Competition Between Internet Search Engines

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Abstract

We develop a model of vertical differentiation in the Internet search engine market. A key property of the model is that users who try out one engine may be dissatisfied with the results, and consult another engine in the same session. This residual demand allows lower quality engines to survive in equilibrium. We consider a two-period game between an incumbent and an entrant who enters in the second period. Since users prefer to try out a higher quality engine first, the demand for an engine is discontinuous in quality, depending on whether the engine is a leader or a follower. We take into account brand loyalty for the incumbent. The interaction of brand loyalty and a cost advantage for the entrant determines which engine is the leader in equilibrium.

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1 Introduction

Internet search engines have emerged as the gatekeepers of electronic commerce. Internet users often begin their session by visiting these engines. The widespread use of these portal sites and their central role as information disseminators has led to high stock price valuations for these firms.[1]

We present a model of vertical differentiation in the search engine market, using two key properties of search engines. First, these engines are, for the most part, offered free to consumers (that is, consumers do not directly pay for the use of the engine). Whereas users do incur a cost in using a search engine (in terms of time and effort), the revenues for these engines typically come from advertisers rather than consumers. Second, users often sample more than one search engine during a single session. On any given visit to a search engine, a user may be dissatisfied with the output. That is, the user may not find a specific site she is searching for, or may not find the information she is looking for. This often induces a visit to another search engine, which restarts the search process.

As Lawrence and Giles (1998) and Bradlow and Schmittlein (1999) mention, search engines maintain databases that contain only a fraction of the information in the universe in their database. Therefore, on any given search, there is a probability that a user is not satisfied with the results. Telang, Mukhopadhyay and Wilcox (2000) show that, although people are loyal to the engines they use, they are not averse to switching when dissatisfied with the results.

Therefore, we allow consumers in our model to use more than one engine in a given period. This represents a point of departure from earlier work on vertical

[1]For example, the stock price of Yahoo!, at its peak, had increased about 3,500% over its initial public offering price of $5.50 per share, over three years ago. In the same period, Excite had seen over a 1,000% increase

differentiation, such as Mussa and Rosen (1978), and Shaked and Sutton (1982, 1983), and leads to some new insights into the market structure of this industry. In most of the earlier work, vertical differentiation is tied to price differentiation. Lower quality products cost less, offering the consumer a tradeoff between quality and price.

In our model, even though consumers incur the same costs for using any search engine,[2] engines of different qualities can survive. If there is no brand loyalty, all consumers choose the highest quality engine. This corresponds to the consumers choosing the highest quality good in the Shaked and Sutton (1982) model, when all goods are offered at the same price. However, in the latter model, this would imply that no lower quality good could exist. In our model, with some probability, consumers then go on to sample a lower quality engine. This residual demand can be high enough to allow a lower quality engine to survive.

We consider a two-period model, with an incumbent present in both periods and an entrant who enters in the second period. In equilibrium, the entrant may be either the leader or the follower (that is, it may have a higher or lower quality), depending on the degree of brand loyalty for the incumbent and the cost advantage of the entrant. When the entrant is a follower, it survives only on residual demand: all users use the higher quality engine first, and a proportion of dissatisfied users switch to the lower quality one.

Technology costs have been rapidly falling in this sector. A late entrant can enter with the newest technology, whereas an incumbent is locked into an earlier technology. Thus the entrant may have a cost advantage over the incumbent. The issue of a strategic investment in technology and its timing has been studied in the Information Systems literature by, among others, Barua, Kriebel, and Mukhopadhyay (1991) and Clemons (1996, 1997). In our model, we study the strategic interaction

[2]Note that these costs are modeled only implicitly, via a demand curve.

of first mover advantage through brand loyalty for the incumbent and technology innovation by the entrant. As we show, the incumbent may strategically choose a quality that forces the entrant to come in as a follower. However, if the cost advantage of the entrant is too high, it may dominate the effects of brand loyalty and the entrant may emerge a leader in equilibrium.

While we do not model switching costs explicitly, their presence can explain brand loyalty for a particular product. Klemperer (1987a,b) and Farrell and Shapiro (1988) demonstrate that switching costs weaken price competition. Gabszewicz, Pepall and Thisse (1992) extend this framework by introducing learning costs for consumers. Carroll (1987) suggests that switching costs due to learning exist in human-computer interaction.

Switching costs allow an incumbent to charge a higher price than otherwise feasible to repeat purchasers. In our model, however, price is not a strategic variable. As with other information goods, consumers do not directly pay providers. Instead, the most important source of revenues for such firms is advertising, which depends on the number of users who visit the particular web site (see, for example, Hoffman and Novak, 1996, and Chatterjee, Hoffman and Novak, 1995). This is akin to the modes of payment for network television programming, considered, for example, by Steiner (1952) and and Beebe (1977). We note that television viewers may also switch from one channel to another. However, television is mainly a broadcast medium whereas the Internet and search engines are interactive in nature.

The rest of this paper is as follows. Section 2 describes the model in detail, and presents a preliminary result that helps us find the equilibrium of the game. In Section 3, we determine the equilibrium when the entrant has no cost advantage, and show that the incumbent is the leader. We then consider the effects of a cost advantage for the entrant and brand loyalty for the incumbent. Section 4 considers the case of low brand loyalty, and Section 5 that of high brand loyalty. Managerial implications of our findings are presented in 6, with some concluding remarks in Section 7. All proofs are relegated to the Appendix, Section 8.

2 Model

Two firms compete in quality in the search engine market. We interpret quality as the ability to satisfy a user. This definition of quality includes the two common attributes of quality amongst search engines:[3] (1) the quantity of information retrieved, and (2) its relevance to the user. Our definition of quality may be interpreted as a reduced form notion that encompasses both these attributes. A higher q is more valued by the user, because it implies either a higher quantity of information, or information of greater relevance, or both.

Using a search engine requires both time and effort on the part of a consumer. Therefore, the lower the quality of an engine, the less likely it is that a consumer will be willing to invest her time and effort. Hence, we model demand for the search engine as increasing in quality. If a firm is a monopolist and offers quality q, the demand in a given period is $D(q)$. We assume that the demand curve is linear, with $D(q) = a + bq$, where $a \leq 0$ (so that a firm offering a zero quality has no customers) and $b > 0$.

Our model encompasses two periods, 0 and 1. The demand curve is assumed to be the same in each period. In period 0, firm 1 (the incumbent) is the only firm in the market, and chooses a quality level q_0. In period 1, firm 2 (the entrant) enters, and offers quality level q_2. The incumbent may increase its quality level from q_0 to q_1. However, we assume that quality cannot decrease, so that $q_1 \geq q_0$.

[3] See, for example, www.searchenginewatch.com.

Setting up a search engine incurs significant hardware and software development costs (see, for example, Jones and Mendelson, 1998). Typically, an engine has web robots crawl the world-wide web, and index the information in a local database. When a user inputs a search term, queries are run on the local database, and results are returned. We assume that the databse updating cost is small, so that engine 1 can remain at its original quality level q_0. Hence, we restrict q_1 to be at least as high as q_0.

Firm 1 has a cost function $C(q)$ that is assumed to be strictly increasing and strictly convex. Firm 2 has a cost function $\lambda C(q)$, where λ ranges between zero and 1. When $\lambda = 1$, neither firm has a cost advantage. When $\lambda < 1$, the entrant has a cost advantage. λ captures the idea that the costs associated with technology are constantly falling (or, alternatively, that the technology itself is improving). The incumbent, since it established its infrastructure in period 0, is locked into the old technology. These costs are one-time set-up costs that enable repeated use of the search engine.

On any given search, a user of a search engine is satisfied with the result with probability $p(q)$. Suppose a user in period 1 tries out the engine of firm 1 first. Then, with probability $p(q_1)$, she finds the site she needs and her search ends. With probability $(1 - p(q_1))$, she is dissatisfied with the results from the first search engine. Now, if the engine of firm 2 is of sufficiently high quality, she proceeds to use it. Otherwise, her search remains unsuccessful.

To facilitate analytical solutions, we assume that $p(q) = q$, and $C(q) = kq^2$. Parameter restrictions are imposed to insure that chosen quality levels satisfy $0 < q < 1$. As shown below, firms' demand and profit functions are discontinuous, depending on whether the firm is a leader or a follower. Therefore, though equilibria exist with more general probability or cost functions, characterizing these is more difficult.

