actors

The model involves four types of actors:

1. Noise traders (N) submit a stochastic aggregate order flow $\tilde{C}_0^N$ with realization C_0^N at $t = 0$; they do not participate at $t = 1$. We assume that $\tilde{C}_0^N$ is uniformly distributed on $\left[C_0^{N-}, C_0^{N+}\right]$. The assumption that noise traders participate only in the initial period is for expositional simplicity and is not necessary to obtain these results.[11]

[9] While the assumption of linear demand is critical to obtain simple analytic results, the same intuition obtains with a convex demand curve. While losing analytic tractability, these demand functions have the advantage that the monopolist has a strict benefit from participating in the futures market. When demand is linear, the increased profits in the futures market that come with futures market participation are exactly offset by lower spot market monopoly profits.

[10] Note that a futures contract whose payoff is based on the price of another futures contract is unusual. However, there are many options whose payoff is based on a futures contract. While we use a linear futures contract and not an options contract at $t = 0$ for analytical tractability, our result that manipulative trading exists in equilibrium is robust to changes in the contractual structure. Furthermore, many futures markets based on the spot price of a storable commodity are effectively a future on a future, as storability links current spot and forward prices.

[11] For analytical tractability, we assume that these traders act mechanically, as "noise traders" do in the Kyle model. Introducing optimal behavior on their part as in Section 2 would not change our result that manipulative trading is possible in equilibrium.

2. Monopolist (M) submits a futures market order C_1^M at $t = 1$ (which in equilibrium will be a draw from the random variable $\tilde{C}_1^M(\theta_0)$ given aggregate futures market order flow of θ_0 at $t = 0$), and then sets prices and quantities optimally at $t = 2$. To simplify the problem, the monopolist is assumed not to participate in the derivatives market at $t = 0$.

3. Manipulator (H denotes "hider") submits a derivatives market order C_0^H at $t = 0$ (which in equilibrium will be a draw from the random variable $\tilde{C}_0^H$), and a futures market order C_1^H at $t = 1$ (which in equilibrium will be a function of θ_0 and C_0^H).

4. Market makers, as before, are risk-neutral and act competitively to set strike prices k_0 and k_1. They observe aggregate order flow $\theta_1 \equiv C_1^M + C_1^H$ at $t = 1$ and $\theta_0 \equiv C_0^H + C_0^N$ at $t = 0$ and make rational inferences about the positions of various agents and their impact on contract payoffs. Therefore, $k_1(\theta_1) = E[P^*|\theta_1]$ and $k_0(\theta_0) = E[k_1|\theta_0]$.

Figure 2 provides a timeline showing which agents participate in each market.

[Insert Figure 2 here]

The monopolist is willing to participate in the futures market for the same strategic reason outlined in Section 2. He earns profits by setting spot market prices to make his futures market position profitable. While the monopolist's spot market profit at $t = 2$ is lower than it would be had he not participated in the futures market, futures market profits in $t = 1$ are high enough (at least weakly) to offset these reduced profits.

The manipulator is willing to accept expected losses in the futures market at $t = 1$ for the same reason that the monopolist is willing to accept lower expected profits in the spot market at $t = 2$. Just as the monopolist sets spot prices at $t = 2$ to make his futures market position at $t = 1$ profitable, the manipulator trades in the futures market at $t = 1$ in order to move futures prices, thereby making his derivatives market position at $t = 0$ profitable. Just as the monopolist earns expected profits at the expense of the manipulator at $t = 1$, the manipulator earns expected profits at the expense of noise traders at $t = 0$.

3.1.3 Spot Market Prices

For a given futures market position (C_1^M) and the realization of demand (a) the monopolist's profit is given by (1) so that prices and quantities are set optimally according to (2). Adding appropriate time subscripts to equation (3) yields the monopolist's profit in this three-period setting

$$\pi^M = C_1^M (P^* - k_1) + P^* Q^* = -C_1^M k_1 + \frac{1}{4b} \left(a + bC_1^M\right)^2 . \tag{6}$$

As in Section 2, futures market participation causes the monopolist to deviate from the spot market monopoly optimum. He moves prices to make the futures market position profitable.

3.2 Equilibrium with Manipulation

In this setup, a perfect Bayesian equilibrium with manipulation consists of the following sets of actions and beliefs at $t = 0$ and at $t = 1$, respectively: $\left\{\tilde{C}_0^N, \tilde{C}_0^H, k_0(.)\right\}$ and $\left\{C_1^H\left(\theta_0, C_0^H\right), \tilde{C}_1^M(\theta_0), k_1(. \mid \theta_0)\right\}$. Again, all agents believe that other agents' actions conform to this equilibrium and all agents' actions are optimal given those beliefs. We propose a perfect Bayesian equilibrium with manipulation in which $\tilde{C}_0^H$ takes the values $+x$ and $-x$ (with x normalized to $x \geq 0$) with equal probability, i.e. the manipulator finds it optimal to randomize his futures market orders in period $t = 0$ between $+x$ and $-x$. The actions and associated beliefs of this equilibrium are defined as follows:

$t = 0$: $\tilde{C}_0^H$ takes the values $+x$ and $-x$ with equal probability where

$$x = \frac{2}{5}\left(C_0^{N+} - C_0^{N-}\right)$$

Market makers set

$$k_0(\theta_0) = \frac{1}{2}E[\tilde{a}] \tag{7}$$

independent of the aggregate order flow submitted.[12]

$t = 1$: For the equilibrium actions and beliefs $\left\{C_1^H\left(\theta_0, C_0^H\right), \tilde{C}_1^M(\theta_0), k_1(. \mid \theta_0)\right\}$, in period $t = 1$, there are three possible subgames (denoted SG1, SG2, and SG3) depending on the aggregate order flow, $\theta_0 = C_0^H + C_0^N$, at $t = 0$:

SG1. If $\theta_0 > C_0^{N+} - x$ then market makers know that the manipulator must have submitted $C_0^H = +x$. In this case, the monopolist will not participate in the futures market, i.e. $\tilde{C}_1^M(\theta_0) = 0$. The manipulator submits the same order as in the previous period, i.e. $C_1^H\left(\theta_0, C_0^H\right) = C_0^H$, and market makers set the futures price schedule as

$$k_1(\theta_1 \mid \theta_0) = E[P^*|\theta_1] = \frac{1}{2}E[\tilde{a}] + \frac{1}{2}b(\theta_1 - x). \tag{8}$$

Since the monopolist does not participate in the futures market in this subgame, he will set prices at the spot market monopoly optimum at $t = 2$, $P = \frac{1}{2}a$ from equation (2). The equilibrium

[12] Here, we propose a specific equilibrium with a specific value for x; this is in contrast to Section 2 in which we presented a set of equilibria, each with its own x.

futures market order flow, $\theta_1 = \tilde{C}_1^M + C_1^H = 0 + C_0^H = +x$, ensures that the futures strike price is set at the expected monopoly optimum, $k_1(+x) = \frac{1}{2}E[\tilde{a}]$.

SG2. If $C_0^{N-} + x \leq \theta_0 \leq C_0^{N+} - x$ then the manipulator has successfully hidden his order flow at $t = 0$. The monopolist randomizes his $t = 1$ order $\tilde{C}_1^M(\theta_0)$ over $C_1^M = -\frac{1}{2}x$ and $C_1^M = +\frac{1}{2}x$ with equal probability, and the manipulator sets $C_1^H\left(\theta_0, C_0^H\right) = \frac{1}{2}C_0^H$. Market makers set the futures price as

$$k_1(\theta_1 \mid \theta_0) = E[P^*|\theta_1] = \frac{1}{2}E[\tilde{a}] + \frac{1}{4}b\theta_1. \tag{9}$$

SG3. If $\theta_0 < C_0^{N-} + x$ then market makers know that the manipulator must have submitted $C_0^H = -x$. The monopolist will not participate, i.e. $\tilde{C}_1^M(\theta_0) = 0$, and the manipulator submits the same order as in the previous period, i.e. $C_1^H\left(\theta_0, C_0^H\right) = C_0^H$. The futures price is set as

$$k_1(\theta_1 \mid \theta_0) = E[P^*|\theta_1] = \frac{1}{2}E[\tilde{a}] + \frac{1}{2}b(\theta_1 + x). \tag{10}$$

This subgame mirrors SG1, but with reversed signs.

$t = 2:$ The monopolist sets prices and quantities according to (2).

Proposition 4 *The actions and beliefs described above constitute a perfect Bayesian equilibrium.*

Proof. See Appendix A.3. ■

Financial market manipulation is possible when agents without market power reveal the trades of those with market power. A manipulator without market power can profit by taking a random position in the derivatives market at $t = 0$. When the manipulator's $t = 0$ position is identified (SG1 and SG3), his subsequent trades can be inferred perfectly and he earns no profit. In this case, the monopolist does not participate in the futures market and therefore earns zero expected profits (relative to the spot market monopoly optimum). More interestingly, the manipulator is able to earn strictly positive profits only when his $t = 0$ position is hidden within the noise traders' order flow (SG2). The manipulator's expected profit given that he is able to hide at $t = 0$ ($\frac{1}{16}bx^2$) is increasing in the absolute size of his $t = 0$ derivatives position (x); however, the probability that he hides successfully at $t = 0$ ($1 - 2x/\left(C_0^{N+} - C_0^{N-}\right)$) is decreasing in x.[13] Appendix A.3 shows that the absolute size of the manipulator's $t = 0$ equilibrium trade ($x = \frac{2}{5}\left(C_0^{N+} - C_0^{N-}\right)$) solves the optimal trade-off between these two competing effects.

[13] The probability of hiding is strictly positive only for $x < \frac{1}{2}\left(C_0^{N+} - C_0^{N-}\right)$; note that this condition is satisfied trivially by our proposed equilibrium $x = \frac{2}{5}\left(C_0^{N+} - C_0^{N-}\right)$.

When the manipulator is able to hide at $t = 0$ (SG2) , the monopolist and manipulator both independently randomize between $+\frac{1}{2}x$ and $-\frac{1}{2}x$ at $t = 1$. (Though the manipulator deterministically submits half his initial order, $C_1^H = \frac{1}{2}C_0^H$, this order appears random to the monopolist and market makers.) Half the time they submit the same order (either both submitting $+\frac{1}{2}x$ or both submitting $-\frac{1}{2}x$) and the monopolist's position is spotted at $t = 1$; half the time they submit opposite orders and the monopolist's position is hidden.

In SG2 when the monopolist and manipulator submit the same order at $t = 1$, the monopolist's trade is identified, and the impact of their position on the subsequent spot price is incorporated into the strike price. The monopolist earns zero profits from the $t = 0$ derivatives market (since he did not participate), zero expected profits from the $t = 1$ futures market (since his position is spotted by marker makers), and an expected loss of $\frac{1}{16}bx^2$ from the $t = 2$ spot market (relative to the spot market monopoly optimum). The manipulator earns $\frac{1}{4}bx^2$ from the $t = 0$ derivatives market (since the monopolist's $t = 1$ trades are identified and move $t = 1$ futures market prices to make the manipulator's $t = 0$ derivatives market position profitable), an expected profit of zero from the $t = 1$ futures market (since the monopolist's trades which coincide with his own are spotted), and profit of zero from the $t = 2$ spot market (since the manipulator does not participate).