Revenues of each firm are directly linked to its demand; we assume that a firm earns an advertising revenue of r in each period per consumer that visits it. As mentioned by Hoffman and Novak (1996), advertising rates depend on the number of "hits" a web site obtains. For convenience, we define $c = \frac{k}{r}$, and renormalize the profit function by dividing by r. We can then state revenues in a period as simply the demand, and the cost as cq^2. This renormalization affects the level of profit earned by a firm, but not its profit-maximizing quality level (i.e., its strategy). In the analysis below, our main concern is with the strategies of the firms, and whether profits are greater or less than zero. Neither of these are affected by the renormalization.

We allow for brand loyalty in the search engine market. In the marketing literature, loyalty has been shown to be an important factor in brand choices.[4] Among Internet related products, Telang et.al. (2000) show that loyalty affects users' search engine choices and Smith et. al. (1999) report similar results for on-line book stores. In period 0, $D(q_0) = a + bq_0$ consumers use firm 1, as it is the only search engine in the market. Of these consumers, a fraction $\beta \in [0, 1]$ remain loyal and return to use engine 1 in period 1, regardless of q_1 and q_2. In our model, we analyze the strategic interaction between the firms for different levels of β.

To illustrate the demand of both firms in the presence of brand loyalty, consider the following two cases.

Case 1:

Let $q_2 > q_1 > q_0$. Then, firm 2 is the leader and firm 1 the follower in period 1. Figure 1 shows the demand curve.

Consider the demand faced by firm 2 in period 1. The users $D_2 - D_0 = b(q_2 - q_0)$ are all new users in this period (that is, they did not use a search engine at time 0). We assume that all new users in period 1 use the higher quality engine first. In this

[4] Guadagni and Little (1983) is a seminal paper in this field.

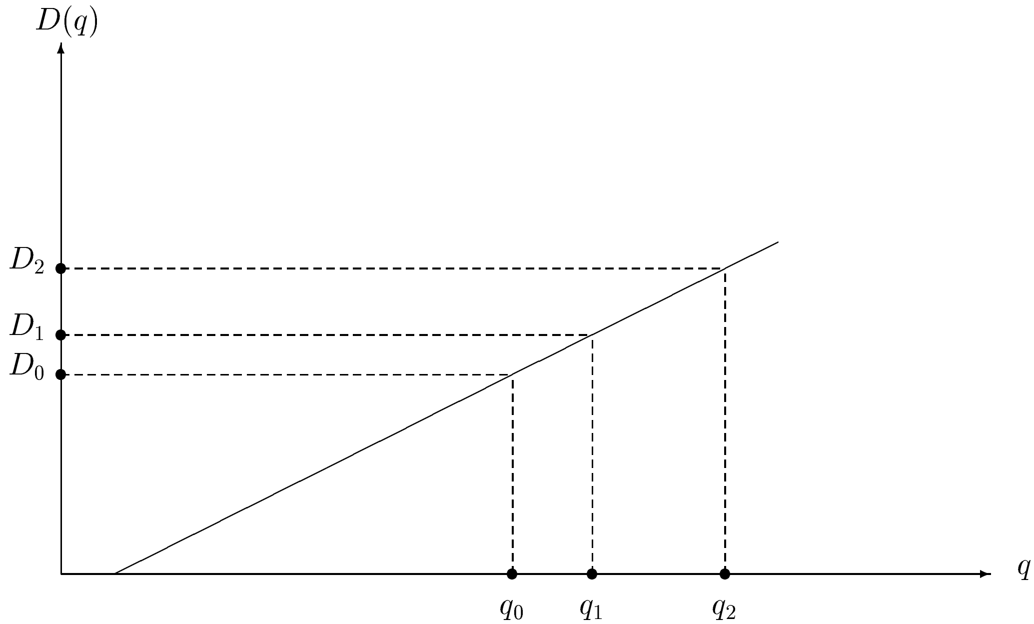

Figure 1: **Consumer demand when** $q_2 > q_1 > q_0$**.**

case, since $q_2 > q_1$, all these users use engine 2 as their first choice in period 1. In addition, of the users D_0 who used engine 1 in period 0, the fraction $1-\beta$ are disloyal, and switch to the higher quality engine in period 1. Hence, the total demand for firm 2 as a first choice is $(1-\beta)(a+bq_0)+b(q_2-q_0)$. With probability $(1-q_2)$, each of these users will be dissatisfied with the output of firm 2. Users in the region $[D_1, D_2]$ end their search, since q_1 is too low for them. The remainder will switch to firm 1.

Of the period 0 users, D_0, the fraction β continue to use engine 1 as the first choice engine in period 1. These users switch to engine 2 if dissatisfied (that is, with probability $(1-q_1)$). Table 1 represents the demand for both engines in period 1 alone.

The profit of firm 2 is

$$\pi_2^l(q_0, q_1, q_2) \quad = \quad D_2(q_0, q_1, q_2) - C(q_2) = a(1-\beta q_1) + b(q_2 - \beta q_0 q_1) - \lambda c q_2^2, \quad (1)$$

	First Choice	Second Choice	Total Users
Firm 1	$\beta(a+bq_0)$	$(1-q_2)\left((1-\beta)(a+bq_0)+ b(q_1-q_0)\right)$	$a(1-q_2+\beta q_2)+ b(q_1+q_0(\beta q_2-q_2))$
Firm 2	$(1-\beta)(a+bq_0)+ b(q_2-q_0)$	$(1-q_1)\beta(a+bq_0)$	$a(1-\beta q_1)+ b(q_2-\beta q_0 q_1)$

Table 1: **Demand for firms in period 1**

where we use the normalization $c=\frac{k}{r}$. The profit of firm 1 is the sum of its profits in periods 0 and 1, with no discounting.[5] The cost of firm 1 is cq_0^2 in period 0, and $c(q_1^2-q_0^2)$ in period 1, for a total cost of cq_1^2. Its demand in period 0 is $a+bq_0$, and demand in period 1 comes from the above table. Hence,

$$\pi_1^f(q_0,q_1,q_2) \quad = \quad a(2-q_2+\beta q_2)+b(q_1+q_0(1+\beta q_2-q_2))-cq_1^2. \tag{2}$$

Case 2:

Now firm 1 is the leader in period 1, and firm 2 the follower. In particular, suppose that $q_1>q_2$, and $q_1\geq q_0$, where q_0 can be greater or less than q_2. Then, the first and second choice demands for the search engines change in an appropriate manner. The profit function of firm 1 is now determined to be

$$\pi_1^l(q_0,q_1,q_2) \quad = \quad (a+bq_0)+(a+bq_1)-cq_1^2. \tag{3}$$

Notice that, in this situation, all users who used engine 1 in period 0 come back to it as the first choice in period 1, regardless of β, since it is the higher quality engine.

[5]Adding a discount factor affects the levels of profit, but does not change any of the key insights provided by the model. When period 1 profits are discounted, firm 1 is less eager to be a leader at time 1. This changes the parameter regions in which different equilibria emerge, but not the characteristics of those equilibria.

Hence, there is no residual demand for firm 1: any user dissatisfied with the output of engine 2 has already tried engine 1, and found it unsatisfactory. The profit function of firm 2 is

$$\pi_2^f(q_0, q_1, q_2) \quad = \quad (1 - q_1)(a + bq_2) - \lambda c q_2^2. \tag{4}$$

If $q_2 = q_1$, then firms are assumed to share new customers equally (that is, those consumers in the region $b(q_1 - q_0)$). As before, of the experienced consumers, in the region $a + bq_0$, the fraction β use engine 1 as the first choice.

Notice the discontinuity in the profit functions associated with being a leader instead of a follower.[6] This discontinuity has the same flavor as the discontinuity in the Bertrand model of duopoly. The difference is that, in a standard Bertrand model, if one firm has a higher price than the other, its demand falls to zero. In our model, if one firm has a lower quality than the other, its demand does fall, but can remain strictly positive because no firm can perfectly satisfy its consumers. Dissatisfied consumers will try out other search engines. This feature of the search engine market, we argue, is precisely one of the reasons that allows a large number of search engines of differing qualities to remain in the market.

The solution concept that we use is subgame perfect equilibrium. This is a Nash equilibrium with the additional property that firms behave optimally at period 1, after any q_0 chosen by firm 1 in period 0.[7] First, we show that, in equilibrium, firm 1 sets $q_1 = q_0$. Suppose, instead, that $q_1 > q_0$. Then, firm 1 incurs a total cost of cq_1^2 over the two periods. Increasing first period quality to q_1 leads to additional demand (hence additional revenue) in period 0, at no extra cost (since the amount $c(q_1^2 - q_0^2)$

[6] This discontinuity implies, among other things, that in equilibrium it cannot be that $q_1 = q_2$. If $q_1 = q_2$, each firm has an incentive to increase its quality by $\epsilon > 0$.

[7] See, for example, Fudenberg and Tirole, 1991, page 74.

is incurred anyway in period 1). Hence, a strategy that has $q_0 < q_1$ is dominated by one that has $q_0 = q_1$.

Note that the proposition below holds for all values of λ and β. All proofs are in the appendix, in Section 8.

Proposition 1 *In any pure strategy subgame perfect equilibrium, the incumbent will set $q_1 = q_0$.*

For the rest of the paper, therefore, we set $q_1 = q_0$. Therefore, in equilibrium, there will be new users in period 1 (that is, users who did not attempt a search in period 0) only if $q_2 > q_1$. By assumption, these users will first visit the engine of firm 2. Relaxing this assumption, therefore, would make it more difficult for firm 2 to emerge as a leader in equilibrium. That is, it will need a higher cost advantage, or a lower brand loyalty on the part of period 0 users, to be the leader.

Next, we define the quality levels the incumbent and entrant would offer if each were a monopolist. Let q_1^m be firm 1's monopoly level, and q_2^m that of firm 2. Then, q_1^m solves the problem $\max_q \pi_1(q) = 2(a + bq) - cq^2$. The first order condition is $2bq - 2cq = 0$, from which we have $q_1^m = \frac{b}{c}$. The monopoly level of the entrant, q_2^m, is found as the solution to $\max_q \pi_2(q) = a + bq - \lambda cq^2$. The first order condition is $bq - 2\lambda cq = 0$, from which we have $q_2^m = \frac{b}{2\lambda c}$.

We maintain the following assumptions on the parameters:

Assumption 1 (i) $\lambda c > (a + b)$. (ii) $a + \frac{b^2(1-\frac{b}{c})}{4c} > 0$.

Part (i) of the assumption ensures that the cost stays within a range such that the entrant will not be profitable at a quality of 1. Therefore, it will choose a quality strictly less than 1. Since $\lambda \leq 1$ in this model, this also ensures that the incumbent will not choose a quality of 1. Part (ii) ensures that the entrant makes a strictly positive profit when $\lambda = 1$, the incumbent offers its monopoly quality $q_1^m = \frac{b}{c}$, and the entrant plays a best response. If λ is strictly less than 1, then this condition is more likely to be satisfied, so it is sufficient to assume it at $\lambda = 1$. As explained below, this condition is required in Proposition 2.

3 No Cost Advantage for Entrant

First, consider the case of $\lambda = 1$; that is, the entrant has no cost advantage over the incumbent. Under the assumptions made, we show that the incumbent, firm 1, will offer its monopoly quality level, and the entrant will be the follower at time 1. In this situation, the presence of the entrant does not affect firm 1 at all. It continues to offer a monopoly quality and earn a monopoly profit. However, firm 2 still finds it profitable to enter the market, because of demand from customers of firm 1 that are dissatisfied with the results of their search query.

Proposition 2 *Suppose $\lambda = 1$. Then, the incumbent sets $q_1 = q_0 = q_1^m$, and the entrant chooses $q_2 = \frac{b(1-q_1^m)}{2c} < q_1$.*

This proposition holds for all values of β. When $\beta = 1$, the incumbent enjoys complete brand loyalty, and is not affected by the entrant's strategy. However, even at the other extreme of zero brand loyalty, when $\beta = 0$, the incumbent has a first-mover advantage, and the entrant is the follower in equilibrium. When the entrant is a follower, its demand (and profit) are unaffected by β: all consumers try out engine 1 before they come to the entrant.

Part (ii) of Assumption 1 is necessary to ensure that the entrant can enter the market at a low quality level and still make a positive profit. If this assumption is violated, then there is no quality level at which the entrant earns a profit, and it chooses to just stay out of the market.

To demonstrate the nature of the strategic interaction between the firms, we consider a numeric example and depict the best response of the entrant to every possible $q \in [0,1]$ that the incumbent might offer. This example represents a base case, and all other examples in the paper consider variants of the same parameters. The parameters for this example are $a = -0.05, b = 1, c = 2, \lambda = 1, \beta = 0$.

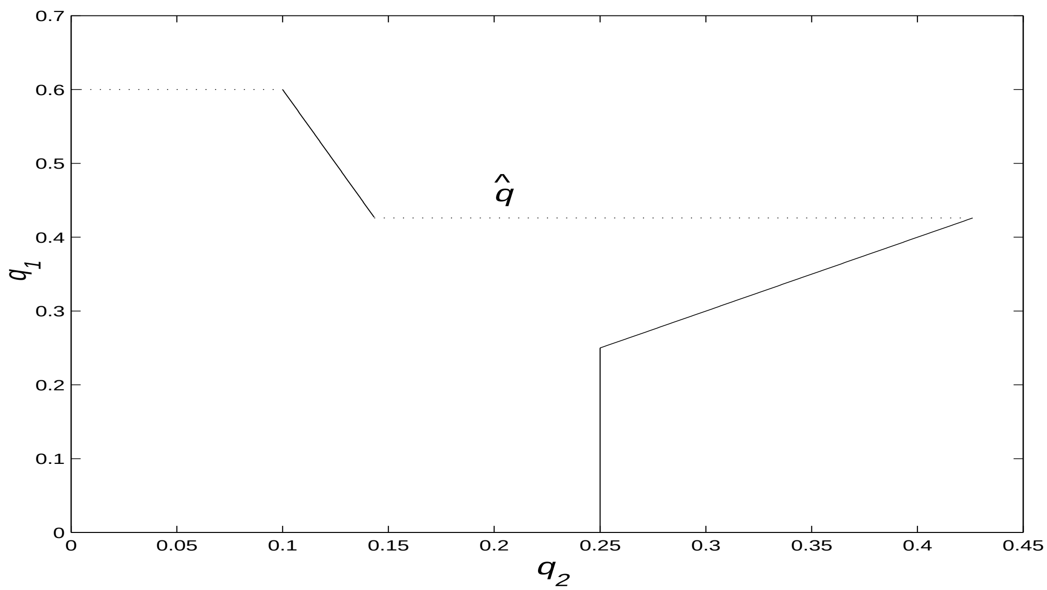

Figure 2: **Reaction function of Firm 2**

Figure 2 demonstrates the reaction function of firm 2, given a quality level q_1 chosen by firm 1. When q_1 is low, close to zero, firm 2 is the leader, and chooses its monopoly level, $q_2^m = 0.25$. As q_1 rises to just above 0.25, firm 2 prefers to compete with firm 1, and choose $q_2 = q_1 + \epsilon$. However, at $q_1 = \hat{q}$ (0.43 in this example), firm 2 is indifferent between being a leader and a follower. For $q_1 > \hat{q}$, firm 2 is the follower, and its quality falls as q_1 increases. Finally, when q_1 rises above 0.6, firm 2 exits the market altogether (i.e., sets $q_2 = 0$).

The reaction function, therefore, has two points of discontinuity. The first is at $\hat{q}$, as mentioned before. The second occurs when firm 1 chooses a quality level against which firm 2 can no longer make a positive profit. Since firm 1 sets $q_0 = q_1$, we can model the best response of firm 1 as choosing an optimal point on firm 2's reaction function. That is, firm 1 computes its profit for each point on the reaction function, and chooses the q_1 at which its profits are maximized.

When firm 1 chooses $q_1 = q_1^m = \frac{b}{c} = \frac{1}{2}$, firm 2 chooses $q_2 = 0.125 < q_1$, which is the equilibrium described in Proposition 2. Notice that when firm 1 chooses $q_1 = 0.43$, firm 2 is exactly indifferent between being a follower ($q_2 = 0.14$) and a leader ($q_2 = 0.43 + \epsilon$ for some small $\epsilon > 0$).