In SG2 when the monopolist and manipulator submit orders of opposite signs at $t = 1$, the monopolist's trade is hidden behind the manipulator's order flow. The monopolist earns zero profits from the $t = 0$ derivatives market (since he did not participate), an expected profit of $\frac{1}{8}bx^2$ from the $t = 1$ futures market (profiting at the expense of the manipulator's unprofitable position), and an expected loss of $\frac{1}{16}bx^2$ from the $t = 2$ spot market (relative to the spot market monopoly optimum). The manipulator earns zero profits from the $t = 0$ derivatives market (since the $t = 1$ futures trades do not move the $t = 1$ prices that determine the payoff of the $t = 0$ derivatives market), an expected loss of $\frac{1}{8}bx^2$ from the $t = 1$ futures market (since the monopolist moves prices against him), and profit of zero from the $t = 2$ spot market (since the manipulator does not participate).

Note that for the manipulator, this randomization at $t = 1$ leads to a net expected gain of $\frac{1}{16}bx^2$ in SG2: a gain of $\frac{1}{4}bx^2$ in $t = 0$ when the monopolist is spotted and an expected loss of $\frac{1}{8}bx^2$ in $t = 1$ when the monopolist is hidden. For the monopolist, this randomization leads to a net expected gain of zero in SG2: an expected gain of $\frac{1}{8}bx^2$ in $t = 1$ when he is spotted and an expected loss of $\frac{1}{16}bx^2$ at $t = 2$ both when he is spotted and when he is hidden.[14]

[14] Here the monopolist is indifferent between participating and not participating in the futures market. This result is obtained because we make the assumptions that there is no noise trading at $t = 1$ and demand is linear. If we either allow for noise trading at $t = 1$ or a convex demand function the monopolist would have a strict incentive to strategically randomize at $t = 1$.

In Section 2, a monopolist's trades could not be differentiated from those submitted by risk-averse agents. The monopolist was able to profit because the futures market trades he submitted moved prices by *less* than they would have had his trades been perfectly observable. In this section, a manipulator's trades serve to reveal the trades of the monopolist. The manipulator is able to profit because the futures market trades he submits move prices by *more* than they would had the monopolist's trades been perfectly observable. As in Section 2, the monopolist profits from the manipulator's presence at $t = 1$ since this causes the trades he submits to move prices by *less* than they would otherwise.

4 Conclusions

In this paper, we have shown how monopoly power impacts futures market behavior when futures market participation is not observable. Spot market monopolists will trade in the futures market – trying to hide behind the trades of agents without market power – and then strategically set spot prices to make their futures positions more profitable. This makes hedging expensive, and therefore reduces futures market participation for agents without market power. Agents without market power may manipulate futures prices by strategically revealing the trades of the monopolist to make their earlier derivatives market positions profitable. As in the case of models with an informed trader instead of a monopolist, we have shown that both strategic and manipulative trading can exist in equilibrium.

Many existing futures markets with imperfectly competitive underlying spot markets exhibit very low levels of participation relative to their importance. In particular, markets for longer term contracts tend to be illiquid. For example, the trading activity in futures markets for oil is relatively low. Furthermore, most trading in crude oil futures is based on West Texas Intermediate or Brent Crude oil. While Middle East crudes account for nearly 30 percent of world oil production, these account for little of the trading in oil futures. By contrast, WTI and Brent, which are produced by smaller, more competitive firms, account for roughly one-fifth of world oil production but the majority of futures market positions. Our paper suggests an explanation based on the imperfectly competitive nature of the oil spot market. Given the moral hazard problems discussed in this paper, several markets – including weather derivatives – have emerged to avoid the inefficiencies caused by market power. Weather derivatives provide an index-hedge against extreme temperatures, and therefore against oil demand risk. Despite large basis risk, these contracts are not susceptible to moral hazard.

A Appendix: Proofs

A.1 Proof of Proposition 1

When submitting C^M, there are 7 ranges the monopolist's order flow could be in. These are categorized according to which possible prices, $k_x(\theta)$, in ranges $A1 - A5$ the monopolist could face, depending upon the realization of $\tilde{N}$. For each range, we derive the monopolist's expected profits associated with the optimal order flow in this range. His expected profits are given by

$$E\left[\pi|C^M\right] = -C^M E\left[k_x(\theta)\,|\tilde{N}, C^M\right] + \frac{1}{4b}E\left[\left(\tilde{a} + bC^M\right)^2\right]$$

M1 $C^M > x + c_x$: always $A1$

$$\begin{aligned} E\left[\pi|C^M\right] &= -C^M \int_0^1 \left(\frac{1}{2}E[\tilde{a}] + \frac{1}{2}b\left(C^M + Nc_x\right)\right) dN + \frac{1}{4b}E\left[\left(\tilde{a} + bC^M\right)^2\right] \\ &= \frac{1}{4b}E\left[\tilde{a}^2\right] - \frac{1}{2}bC^M\left(\frac{1}{2}C^M + \frac{1}{2}c_x\right) \\ &< E[\pi|0] = \frac{1}{4b}E\left[\tilde{a}^2\right] \text{ if } c_x > -C^M \end{aligned}$$

M2 $-x + c_x < C^M \leq x + c_x$: either

(a) $A1$ if $c_x + x - C^M < Nc_x \leq c_x$ or

(b) $A2$ if $0 \leq Nc_x \leq c_x + x - C^M$

$$\begin{aligned} E\left[\pi|C^M\right] &= \begin{bmatrix} -C^M \frac{1}{2}E[\tilde{a}] - C^M \int_{1+\frac{x-C^M}{c_x}}^1 \frac{1}{2}b\left(C^M + c_x N\right) dN \\ -C^M \int_0^{1+\frac{x-C^M}{c_x}} \frac{1}{2}bx dN + \frac{1}{4b}E\left[\left(\tilde{a} + bC^M\right)^2\right] \end{bmatrix} \\ &= \frac{1}{4b}E\left[\tilde{a}^2\right] - \frac{1}{4}bC^M\left(C^M + \frac{\left(x - C^M\right)^2}{c_x}\right) \\ &< E[\pi|0] = \frac{1}{4b}E\left[\tilde{a}^2\right] \text{ if } C^M > 0 \end{aligned}$$

M3 $x < C^M \leq -x + c_x$: either

(a) $A1$ if $c_x + x - C^M < c_x N \leq c_x$ or

(b) $A2$ if $c_x - x - C^M \leq c_x N \leq c_x + x - C^M$ or

(c) $A3$ if $0 \leq c_x N \leq c_x - x - C^M$

$$\begin{aligned} E\left[\pi|C^M\right] &= \begin{bmatrix} -C^M \frac{1}{2}E[\tilde{a}] - C^M \int_{1+\frac{x-C^M}{c_x}}^1 \frac{1}{2}b\left(C^M + c_x N\right) dN \\ -C^M \int_{1-\frac{x+C^M}{c_x}}^{1+\frac{x-C^M}{c_x}} \frac{1}{2}bx dN + \frac{1}{4b}E\left[\left(\tilde{a} + bC^M\right)^2\right] \end{bmatrix} \\ &= \frac{1}{4b}E\left[\tilde{a}^2\right] - \frac{1}{2}bC^M\left(\frac{1}{2}C^M - \frac{1}{2}\frac{x^2 - \left(C^M\right)^2}{c_x} - x + 2\frac{x^2}{c_x}\right) \end{aligned}$$

Furthermore,

$$\frac{dE\left[\pi|C^M\right]}{dC^M} = -\frac{1}{2}b\left(C^M - x + \frac{3}{2}\frac{x^2}{c_x} + \frac{3}{2}\frac{C^M}{c_x}\right) < 0 \;\forall\; C^M > x$$

M4 $-x \leq C^M \leq x$: either

(a) $A2$ if $c_x - x - C^M \leq c_x N \leq c_x$ or

(b) $A3$ if $x - C^M \leq c_x N \leq c_x - x - C^M$ or

(c) $A4$ if $0 \leq c_x N < x - C^M$

$$\begin{aligned} E\left[\pi|C^M\right] &= -C^M \frac{1}{2} E\left[\tilde{a}\right] - C^M \int_{1-\frac{x+C^M}{c_x}}^{1} \frac{1}{2} bx dN + C^M \int_0^{\frac{x-C^M}{c_x}} \frac{1}{2} bx dN + \frac{1}{4b} E\left[\left(\tilde{a} + bC^M\right)^2\right] \\ &= \frac{1}{4b} E\left[\tilde{a}^2\right] + b\left(C^M\right)^2 \left(\frac{1}{4} - \frac{1}{c_x} x\right) \end{aligned}$$

For $0 < x < \frac{1}{4} c_x$ – as in the proposed — $E\left[\pi|C^M\right]$ is maximized at $C^M = x$ and $C^M = -x$.

M5 $x - c_x \leq C^M < -x$: either

(a) $A3$ if $x - C^M \leq c_x N \leq c_x$ or

(b) $A4$ if $-x - C^M \leq c_x N < x - C^M$ or

(c) $A5$ if $0 \leq c_x N < -x - C^M$

By analogy to $M3$, $\frac{d}{dC^M} E\left[\pi|C^M\right] > 0 \ \forall \ C^M < -x$

M6 $-x - c_x \leq C^M < x - c_x$: either

(a) $A4$ if $-x - C^M \leq c_x N < c_x$ or

(b) $A5$ if $0 \leq c_x N < 0 - x - C^M$ or

By analogy to $M2$, $E\left[\pi|C^M\right] < E\left[\pi|0\right]$

M7 $C^M < -x - c_x$: always $A5$

By analogy to $M1$,$E\left[\pi|C^M\right] < E\left[\pi|0\right]$.

Given the values of $E\left[\pi|C^M\right]$ above, $E\left[\pi\right]$ is maximized for $C^M = x$ and $C^M = -x$ if $0 < x < \frac{1}{4} c_x$, so that

$$E\left[\pi|x\right] = E\left[\pi| - x\right] = \frac{1}{4b} E\left[\tilde{a}^2\right] + \frac{1}{4} bx^2 - \frac{b}{c_x} x^3 > E\left[\pi|0\right].$$

Therefore, the monopolist is indifferent between submitting $C^M = x$ and $C^M = -x$, and the market makers can rationally believe that the monopolist randomizes between these two values with equal probability. Given these beliefs, prices are set competitively and no market maker has an incentive to change $k_x(\theta)$.