Define $\hat{q}$ to be the level of q_1 at which firm 2 is indifferent between being a leader and a follower. As brand loyalty, β, increases above zero, the profit of firm 2 as a leader falls (since firm 1 captures some loyal users from period 0), whereas its profit as a follower stays the same (all users use firm 1 as the first choice in period 1). Hence, $\hat{q}$ falls, and, in equilibrium, firm 1 continues to offer the monopoly level. Therefore, the equilibrium in Proposition 2 continues to hold for all values of β.

4 Entrant Cost Advantage, Low Brand Loyalty

As shown above in Proposition 2, when there is no cost advantage to the entrant, the incumbent is the leader in equilibrium. However, when brand loyalty is low, its effect can be dominated by a high cost advantage for the entrant. That is, under some conditions, the entrant can also be the leader in equilibrium.

In this section, we analyze the countervailing effects of brand loyalty for the incumbent (β), and cost advantage to the entrant (λ). We first analyze their effects in isolation, and then demonstrate that, depending on the strengths of these two effects, either the entrant or the incumbent could be the period 1 leader in equilibrium.

The incumbent in our model has a first-mover advantage. By choosing $q_1 > \hat{q}$

(where $\hat{q}$ is the quality level at which the entrant is indifferent between being a leader or a follower, see Section 3), it can always force the entrant to be the follower in equilibrium. Hence, the effects of the parameters λ and β can be analyzed through their effect on $\hat{q}$. If $\hat{q}$ increases, this is costly for the incumbent: it has to offer a higher quality than otherwise, to force the entrant to be a follower. If $\hat{q}$ falls, the incumbent is better off.

First, consider the effect of increasing brand loyalty on firm 2's best response, for a fixed value of λ. As β increases above zero, $\hat{q}$ will fall. At some β, which we denote as $\hat{\beta}$, we will have $\hat{q} = q_2^m = \frac{b}{2\lambda c}$ (notice that $\hat{\beta}$ is a function of λ). When $\beta = \hat{\beta}$, if firm 1 offers $q_1 = \hat{q}$, firm 2 will choose $q_2^m < \hat{q}$. That is, firm 1 will be the leader in equilibrium. Now, if β increases beyond this, brand loyalty is so strong that the incumbent will continue to be the leader in period 1. In this section, we analyze the case of $\beta \in [0, \hat{\beta})$, which we term low brand loyalty. In the next section, we consider high brand loyalty.

Define the critical value of brand loyalty, $\hat{\beta}$, to be the level at which $\hat{q} = q_2^m = \frac{b}{2\lambda c}$. As an aside, we note that we can explicitly determine the value of $\hat{\beta}$ in our model.

Lemma 1 $\hat{\beta} = 1 - \frac{b^3}{4\lambda c(b^2 + 2a\lambda c)}$.

We show formally that, when brand loyalty is low, $\hat{q}$ declines in β. Note that since $\hat{\beta}$ as defined is a function of λ, the cost advantage of the entrant, we can interpret the condition $\beta \in [0, \hat{\beta})$ as a joint restriction on β and λ.

Proposition 3 *Fix a value of λ. Suppose $\beta \in [0, \hat{\beta})$. Then, as β increases, $\hat{q}$ falls.*

Since $\hat{q}$ is the quality level of the incumbent at which the entrant is indifferent between being a follower and a leader, this Proposition formalizes the advantage to the incumbent of brand loyalty. As brand loyalty increases, it is less costly for the incumbent to choose a quality level that forces the entrant to be a follower. That is, the quality the incumbent needs to offer is lower.

Next, consider the effect of increasing cost advantage to the entrant, for a fixed level of brand loyalty, $\beta \in [0, \hat{\beta})$. As λ falls below 1, $\hat{q}$ begins to rise.

Proposition 4 *Fix a value of* $\beta \in [0, \hat{\beta})$. *Then, as* λ *falls,* $\hat{q}$ *rises.*

Now, suppose that λ begins to fall below 1. Since $\hat{q} < q_1^m$ at $\lambda = 1$, there will be a value of λ, λ_m, at which $\hat{q} = q_1^m$. Until this value of λ is reached, it is clear that the equilibrium of Proposition 2 will continue to hold. The incumbent is the leader, and the entrant will remain the follower.

The entrant will offer a real competitive threat to the incumbent only for $\lambda < \lambda_m$. When $\hat{q} > q_1^m$, it becomes costly for the incumbent to force the entrant to become a follower. The incumbent now compares its own profit as a leader (as a leader, it must offer $\hat{q}$ now) and as a follower (in which case it plays some best response strategy, with the entrant as the leader). We show first that, if the entrant is a leader, then it offers its own monopoly quality level, $q_2 = \frac{b}{2\lambda c}$.

Proposition 5 *Suppose* $\beta < 1$ *and, in equilibrium, the entrant is the leader in period 1. Then,* $q_2 = \frac{b}{2\lambda c}$, *and* $q_1 = \frac{b}{c} - \frac{b^2(1-\beta)}{4\lambda c^2}$.

Therefore, when the entrant is a leader in equilibrium, the quality of the incumbent is increasing in λ. As λ falls, q_2 rises. The demand for the incumbent is now the residual demand created by dissatisfied users of firm 2. As this demand shrinks, the quality offered by firm 1 falls as well.

If λ is sufficiently low, then $\hat{q}$ can be substantially higher than q_1^m. At this point, exercising the first-mover advantage is costly to the incumbent, and it may prefer to be the follower in equilibrium. This is formalized in the next Proposition, which shows that for high values of λ, the incumbent will remain the leader, but, for low values of λ, the cost advantage of the entrant can dominate the effects of brand loyalty to an extent that the entrant becomes the leader.

Proposition 6 *Fix a value of* $\beta \in [0, \hat{\beta})$. *Then, there exist a* $\lambda_l, \lambda_h \in (0,1)$ *such that, if* $\lambda > \lambda_h$, *the incumbent remains the leader, but for* $\lambda < \lambda_l$, *the entrant is the leader.*

In our base example, with $a = -0.05, b = 1, c = 2$, and $\beta = 0$, $\lambda_l = \lambda_h = 0.57$. Figure 3 below shows firm 1's profit as a leader less its profit as a follower.

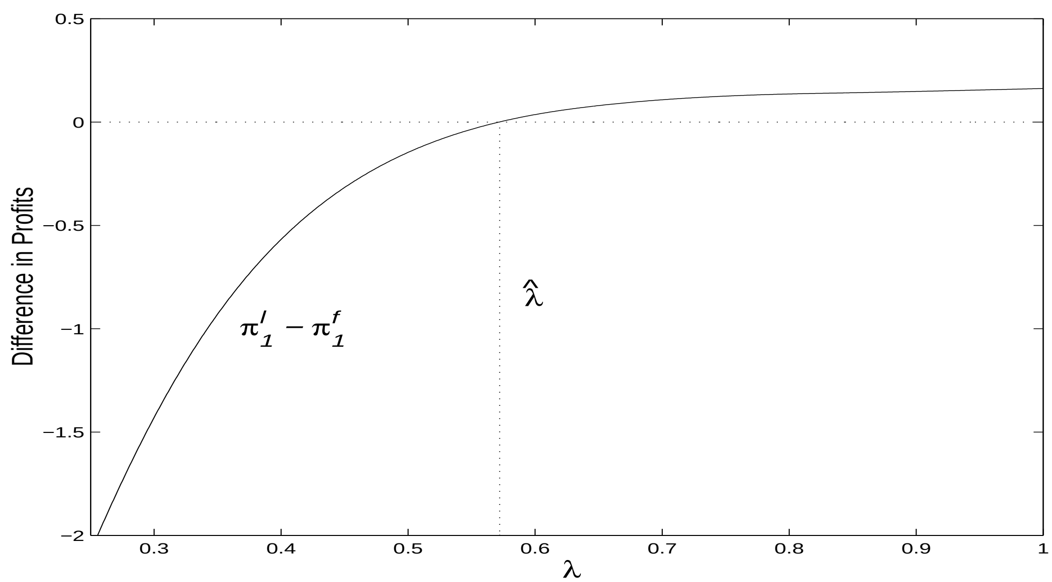

Figure 3: **Difference in Profit of Firm 1, Leader and Follower**

When the difference in profits is positive (for $\lambda > 0.57$), firm 1 chooses to be a leader in equilibrium; that is, it sets $q_1 = \max(\hat{q}, q_1^m)$. When this difference is negative, it is too costly for firm 1 to be a leader (that is, $\hat{q}$ is too high), and it chooses to be a follower. At the critical value of λ, firm 1 is indifferent between being a leader and a follower, and both equilibria exist.