A.2 Proof of Proposition 2

Risk-averse agents maximize the following objective function:

$$\begin{aligned} c_x^n &= \arg\max_c E\left[u\left(c\left(P^* - k_x\left(\tilde{N} c_x + \tilde{C}_x^M\right)\right) + \alpha - \beta P^*\right)\right] \\ &= \arg\max_c E\left[u\left(c\left(\frac{1}{2}\left(\tilde{a} + b\tilde{C}_x^M\right) - k_x\left(\tilde{N} c_x + \tilde{C}_x^M\right)\right) + \alpha - \beta \frac{1}{2}\left(\tilde{a} + b\tilde{C}_x^M\right)\right)\right] \end{aligned}$$

where the risk averse agent takes as given the order flow submitted by all other risk-averse agents, c_x, and by the monopolist, $\tilde{C}_x^M$. The first and second derivative of expected utility are given by

$$\begin{aligned} \frac{\partial Eu}{\partial c} &= E\left[\left(\frac{1}{2}\left(\tilde{a} + b\tilde{C}_x^M\right) - k_x\left(\tilde{N} c_x + \tilde{C}_x^M\right)\right) u'\left(c\left(\frac{1}{2}\left(\tilde{a} + b\tilde{C}_x^M\right) - k_x\left(\tilde{N} c_x + \tilde{C}_x^M\right)\right) + \alpha - \beta \frac{1}{2}\left(\tilde{a} + b\tilde{C}_x^M\right)\right)\right] \\ \frac{\partial^2 Eu}{\partial c^2} &= E\left[\left(\frac{1}{2}\left(\tilde{a} + b\tilde{C}_x^M\right) - k_x\left(\tilde{N} c_x + \tilde{C}_x^M\right)\right)^2 u''\left(c\left(\frac{1}{2}\left(\tilde{a} + b\tilde{C}_x^M\right) - k_x\left(\tilde{N} c_x + \tilde{C}_x^M\right)\right) + \alpha - \beta \frac{1}{2}\left(\tilde{a} + b\tilde{C}_x^M\right)\right)\right] < 0 \end{aligned}$$

Expected utility of risk averse agents is therefore a concave function in c. For the first derivative we further deduce

$$\frac{\partial Eu}{\partial c} \tag{11}$$

$$= \left[\begin{array}{c} \frac{1}{2}\int E\left[\left(\frac{1}{2}(\tilde{a}+bx)-k_x\left(\tilde{N}c_x+x\right)\right)u'\left(c\left(\frac{1}{2}(\tilde{a}+bx)-k_x\left(\tilde{N}c_x+x\right)\right)+\alpha-\beta\frac{1}{2}(\tilde{a}+bx)\right)\right]2NdN \\ +\frac{1}{2}\int E\left[\left(\frac{1}{2}(\tilde{a}-bx)-k_x\left(\tilde{N}c_x-x\right)\right)u'\left(c\left(\frac{1}{2}(\tilde{a}-bx)-k_x\left(\tilde{N}c_x-x\right)\right)+\alpha-\beta\frac{1}{2}(\tilde{a}-bx)\right)\right]2NdN \end{array} \right]$$

$$= \left[\begin{array}{c} \frac{1}{2}\left(1-\left(1-\frac{2x}{c_x}\right)^2\right)E\left[\left(\frac{1}{2}\tilde{a}-\frac{1}{2}E[\tilde{a}]\right)u'\left(c\left(\frac{1}{2}\tilde{a}-\frac{1}{2}E[\tilde{a}]\right)+\alpha-\beta\frac{1}{2}(\tilde{a}+bx)\right)\right] \\ +\frac{1}{2}\left(1-\frac{2x}{c_x}\right)^2E\left[\left(\frac{1}{2}(\tilde{a}+bx)-\frac{1}{2}E[\tilde{a}]\right)u'\left(c\left(\frac{1}{2}(\tilde{a}+bx)-\frac{1}{2}E[\tilde{a}]\right)+\alpha-\beta\frac{1}{2}(\tilde{a}+bx)\right)\right] \\ +\frac{1}{2}\left(1-\left(\frac{2x}{c_x}\right)^2\right)E\left[\left(\frac{1}{2}(\tilde{a}-bx)-\frac{1}{2}E[\tilde{a}]\right)u'\left(c\left(\frac{1}{2}(\tilde{a}-bx)-\frac{1}{2}E[\tilde{a}]\right)+\alpha-\beta\frac{1}{2}(\tilde{a}-bx)\right)\right] \\ +\frac{1}{2}\left(\frac{2x}{c_x}\right)^2E\left[\left(\frac{1}{2}\tilde{a}-\frac{1}{2}E[\tilde{a}]\right)u'\left(c\left(\frac{1}{2}\tilde{a}-\frac{1}{2}E[\tilde{a}]\right)+\alpha-\beta\frac{1}{2}(\tilde{a}-bx)\right)\right] \end{array} \right]$$

a) Evaluated at $c=\beta$ yields

$$\frac{\partial Eu}{\partial c}|_{c=\beta} = -\frac{bx^2}{c_x}\left(1-\frac{2x}{c_x}\right)u'\left(\alpha-\frac{1}{2}\beta E[\tilde{a}]\right) < 0$$

for all $0<x<\frac{1}{2}c_x$ which implies $c_x^n<\beta$ for all $0<x<\frac{1}{2}c_x$.

b) Note that when $c_x=0$ and $x=0$, then $c_x^n=\beta>0$. Since c_x^n is smooth in x, $c_x^n>0$ for sufficiently small values of x.

c) The FOC (11) above defines $c_x^n=c_x^n(c_x)$. We have to show that for some $x>0$ there exists c_x such that

$$c_x^n(c_x)=c_x. \tag{12}$$

Note that (12) is satisfied for $x=0$ and $c_x=\beta$. From above we know that $c_x^n(\beta)<\beta$ and $c_x^n(0)>0$ for sufficiently small x. Since $c_x^n(c_x)$ – as defined by the FOC (11) – is smooth in c_x, there exists c_x with $0<c_x<\beta$ such that $c_x^n(c_x)=c_x$.