5 Entrant Cost Advantage, High Brand Loyalty

First, we consider the case of complete brand loyalty; that is, $\beta = 1$. When $\beta = 1$, the incumbent can completely ignore the entrant. By Proposition 1, in equilibrium, it must be that $q_0 = q_1$. In this case, the entrant's quality level has absolutely no effect on the incumbent's demand. Therefore, firm 1 chooses its monopoly quality level, $q_1 = \frac{b}{c}$.

Now, firm 2 can choose to be either the leader or the follower, depending on λ. As shown in the next Proposition, $\lambda = \frac{1}{2} - \frac{b}{4c}$ is a critical value. For higher values of λ, the entrant is a follower, and for lower values a leader.

Proposition 7 *Suppose $\beta = 1$. Then, in equilibrium, $q_0 = q_1 = q_1^m = \frac{b}{c}$, regardless of λ. Further, the entrant chooses $q_2 = \frac{b(1-q_1^m)}{2\lambda c} < q_1^m$ if $\lambda \leq \frac{1}{2} - \frac{b}{4c}$, and $q_2 = q_2^m = \frac{b}{2\lambda c} > q_1$ otherwise.*

When $\lambda < \frac{1}{2}$, $q_2^m = \frac{b}{2\lambda c} > q_1^m = \frac{b}{c}$. However, this is not sufficient for the entrant to be the leader in equilibrium. Consider a value $\tilde{\lambda} \in (\frac{1}{2} - \frac{b}{4c}, \frac{1}{2})$. At $\tilde{\lambda}$, the entrant is better off being a follower, and not choosing $q_2 = \frac{b}{2\lambda c}$. Because brand loyalty is complete,[8] the entrant only attracts new demand, $b(q_2 - q_1)$. All users from period 0 return to firm 1 as their first choice.

Finally, we consider intermediate values of high brand loyalty; that is, $\beta \in (\hat{\beta}, 1)$. There are two main points of departure from the case of $\beta < \hat{\beta}$. First, $\hat{q}$ is now less than q_2^m, which affects the reaction function of the entrant. If firm 1 chooses $q < \hat{q}$, the entrant will choose q_2^m as its best response. This further affects the profit of firm 1 as a follower, and hence has a feedback on $\hat{q}$ itself. Consider the best response of firm 2 as a leader, when firm 1 offers $q_1 = \hat{q}$. If $\beta < \hat{\beta}$, then $\hat{q} > q_2^m$. Therefore, as

[8] By continuity, a similar argument holds for high, but not complete, brand loyalty; that is, values of β close to 1.

a leader, firm 2 should offer $\hat{q} + \epsilon$, for some small $\epsilon > 0$. As argued in the proof of Proposition 2 above, the profit of firm 2 is maximized in the limit as ϵ goes to zero. This profit is $\pi_2^l = (1 - \beta\hat{q})(a + b\hat{q}) - \lambda c\hat{q}^2$.

When $\beta > \hat{\beta}$, however, $\hat{q} < q_2^m$. Now, if firm 1 chooses $q_1 = \hat{q}$, the best response of firm 2 as a leader is to offer $q_2 = q_2^m = \frac{b}{2\lambda c}$. Hence, its profit as a leader is $\pi_2^l = (1 - \beta\hat{q})(a + b\hat{q}) + b(\frac{b}{2\lambda c} - \hat{q}) - \frac{b^2}{4\lambda c}$.

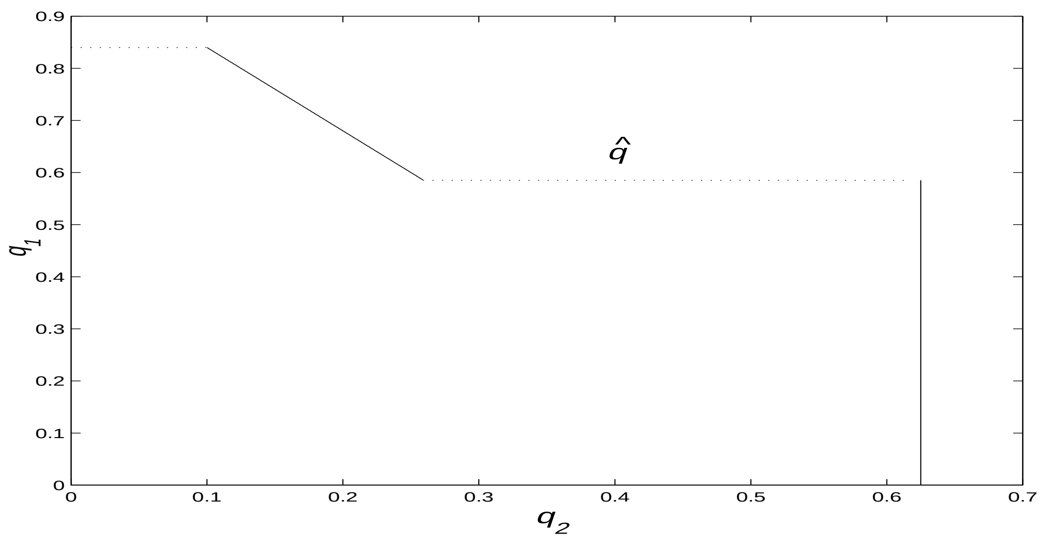

Figure 4: **Reaction function of Firm 2**

Therefore, the equation that determines $\hat{q}$ changes, when $\beta > \hat{\beta}$. As an example, in Figure 4 above, we demonstrate the reaction function for our base case, with $a = -0.05, b = 1, c = 2$, and $\lambda = 0.4, \beta = 0.8$. Since $\hat{q} < q_2^m$, as compared to Figure 2, the upward sloping segment of the reaction function no longer exists.

Second, as compared to the case of $\beta < \hat{\beta}$, the critical value of λ below which the entrant is a leader, $\hat{\lambda}$, is not monotonic in β. While analytical results are difficult to obtain here, we offer a numeric example which demonstrates this point, and offers some insight into the nature of the competition between the two firms in this region.

As before, in this example, we use $a = -0.05, b = 1, c = 2$. In Figure 5 below, the upward-sloping line indicates $\hat{\beta}$. The region to the right of this line is the region where $\beta > \hat{\beta}$. For $\beta < \hat{\beta}$, the dashed line indicates the value of $\hat{\lambda}$. When $\lambda > \hat{\lambda}$, the incumbent is the leader in equilibrium, and when $\lambda < \hat{\lambda}$, the entrant is the leader. Finally, the continuation of this dashed line in the region $\beta > \hat{\beta}$ is indicated by a solid line. As seen from the figure, $\hat{\lambda}$ is not monotonic in β in this region.

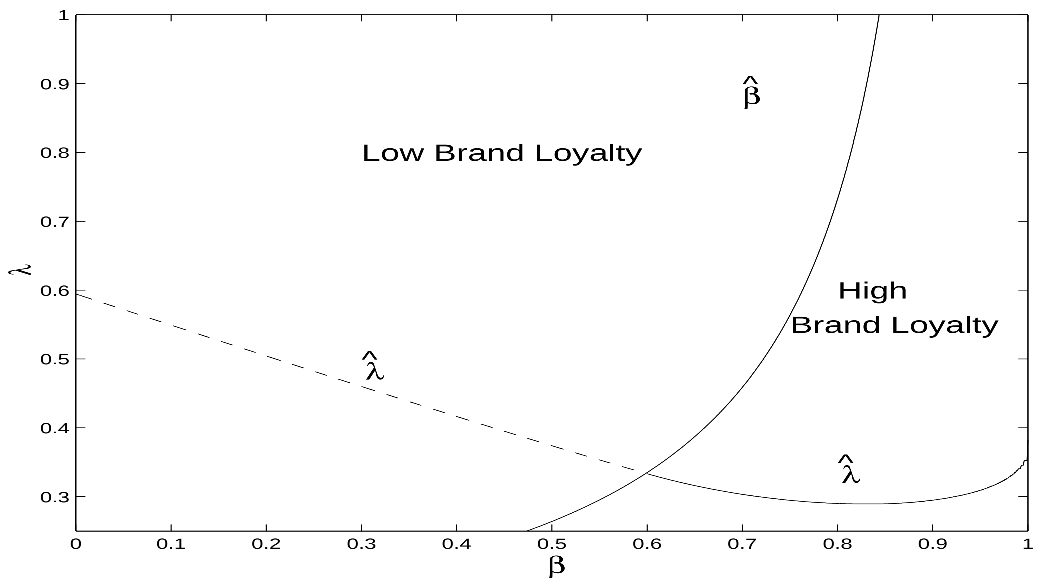

Figure 5: $\hat{\beta}$ **and** $\hat{\lambda}$

As shown in Proposition 7, when $\beta = 1$, $\hat{\lambda} = \frac{1}{2} - \frac{b}{4c}$. Now suppose that β falls by some $\epsilon > 0$. The critical value of λ, $\hat{\lambda}$, is found by equating the profits of firm 1 as a leader to its profit as a follower.