A.3 Proof of Proposition 4

A.3.1 Monopolist

Here we consider optimal behavior by the monopolist given his beliefs about the manipulator's behavior.

We showed that at $t=2$ the monopolist maximizes his profits by setting price and quantity as $P^*=\frac{1}{2}\left(a+bC_1^M\right)$ and $Q^*=\frac{1}{2b}\left(a-bC_1^M\right)$. At $t=1$, the monopolist maximizes expected profits given the price schedule he faces and the beliefs he holds about the manipulator's trading behavior. His objective function at $t=1$ depends on the aggregate order flow $\theta_0=C_0^H+C_0^N$ at $t=0$.

In SG1, if $\theta_0>C_0^{N+}-x$, the monopolist's expected profits can be found by substituting aggregate order flow ($\theta_1=C_1^M+\tilde{C}_1^H$) and the strike price from equation (8) into equation (6) and taking expectations:

$$\begin{aligned} E\left[\pi^M\right] &= E\left[-C_1^Mk_1+\frac{1}{4b}\left(\tilde{a}+bC_1^M\right)^2\right] \\ &= E\left[-C_1^M\left(\frac{1}{2}E[\tilde{a}]+\frac{1}{2}b(\theta_1-x)\right)+\frac{1}{4b}\left(\tilde{a}+bC_1^M\right)^2\right] \\ &= E\left[-C_1^M\left(\frac{1}{2}E[\tilde{a}]+\frac{1}{2}b\left(C_1^M+\tilde{C}_1^H-x\right)\right)+\frac{1}{4b}\left(\tilde{a}+bC_1^M\right)^2\right]. \end{aligned}$$

The FOC is

$$\frac{\partial E\left[\pi^M\right]}{\partial C_1^M} = E\left[-\frac{1}{2}b\left(\tilde{C}_1^H-x\right)-\frac{1}{2}bC_1^M\right]=0.$$

Note that given $\theta_0 > C_0^{N+} - x$ and the beliefs about the manipulator's trade at $t = 0$ we have $C_1^H = +x$, which implies $C_1^M = 0$. The SOC

$$\frac{\partial^2 E\left[\pi^M\right]}{\partial \left(C_1^M\right)^2} = -\frac{1}{2}b < 0$$

is satisfied.

In SG2, if $C_0^{N-} + x \leq \theta_0 \leq C_0^{N+} - x$, the monopolist's expected profits can be found by substituting aggregate order flow ($\theta_1 = C_1^M + \tilde{C}_1^H$) and the strike price from equation (9) into equation (6) and taking expectations:

$$\begin{aligned} E\left[\pi^M\right] &= E\left[-C_1^M k_1 + \frac{1}{4b}\left(\tilde{a} + bC_1^M\right)^2\right] \\ &= E\left[-C_1^M\left(\frac{1}{2}E\left[\tilde{a}\right] + \frac{1}{4}b\theta_1\right) + \frac{1}{4b}\left(\tilde{a} + bC_1^M\right)^2\right] \\ &= E\left[-C_1^M\left(\frac{1}{2}E\left[\tilde{a}\right] + \frac{1}{4}b\left(C_1^M + \tilde{C}_1^H\right)\right) + \frac{1}{4b}\left(a + bC_1^M\right)^2\right] \\ &= \frac{1}{4b}E\left[\tilde{a}^2\right] - \frac{1}{4}bC_1^M E\left[\tilde{C}_1^H\right] \end{aligned}$$

The FOC is

$$\frac{\partial E\left[\pi^M\right]}{\partial C_1^M} = E\left[-\frac{1}{4}b\tilde{C}_1^H\right] = 0.$$

Note that the monopolist believes that the manipulator will randomize between $+\frac{1}{2}x$ and $-\frac{1}{2}x$ with equal probability, so that $E\left[\tilde{C}_1^H\right] = 0$, and $\frac{\partial E\left[\pi^M\right]}{\partial C_1^M} = 0$. Therefore, the monopolist is indifferent between submitting any order flow and therefore willing to submit $C_1^M = +\frac{1}{2}x$ and $C_1^M = -\frac{1}{2}x$ with equal probability.

In SG3, if $\theta_0 < C_0^{N-} + x$, the monopolist's expected profits can be found by substituting aggregate order flow ($\theta_1 = C_1^M + \tilde{C}_1^H$) and the strike price from equation (10) into equation (6) and taking expectations:

$$E\left[\pi^M\right] = E\left[-C_1^M\left(\frac{1}{2}a + \frac{1}{2}b\left(C_1^M + \tilde{C}_1^H + x\right)\right) + \frac{1}{4b}\left(a + bC_1^M\right)^2\right]$$

In this case, the FOC is

$$\frac{\partial E\left[\pi^M\right]}{\partial C_1^M} = E\left[-\frac{1}{2}b\left(\tilde{C}_1^H + x\right) - \frac{1}{2}bC_1^M\right] = 0.$$

Note that given $\theta_0 < C_0^{N-} + x$ and the beliefs about the manipulator's trade at $t = 0$ we have $C_1^H = -x$. This implies $C_1^M = 0$. The SOC

$$\frac{\partial^2 E\left[\pi^M\right]}{\partial \left(C_1^M\right)^2} = -\frac{1}{2}b < 0$$

is satisfied. We have thus shown that the monopolist has no incentive to deviate from the proposed order flow given the price schedule and his beliefs about the manipulator's actions. Next, we examine the optimal behavior of the manipulator.

A.3.2 Manipulator

At $t = 1$ the manipulator submits an order flow to maximize his expected profits which depend on the aggregate order flow $\theta_0 = C_0^H + C_0^N$ at $t = 0$. His expected profits are the sum of his profits at times $t = 0$ and $t = 1$:

$$E\left[\pi^H\right] = E\left[C_0^H\left(k_1 - k_0\right) + C_1^H\left(P^* - k_1\right)\right]. \tag{13}$$

In SG1, if $\theta_0 > C_0^{N+} - x$, the manipulator's expected profits can be found by substituting the monopolist's optimal spot price from equation (2), the strike prices from equations (8) and (7), and aggregate order flow

$(\theta_1 = \tilde{C}_1^M + C_1^H = C_1^H)$ into equation (13):

$$\begin{aligned}
E\left[\pi^H|SG1\right] &= E\left[C_0^H(k_1-k_0)+C_1^H(P^*-k_1)\right] \\
&= E\left[C_0^H\left(\frac{1}{2}E[\tilde{a}]+\frac{1}{2}b(\theta_1-x)-\frac{1}{2}E[\tilde{a}]\right)+C_1^H\left(\frac{1}{2}\left(a+bC_1^M\right)-\left(\frac{1}{2}E[\tilde{a}]+\frac{1}{2}b(\theta_1-x)\right)\right)\right] \\
&= E\left[\frac{1}{2}b\left(C_0^H-C_1^H\right)(\theta_1-x)+C_1^H\left(\frac{1}{2}\left(bC_1^M\right)\right)\right] \\
&= E\left[\frac{1}{2}b\left(C_0^H-C_1^H\right)\left(C_1^H-x\right)\right].
\end{aligned}$$

The FOC is

$$\frac{\partial E\left[\pi^H|SG1\right]}{\partial C_1^H} = E\left[\frac{1}{2}b\left(C_0^H-2C_1^H+x\right)\right]=0$$

and the SOC is satisfied. This implies $C_1^H = \frac{1}{2}\left(x+C_0^H\right)$. Note that if the manipulator does randomize between $+x$ and $-x$ at $t=0$, then $\theta_0 > C_0^{N+}-x$ is only true if $C_0^H = x$. Thus $C_1^H = x$ and his profits are $E\left[\pi^H|SG1\right]=0$.

In SG2, if $C_0^{N-}+x \leq \theta_0 \leq C_0^{N+}-x$, the manipulator's expected profits can be found by substituting the monopolist's optimal spot price from equation (2), the strike prices from equations (9) and (7), and aggregate order flow $(\theta_1 = \tilde{C}_1^M + C_1^H)$ into equation (13):

$$\begin{aligned}
E\left[\pi^H|SG2\right] &= E\left[C_0^H(k_1-k_0)+C_1^H(P^*-k_1)\right] \\
&= E\left[C_0^H\left(\frac{1}{2}E[\tilde{a}]+\frac{1}{4}b\theta_1-\left(\frac{1}{2}E[\tilde{a}]\right)\right)+C_1^H\left(\frac{1}{2}\left(a+b\tilde{C}_1^M\right)-\left(\frac{1}{2}E[\tilde{a}]+\frac{1}{4}b\theta_1\right)\right)\right] \\
&= E\left[\frac{1}{4}bC_0^H\left(\tilde{C}_1^M+C_1^H\right)+\frac{1}{4}bC_1^H\left(\tilde{C}_1^M-C_1^H\right)\right].
\end{aligned}$$

The FOC is

$$\frac{\partial E\left[\pi^H|SG2\right]}{\partial C_1^H} = \frac{1}{4}b\left(C_0^H+E\left[\tilde{C}_1^M\right]-2C_1^H\right)=0$$

and the SOC is satisfied. This implies $C_1^H = \frac{1}{2}\left(C_0^H+E\left[\tilde{C}_1^M\right]\right)$. Note that if the manipulator believes that the monopolist randomizes between $+\frac{1}{2}x$ and $-\frac{1}{2}x$ with equal probability at $t=0$, then $C_1^H = \frac{1}{2}C_0^H$. His profits are then

$$E\left[\pi^H|SG2\right] = \frac{1}{16}b\left(C_0^H\right)^2 > 0.$$

In SG3, if $\theta_0 < C_0^{N-}+x$, the manipulator's expected profits can be found by substituting the monopolist's optimal spot price from equation (2), the strike prices from equations (10) and (7), and aggregate order flow $(\theta_1 = \tilde{C}_1^M + C_1^H = C_1^H)$ into equation (13):

$$E\left[\pi^H|SG3\right] = E\left[\frac{1}{2}b\left(C_1^H+x\right)\left(C_0^H-C_1^H\right)\right].$$

The FOC is

$$\frac{\partial E\left[\pi^H|SG3\right]}{\partial C_1^H} = E\left[\frac{1}{2}b\left(C_0^H-2C_1^H-x\right)\right]=0$$

and the SOC is satisfied. This implies $C_1^H = \frac{1}{2}\left(-x+C_0^H\right)$. Again, if the manipulator randomizes between $+x$ and $-x$ at $t=0$, then $\theta_0 < C_0^{N-}+x$ is only true if $C_0^H=-x$. Therefore, $C_1^H=-x$ and his profits are $E\left[\pi^H|SG3\right]=0$.

At $t=0$ the manipulator submits an order flow C_0^H to maximize his expected profits given optimal behavior in subsequent periods.