Suppose firm 1 is a leader. Then, it chooses $q_1 = \max(q_1^m, \hat{q})$. It must choose $q_1 \geq \hat{q}$ to ensure that firm 2 is a follower, and, if its monopoly quality $q_1^m > \hat{q}$, then it makes a higher profit at q_1^m. Let $\tilde{q} = \max(q_1^m, \hat{q})$. Then, its profit as a leader is $\pi_1^l = 2(a + b\tilde{q}) - c\tilde{q}^2$.

Now, at $\beta = 1$, $\hat{\lambda} = \frac{1}{2} - \frac{b}{4c}$, from Proposition 7. At these values, $\hat{q} = q_1^m = \frac{b}{c}$. Suppose now that λ remains $\frac{1}{2} - \frac{b}{4c}$, but β falls to a value just less than 1. Then, $\hat{q}$ increases, and hence π_1^l falls. Proposition 5 shows the equilibrium strategy of firm 1 when it is a follower. Substituting this value back into its profit function, we can see that π_1^f also falls. For β close to 1, π_1^l falls less than π_1^f, and hence $\hat{\lambda}$ falls. As β falls even more, π_1^f falls at a slower rate than π_1^l, and hence $\hat{\lambda}$ rises.

Finally, we show that the flavor of the equilibrium in Proposition 6 stays the same for $\beta > \hat{\beta}$. Note, however, that $\hat{q}$ must be appropriately redefined for this case.

Proposition 8 *Fix a value of $\beta \in (\hat{\beta}, 1]$. Then, there exist a $\lambda_l, \lambda_h \in (0, 1)$ such that, if $\lambda > \lambda_h$, the incumbent remains the leader, but for $\lambda < \lambda_l$, the entrant is the leader.*

As shown in Figure 5, in our example $\lambda_l = \lambda_h$, and we denote this value by $\hat{\lambda}$.

6 Managerial Implications

In this section, we outline some of the business implications of our model. First, our model is predicated on the existence of a residual demand for a search engine; that is, no one search engine can satisfy all users with probability 1. Based on actual usage data, Telang, et. al. (2000) report that users switched to a second engine during 22% of search sessions. For example, the rate of switching was 15% for those who first went to Yahoo!, while the rate for Infoseek was 31%. It should be clear to a search engine firm that some visitors to its site will soon be consulting a competitor. Hence, by either placing a direct link to a competitor (perhaps in the form of an advertisement), a search engine can accrue additional revenue. This phenomenon is evidenced, for example, with Yahoo! and Netscape, which offer links to many other search engines. Therefore, despite the competition between engines, a search engine can increase revenue by itself directing unsatisfied users to other search engines.

Second, firms may benefit from either themselves maintaining multiple engines (perhaps of different qualities), or partnering with other engines so that, when a user switches, the switch is to a partner engine or an engine in the same family. Such partnerships (such as Yahoo! and Google or Lycos and Hotbot or Infoseek and Go) are becoming more common.[9] By offering viable alternatives to the user, more than one firm can capture revenues from the same user. This also opens up the possibility that firms can gain by sharing the same database, and running different search algorithms on it to get different results. The optimal portfolio of engines a firm should offer remains an open question: conceivably, a firm can sometime benefit from offering two lower quality engines instead of one higher quality one.

Third, our model demonstrates that an entrant does not have to compete head-on with an incumbent, but can instead be profitable at a lower quality. To the extent that marginal cost is increasing in quality, one option for an entrant is to enter at a low quality, build up its own brand loyalty, and then increase quality later if need be. At the same time, our model clearly points out the first mover advantage to the incumbent. Many successful firms in this market were first movers (Yahoo! being the classic example).

Finally, our model also considers the role of new technology. Many of the challengers today are firms with some technical superiority. For example, Google is now considered a top engine because of the new technology it uses in indexing pages (http://www.google.com/why_use.html). Eventually, market equilibrium depends on a combination of all the factors mentioned above. Hence, today we see a number of firms in the search engine market which capture some traffic without being either a first mover or having a higher quality than competitors. By allowing lower quality engines to survive in the market, our model points to one explanation for the

[9] The page http://www.searchengineworld.com/engine/partners.htm contains a list of current partnerships.

proliferation of search engines.

7 Conclusion

Our model of vertical differentiation in the search engine market shows that new entrants may enter with low quality, and be viable in this market. This helps to explain the proliferation of search engines. In a relatively short period (Yahoo! was introduced in 1994), there are already close to a thousand engines in the market. In the last three months of 1998 alone, nearly forty new engines entered the market,[10] despite the presence of well-established portal sites such as Yahoo! and Lycos.

Since no single engine can cover the web completely, on any given search there is a possibility that a user will be dissatisfied with the result (i.e., she will not find a site she is looking for). This leads to a residual demand that creates room for other engines to enter the market. Vertical differentiation as a result of this residual demand is different from that considered in the previous literature, where lower prices are used to sustain low quality products.

The incumbent in our model has a first-mover advantage, and, unless the entrant has a cost advantage, will emerge as the leader in equilibrium. When the entrant has a cost advantage, the interaction of this advantage with brand loyalty for the incumbent determines which firm will be the leader. Brand loyalty weakens competition between the firms, and, in the extreme case, allows the incumbent to earn a monopoly profit.

The phenomenon of multiple product sampling is not restricted to search engines, but extends to other information goods on the Internet. For example, users may exhibit similar behavior towards newspaper sites. Some users regularly read more than one newspaper (the content of a single newspaper, or whether a particular

[10] See www.dreamscape.com/frankvad/search.html.

story is in the paper, may be modeled as probabilistic), even for the same news item. Our results extend more generally to such goods, where the consumer can engage in product sampling at little cost.

8 Appendix: Proofs

Proposition 1

By assumption, it cannot be that $q_0 > q_1$. Suppose, then, that firm 1 chooses $q_0 = \tilde{q}$ and $q_1 = \bar{q}$, with $\bar{q} > \tilde{q}$. Consider the effect of choosing $q_0 = \bar{q}$ and retaining $q_1 = \bar{q}$. This has no effect on total cost, which remains $c\bar{q}^2$.

We show that equilibrium at time 1 is unaffected; that is, firm 2's optimal choice of q_2 remains the same. Suppose that $q_1 > q_2$. Then, from equation (4), firm 2's profit function is unchanged as q_0 changes. Hence, its optimal response is unchanged.

Next, suppose that $q_2 > q_1$. In this case, from equation (1), we see that a change in q_0 does affect the level of profit earned by firm 2. However, again it does not affect the optimal choice of q_2. This is observed from the first order condition $\frac{d\pi_2}{dq_2} = 0$, which yields the best response to be $q_2 = \frac{b}{2\lambda c}$, independent of q_0. If this $q_2 < q_1$, then, to be a leader, firm 2 will have to set $q_2 = q_1 + \epsilon$, for some $\epsilon > 0$, which is again unaffected by q_0.

Hence, q_2 remains unchanged, and so equilibrium at time 1 is unchanged when q_0 is set to $\bar{q}$ instead of $\tilde{q}$. Now, consider the demand for firm 1 at time 0. This increases by $b(\bar{q} - \tilde{q}) > 0$. Since the cost of firm 1 does not change, this increase in demand represents an unambiguous increase in profit. Hence, it cannot be an equilibrium for firm 1 to set $q_0 < q_1$. ■

Proposition 2

Clearly, firm 1 can do no better than set $q_1 = q_0 = q_1^m$. This represents the highest profit it can make, even if firm 2 were absent. Hence, to show that this is an equilibrium, we only need to show that firm 2 is playing a best response, and cannot do better by choosing some other q_2.