If $C_0^H \geq 0$ then the game is either in SG1 if $C_0^N > C_0^{N+}-x-C_0^H$ or in SG2 if $C_0^N \leq C_0^{N+}-x-C_0^H$. (If $C_0^H < 0$ then the game is either in SG3 if $C_0^N < C_0^{N-}+x-C_0^H$ or in SG2 if $C_0^N \geq C_0^{N-}+x-C_0^H$.)

Expected profits of the manipulator are

$$E\left[\pi^H\right] = \begin{cases} \frac{1}{C_0^{N+}-C_0^{N-}}\left(\int_{C_0^{N+}-x-C_0^H}^{C_0^{N+}} E\left[\pi^H|SG1\right]dC_0^N + \int_{C_0^{N-}}^{C_0^{N+}-x-C_0^H} E\left[\pi^H|SG2\right]dC_0^N\right) & \text{if } C_0^H \geq 0 \\ \frac{1}{C_0^{N+}-C_0^{N-}}\left(\int_{C_0^{N-}+x-C_0^H}^{C_0^{N+}} E\left[\pi^H|SG2\right]dC_0^N + \int_{C_0^{N-}}^{C_0^{N-}+x-C_0^H} E\left[\pi^H|SG3\right]dC_0^N\right) & \text{if } C_0^H < 0 \end{cases}.$$

Note that expected profits in SG1 and SG3 are zero since the manipulator is spotted. His overall expected profits are therefore the product of the probability that the game is in SG2 ($1-\left(x+C_0^H\right)/\left(C_0^{N+}-C_0^{N-}\right)$ and $1-\left(x-C_0^H\right)/\left(C_0^{N+}-C_0^{N-}\right)$ for $C_0^H \geq 0$ and $C_0^H < 0$, respectively) and his expected profits in SG2 ($\frac{1}{16}b\left(C_0^H\right)^2$):

$$E\left[\pi^H\right] = \begin{cases} \left(1-\left(x+C_0^H\right)/\left(C_0^{N+}-C_0^{N-}\right)\right)\cdot\frac{1}{16}b\left(C_0^H\right)^2 & \text{if } C_0^H \geq 0 \\ \left(1-\left(x-C_0^H\right)/\left(C_0^{N+}-C_0^{N-}\right)\right)\cdot\frac{1}{16}b\left(C_0^H\right)^2 & \text{if } C_0^H < 0 \end{cases}.$$

The first and second derivative of expected profits with respect to C_0^H are

$$\frac{\partial E\left[\pi^H\right]}{\partial C_0^H} = \begin{cases} \frac{2\left(C_0^{N+}-C_0^{N-}-x\right)-3C_0^H}{C_0^{N+}-C_0^{N-}}\cdot\frac{1}{16}bC_0^H & \text{if } C_0^H \geq 0 \\ \frac{2\left(C_0^{N+}-C_0^{N-}-x\right)+3C_0^H}{C_0^{N+}-C_0^{N-}}\cdot\frac{1}{16}bC_0^H & \text{if } C_0^H < 0 \end{cases}$$

and

$$\frac{\partial^2 E\left[\pi^H\right]}{\partial\left(C_0^H\right)^2} = \begin{cases} \frac{2\left(C_0^{N+}-C_0^{N-}-x\right)-6C_0^H}{C_0^{N+}-C_0^{N-}}\cdot\frac{1}{16}b & \text{if } C_0^H \geq 0 \\ \frac{2\left(C_0^{N+}-C_0^{N-}-x\right)+6C_0^H}{C_0^{N+}-C_0^{N-}}\cdot\frac{1}{16}b & \text{if } C_0^H < 0 \end{cases}.$$

This implies that there are two local maxima at $C_0^H = \frac{2}{3}\left(C_0^{N+}-C_0^{N-}-x\right)$ and $C_0^H = -\frac{2}{3}\left(C_0^{N+}-C_0^{N-}-x\right)$ (the second derivative is negative at these values) and a local minimum at $C_0^H = 0$ (the second derivative is positive at this value). The two local maxima are in fact global maxima since $\lim_{C_0^H\to+\infty} E\left[\pi^H\right] = \lim_{C_0^H\to-\infty} E\left[\pi^H\right] = -\infty$ and

$$E\left[\pi^H|C_0^H = \frac{2}{3}\left(C_0^{N+}-C_0^{N-}-x\right)\right] = E\left[\pi^H|C_0^H = -\frac{2}{3}\left(C_0^{N+}-C_0^{N-}-x\right)\right] = \frac{1}{108}b\frac{\left(C_0^{N+}-C_0^{N-}-x\right)^3}{C_0^{N+}-C_0^{N-}}.$$

In equilibrium, $x = C_0^H$ and expected profits are maximized at $C_0^H = +x = \frac{2}{5}\left(C_0^{N+}-C_0^{N-}\right)$ and $C_0^H = -x = -\frac{2}{5}\left(C_0^{N+}-C_0^{N-}\right)$ with value

$$E\left[\pi^H|C_0^H = \frac{2}{5}\left(C_0^{N+}-C_0^{N-}\right)\right] = E\left[\pi^H|C_0^H = -\frac{2}{5}\left(C_0^{N+}-C_0^{N-}\right)\right] = \frac{1}{500}b\left(C_0^{N+}-C_0^{N-}\right)^2 > 0.$$

Maximized expected profits are the same if the manipulator submits $+x$ or $-x$, so he will be willing to randomize with equal probability.

A.3.3 Market Makers

Market makers beliefs are consistent with the actions of the noise traders, the monopolist, and the manipulator. Since prices are set competitively, no market maker has an incentive to change $k_0(.)$ and $k_1(.\mid\theta_0)$.

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