Suppose firm 2 is a follower at time 2. Then, its profit is given by equation (4). Its best response is found from the first order condition, $\frac{d\pi_2}{dq_2} = 0$, which yields $q_2 = \frac{b(1-q_1)}{2\lambda c}$. Substituting for q_2 in equation (4), we have $\pi_2^f = a(1-q_1) + \frac{b^2(1-q_1)^2}{4\lambda c}$.

If firm 2 is a leader, it must set $q_2 \geq q_1$. Since $q_1^m > q_2^m$, its best response is to set $q_2 = q_1 + \epsilon$, for some $\epsilon > 0$. Hence, from equation (1) , (substituting $q_0 = q_1$) $\pi_2^l = a(1-\beta q_1) + b(q_1 + \epsilon - \beta q_1^2) - \lambda c(q_1+\epsilon)^2$. This expression is maximized at $\epsilon = 0$. Consider a decreasing sequence $\epsilon_n \to 0$, where $\epsilon_n > 0$ for each n. Since $q_1^m > q_2^m$, firm 2's profit increases along this sequence. The maximal profit it obtains as a leader, in the limit, is $\pi_2^l = (a + bq_1)(1-\beta q_1)$. Hence,

$$\pi_2^f - \pi_2^l \quad = \quad -aq_1(1-\beta) + \frac{b^2(1-q_1)^2}{4\lambda c} - bq_1 + \beta b q_1^2 + \lambda c q_1^2. \tag{5}$$

At $q_1 = \frac{b}{c}$ and $\lambda = 1$, this reduces to $\pi_2^f - \pi_2^l = -aq_1(1-\beta) + \frac{b^2(1-\frac{b}{c})^2}{4c} + \beta b(\frac{b}{c})^2 > 0$, where the last inequality follows since each of the three terms is strictly positive (recall that $a < 0$). Therefore, firm 2 prefers to be a follower than a leader. Given $q_1 = \frac{b}{c}$, from equation (8), its best response is $q_2 = \frac{b(1-\frac{b}{c})}{2c} = \frac{b}{2c} - \frac{b^2}{2c^2}$, which is less than $\frac{b}{c}$. ■

Lemma 1

Consider equation (5) above, which denotes the difference in profit for firm 2 when it is a leader and a follower. Set $\pi_2^f - \pi_2^\ell$ to zero, and $\hat{q} = \frac{b}{2\lambda c}$, to obtain the desired value of $\hat{\beta}$. ■

Proposition 3

We show that $\frac{\partial \hat{q}}{\partial \beta} < 0$ for $\beta \in [0, \hat{\beta})$. Since $\hat{q} = \frac{b}{2\lambda c}$ at $\beta = \hat{\beta}$, this implies immediately that $\hat{q} > \frac{b}{2\lambda c}$ for all $\beta \in [0, \hat{\beta})$. For convenience, we define $\pi_2^f - \pi_2^l$, from equation (5) above, as $g(q_1, \lambda, \beta)$. That is,

$$g(q_1, \lambda, \beta) \;\; = \;\; -aq_1 + \frac{b^2(1-q_1)^2}{4\lambda c} - bq_1 + a\beta q_1 + b\beta q_1^2 + c\lambda q_1^2.$$

Fix a value of λ. Then, by the implicit function theorem, we have

$$\frac{\partial \hat{q}}{\partial \beta} \;\; = \;\; \frac{2\lambda c\hat{q}(a+b\hat{q})}{2\lambda c(a+b) + b^2(1-\hat{q}) - 2\lambda c\beta(a+2b\hat{q}) - 4\lambda^2 c^2 \hat{q}}$$

The numerator is clearly positive, since $a + b\hat{q} > 0$, otherwise the incumbent is making a loss, and will not offer $\hat{q}$. Consider the denominator. We have $2\lambda c(a+b) - 2\lambda c(a+2b\hat{q}) - 4\lambda^2c^2\hat{q} + (1-\hat{q})b^2 < 0 \iff \hat{q} > \frac{b^2+2\lambda c(a+b)-2a\beta\lambda c}{b^2+4\lambda^2c^2+4b\beta\lambda c}$. In the remainder of the proof, we show that the last inequality is satisfied, and hence the denominator is negative, and $\frac{\partial \hat{q}}{\partial \beta} < 0$.

$\hat{q}$ is the solution to $g(q_1, \lambda, \beta) = 0$, and is computed to be $\hat{q} = \frac{b^2+2\lambda c(a+b)-2a\beta\lambda c}{b^2+4\lambda^2c^2+4b\beta\lambda c} \pm \delta$, where $\delta = \frac{2\sqrt{\lambda c(-b^3-ab(b+2\lambda c)+a^2\lambda c(\beta-1))(\beta-1)}}{b^2+4\lambda^2c^2+4b\beta\lambda c} > 0$.

Next, we show that the negative root can be ignored, and we are only interested in the positive root of the numerator of δ. Consider the equation $g(q_1, \lambda, \beta) = 0$, which has two real roots, as shown above. We have $g(0, \lambda, \beta) = \frac{b^2}{4\lambda c} > 0$ for all values of β.

Consider $g(\frac{b}{2\lambda c}, \lambda, \beta)$, for $\beta \in [0, \hat{\beta})$. By definition of $\hat{\beta}$, $g(\frac{b}{2\lambda c}, \lambda, \hat{\beta}) = 0$. Consider $g(\frac{b}{2\lambda c}, \lambda, 0)$. If $q_1 = q_2^m = \frac{b}{2\lambda c}$, the entrant's best response as a follower is to set $q_2 = \frac{b(1-q_1)}{2\lambda c} < \frac{b}{2\lambda c}$. That is, the entrant as a follower sets a quality less than its monopoly quality. As a leader, it will set a quality just above $q_1 = q_2^m$, and earn a profit very close to its monopoly profit (as we argued, its maximal profit will be in the limit, when it earns exactly the monopoly profit). Clearly, its profit as a follower must be less than that as a leader; that is, $g(q_2^m) < 0$.

Therefore, $g(\frac{b}{2\lambda c}, \lambda, 0) < 0$, and $g(\frac{b}{2\lambda c}, \lambda, \hat{\beta}) = 0$. Further, by inspection, $g(\frac{b}{2\lambda c}, \lambda, \beta)$ is increasing in β.[11] Hence, $g(\frac{b}{2\lambda c}, \lambda, \beta) < 0$ for all $\beta \in [0, \hat{\beta})$. Finally, consider $g(1, \lambda, \beta)$. This is evaluated as $g(1) = -a - b + a\beta + b\beta + \lambda c > 0$, by assumption 1.

Therefore, we have $g(0, \lambda, \beta) > 0$, $g(\frac{b}{2\lambda c}, \lambda, \beta) < 0$, and $g(1, \lambda, \beta) > 0$. Since $g(\cdot)$ is a continuous function, by the Mean Value Theorem, the equation $g(q_1, \lambda, \beta) = 0$ must have a root in the interval $(0, \frac{b}{2\lambda c})$ and the interval $(\frac{b}{2\lambda c}, 1)$.

We ignore the first root, since if $q_1 < \frac{b}{2\lambda c} = q_2^m$, the entrant's best response is clearly q_2^m (that is, it prefers to be a leader than a follower). Define the second root to be $\hat{q}$. Then, it follows that $g(q, \lambda) > 0$ for $q > \hat{q}$.

Hence, we only consider the positive value of δ, and so the denominator of $\frac{\partial \hat{q}}{\partial \beta}$ is negative. Hence, $\hat{q} > \frac{b}{2\lambda c}$ for all $\beta > \hat{\beta}$. ∎

Proposition 4

Consider the function $g(\cdot)$ defined in the proof of Proposition 3 above. $\hat{q}$ is the solution to the equation $g(q, \lambda, \beta) = 0$. From the implicit function theorem, we have

$$\frac{\partial \hat{q}}{\partial \lambda} = \frac{2c\hat{q}\left(2\lambda c\hat{q} - (a+b) + \beta(a + b\hat{q})\right)}{2\lambda c(a+b) + b^2(1-\hat{q}) - 2\lambda c\beta(a + 2b\hat{q}) - 4\lambda^2 c^2 \hat{q}}$$

Consider the numerator. If $\hat{q} = \frac{b}{2\lambda c}$, then the numerator reduces to $4c\hat{q}\left(a(\beta - 1) + \frac{\beta b^2}{2\lambda c}\right) > 0$, where the inequality follows since each term in the parentheses is positive (recall that $a < 0$). Now, from Proposition 3, for all $\beta < \hat{\beta}$, $\hat{q} > \frac{b}{2\lambda c}$. Since the value of the numerator unambiguously increases with $\hat{q}$, the numerator is strictly positive.

The denominator is identical to the denominator of $\frac{\partial \hat{q}}{\partial \beta}$, which was shown to be

[11] As the incumbent's brand loyalty increases, the entrant's demand as a leader, and hence its profit, falls.

negative in the proof of Proposition 3. Hence, $\frac{\partial \hat{q}}{\partial \lambda} < 0$. ∎

Proposition 5

Suppose the entrant is a leader, and sets $q_2 > \frac{b}{2\lambda c}$, that is, higher than its monopoly level. Since the best response of the entrant is above its monopoly level, it must set $q_2 = q_1 + \epsilon$. Above q_2^m, π_2 is declining in q_2, so any higher quality will lead to lower profit. However, for any q_2, the incumbent makes a higher profit by setting $q_1 = q_2 + \epsilon$ than $q_1 = q_2 - \epsilon$. The incumbent's profit at $q_1 = q_2 + \epsilon$ is $\pi_1(q_2 + \epsilon) = 2(a + b(q_2 + \epsilon)) - c(q_2 + \epsilon)^2$, and its profit at $q_1 = q_2 - \epsilon$ is $\pi_1(q_2 - \epsilon) = (2 - q_2(1 - \beta))(a + b(q_2 - \epsilon)) - c(q_2 - \epsilon)^2$.

Since $q_2 > 0$ and $\beta < 1$, in any equilibrium in which the entrant is a leader, for ϵ close enough to zero, $\pi(q_2 + \epsilon) > \pi(q_2 - \epsilon)$. Since there is no equilibrium in which $q_2 = q_1 + \epsilon$, the only other possibility is that the entrant is at its monopoly level,$q_2 = \frac{b}{2\lambda c}$.

Next, consider the best response of the incumbent, given that the entrant is a leader and is choosing $q_2 = \frac{b}{2\lambda c}$. Table 1 shows the demand for firm 1 in period 1 when it is a follower. Substituting $q_0 = q_1$ (using Proposition 1), adding the first period demand of $a + bq_1$, and noting that $q_2 = \frac{b}{2\lambda c}$, the profit of firm 1 as a follower can be represented as $\pi_1^f = (a + bq_1)(2 - \frac{b(1-\beta)}{2\lambda c}) - cq_1^2$. The first order condition, $\frac{\partial \pi_1}{\partial q_1} = 0$, directly yields the optimal value of q_1 to be $q_1 = \frac{b}{c} - \frac{b^2(1-\beta)}{4\lambda c^2}$. ∎

Proposition 6

Suppose the incumbent is a leader. Then, its optimal quality is $\tilde{q} = \max\{q_1^m, \hat{q}\}$. Note that, since $\hat{q}$ is a function of λ, so is $\tilde{q}$. The profit of the incumbent when it is a leader, therefore, is $\pi_1^l(\lambda) = 2(a + b\tilde{q}) - c\tilde{q}^2$.

Suppose the incumbent is a follower. Then, from Proposition 5, the optimal quality of the entrant is $q_2 = \frac{b}{2\lambda c}$. From this, we can calculate the incumbent's best response as $q_1 = \frac{b(2-\frac{b}{2\lambda c}(1-\beta))}{2c}$. Note that $q_1 < q_2$ if and only if $\lambda < \frac{1}{2} + \frac{b}{4c}(1-\beta)$. For such λ, the profit of the incumbent as a follower is computed to be $\pi_1^f(\lambda) = 2a - \frac{ab}{2\lambda c}(1-\beta) + \frac{b^2}{4c}(2 - \frac{b}{2\lambda c}(1-\beta))^2$.

Define the difference function, $h(\lambda) = \pi_1^l(\lambda) - \pi_1^f(\lambda)$ to be

$$h(\lambda) \;\;=\;\; 2b\tilde{q} - c\tilde{q}^2 + \frac{ab}{2\lambda c}(1-\beta) - \frac{b^2}{4c}(2 - \frac{b}{2\lambda c}(1-\beta))^2.$$

Now, we need $\lambda > \frac{b}{2c}$, to ensure that $q_2^m < 1$. We have $\pi_1^f(\frac{b}{2c}) = a + \frac{b^2}{4c}$, which is positive by Assumption 1, part (ii).

Now, consider $\pi_1^l(\frac{b}{2c})$. At $\lambda = \frac{b}{2c}$, $q_2^m = 1$. Further, since $\hat{q} > q_2^m$ for $\beta \in [0, \hat{\beta})$, $\hat{q} > 1$. However, when $q = 1$, the incumbent makes a loss, and the loss is even greater for $q > 1$. Hence, $h(\frac{b}{2c}) < 0$.

Further, it is clear that $h(\lambda_m) > 0$, since at $\lambda = \lambda_m$, the incumbent remains at its monopoly level when it is the leader.

Since $h(\cdot)$ is continuous, there exists at least one value of $\lambda \in (\frac{b}{2c}, \lambda_m)$ such that $h(\lambda) = 0$. If there is only one such value, define this value to be both λ_l and λ_h. If there are multiple values, define the minimal value to be λ_l and the maximal λ_h, and the statement of the proposition follows. ■

Proposition 7

When $q_0 = q_1$, the incumbent's demand in the second period, regardless of q_2, is just $a + bq_1$. Hence, its profit function is $\pi_1(q_1) = 2(a + bq_1) - cq_1^2$, and its optimal strategy is to set $q_0 = q_1 = \frac{b}{c} = q_1^m$.

Next, consider the entrant, given that the incumbent offers $q_0 = q_1 = \frac{b}{c}$. The entrant's profit as a follower is $\pi_2^f(q_2) = (1 - q_1)(a + bq_2) - \lambda c q_2^2$. As discussed, its best response is to set $q_2 = \frac{b(1-q_1)}{2\lambda c}$. Setting $q_1 = \frac{b}{c}$, therefore, its profit as a follower reduces to $\pi_2^f = a(1-\frac{b}{c}) + \frac{b^2(1-\frac{b}{c})^2}{4\lambda c}$.

Now, suppose the entrant is a leader. Its profit as a leader is $\pi_2^l(q_2) = (1-\beta)(a+bq_1) + \beta(1-q_1)(a+bq_1) + b(q_2-q_1) - \lambda c q_2^2$. From the first order condition, it is immediate that the best response is to set $q_2 = \frac{b}{2\lambda c} = q_2^m$. Then, the entrant's profit reduces to $\pi_2^l = a(1-\frac{b}{c}) - \frac{b^3}{c^2} + \frac{b^2}{4\lambda c}$.

Hence, the difference in profits, $\pi_2^l - \pi_2^f$, is greater than zero if $\frac{4\lambda b^2}{c^2}\left(\frac{b}{c} - 2 + 4\lambda\right) < 0$, that is, if $\lambda < \frac{1}{2} - \frac{b}{4c}$. Finally, to complete the proof, note that $\frac{b}{2\lambda c} > \frac{b}{c}$ for all $\lambda < \frac{1}{2}$, and hence including all $\lambda < \frac{1}{2} - \frac{b}{4c}$. ■

Proposition 8

Suppose firm 1 offers $q_1 = \hat{q}$. As noted before, the profit of firm 2 as a leader is $\pi_2^l = (1-\beta\hat{q})(a+b\hat{q}) + b(\frac{b}{2\lambda c} - \hat{q}) - \frac{b^2}{4\lambda c}$, and its profit as a follower remains $\pi_2^f = a(1-\hat{q}) + \frac{b^2(1-\hat{q})^2}{4\lambda c}$.

As before, $\hat{q}$ solves the equation $\pi_2^f - \pi_2^l = 0$, with π_2^l as above. With this redefinition of π_2^l, the proof of Proposition 6 goes through completely. That is, we can now redefine $\tilde{q} = \max\{q_1^m, \hat{q}\}$, as at the start of Proposition 6, and then follow exactly the same steps as in that proof. ■

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